Adaptive Robust Control for Micropositioning of Piezoelectric Actuators with Environment Force Estimation

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Abstract: Piezoelectric actuators are widely used in applications of micropositioning. In the most situations, there is a contact with an environment. Existence of external forces is a challenging problem in controlling mechanical system, especially in micro scales such as piezoelectric actuators. In such a situation, dual hysteresis nonlinearity effects could disrupt the actuator micropositioning accuracy. In this paper a modified Prandtl-Ishlinskii (PI) operator and its inverse is utilized for both identification and real time compensation of the hysteresis effect. A variable structure controller is proposed for trajectory tracking of the actuator position. A sliding-based observer would estimate full states from the only measurable position trajectory. Proposing an adaptive perturbation estimation approach, the pure external forces would be observed. The stability of the controller in the presence of the estimated states and external forces is demonstrated analytically. The experimental results depict that the proposed approach is achieved in precise trajectory tracking and external force estimation.

Keywords: Piezoelectric Actuator, Adaptive Robust Control, Force estimation, Hysteresis Identification

1. INTRODUCTION

Smart actuators, such as piezoceramic actuators, magnetostrictive actuators, and shape memory alloy (SMA) actuators are widely used in applications of micropositioning and vibration control. Due to hysteresis nonlinearity effects, micropositioning of these actuators has been a challenging topic in the recent decades.

Hysteresis is a nonlinear relation between input voltage and position of the actuators. It expresses that the current output of the actuators depends on the current input and background of input and output additionally (Al Janaideh et al. (2008)). This effect not only cause inaccuracy in the output response, but could eventually lead to the instability of the closed loop system.

Several models such as Preisach, Krasnosel’skii-Pokrovskii, Prandtl-Ishlinskii, Duhem and Bouc-Wen models have been proposed to identify the hysteresis effect. By construction of the inverse hysteresis models and applying it as a feedforward compensator, position control of actuators would be facilitated (Ang et al. (2007)).

Several methods have been proposed for free micropositioning and control of piezoelectric actuators such as impedance control (Garcia-Valdovinos et al. (2007)), sliding mode control (Bashash et al. (2007)) and robust control coupled with adaptive approaches (Bashash et al. (2009) and Li et al. (2009)). All of them assume that full states of the actuator are existed though the only measurable state is position. But, the challenging problem is contacting with an environment and existence of external forces. This induced environment forces could disarrange the positioning and therefore the control structure. But in piezoelectric actuators, these forces have an extra undesired effect. The external forces could generate a reverse induced voltage, too. This voltage could contact with the actuator activation voltage. Therefore it would be such an extra perturbation in the system.

Instead of expensive force sensors, estimation algorithms for contact forces have been widely developed. Also, different approaches have been proposed for all disturbances estimation. Considering the error in the actuator position estimation with a linear observer, Alcocera et al. (2003) presented an external force estimation approach based on the estimation error. Also, Abidi et al. (2004) proposed an observer with the similarity in the dynamics of the system and observer. A new force estimation method by eliminating the system uncertainties effect is introduced by Bhattacharjee et al. (2008).

None of the mentioned approaches represented a dynamic error for proposed observer. It means that in these methods, the stability of the observer and controller should be proved separately. Therefore the stability of the controller in the presence of the estimated forces could not be analytically proved. Daly et al. (2009) proposed a sliding-based disturbance observer with an analytical error dynamic. The most problem is that all external forces and perturbations are observed together.

In this paper a generalized Prandtl-Ishlinskii model and its inverse is utilized for identification and online feedforward compensation of hysteresis. As a result, the actuator dynamic
model would be transformed to the second order linear dynamic model. Considering the parametric uncertainties, PI estimation error, probably unmodeled dynamics and dual hysteresis effect, a variable structure controller is proposed for trajectory tracking of the piezoelectric. A sliding-based observer would estimate full states from the only measurable position trajectory. Proposing an adaptive perturbation estimation approach, the pure external forces could be estimated separately. The stability of the controller in the presence of the estimated state and environment force is demonstrated with the Lyapunov criterion. Finally, the experimental results demonstrate that the proposed controller achieves precise trajectory tracking and external force estimation.

2. DYNAMIC MODEL OF PIEZOELECTRIC ACTUATORS

A second-order dynamic has been utilized for piezoelectric actuators. The model was divided to two parts. The first part, proposes a second-order linear dynamic that refers to the mass-spring-damper system. The second part, describes the nonlinear portion of the actuator, i.e. hysteresis nonlinearity effect.

Fig. 1 shows the linear second-order dynamics of the piezoelectric actuator that would be added by nonlinearity effect of hysteresis in the input.

Governing equation is represented as (1)

\[
m\ddot{x}(t) + c\dot{x}(t) + kx(t) = H_F(v(t)) - F_e
\]

where \(x(t)\) and \(v(t)\) represent the actuator displacement and input voltage, respectively. \(m\) and \(c\) and \(k\) denote mass, damping and stiffness gains, respectively. \(F_e\) is the external forces is imported by environment. \(H_F(v(t))\) expresses the hysteretic relation between the input voltage and the excitation force (Bashash et al. (2007)).

Having high stiffness, the piezoelectric actuators possess very high natural frequency. So this situation causes that, in the low frequency operation the inertia and damping effects could be neglected. As a result, (1) would be transformed to (2) in a free motion.

\[
x(t) = \frac{1}{k}H_F(v(t)) = H_x(v(t))
\]

This equation relates the input voltage and actuator displacement together. The advantage of this facilitation is that instead of identification of nonlinear relation of input voltage and exciting force, the nonlinear relation of input voltage and displacement would be identified. Hence the necessity of an accurate force sensor would be eliminated and the position sensor of the actuator would prepare the required position signals.

By identification of the hysteresis map between the input voltage and the actuator displacement \((H_x(v(t)))\), it scales up with a factor of \(k\) to obtain \(H_F(v(t))\).

Considering the parameter uncertainties, hysteresis estimation error and probably unmodeled dynamics, a perturbation term \(P(t)\) would be added to the dynamic model. Finally, the dynamic model of the system would be such as (3).

\[
m\ddot{x}(t) + c\dot{x}(t) + kx(t) = kH_x(v(t)) + P(t) - F_e
\]

3. GENERALIZED PRANDTL-ISLINSKII (PI) HYSTERESIS MODEL

A generalized Prandtl-Ishlinskii model is utilized for both hysteresis identification and compensation (Zareinejad et al. (2009)). The most important advantage of this model is its simplicity and that its inverse could be calculated analytically. In addition, this approach could be utilized in open-loop systems where the feedback of signals is not accessible.

The key idea of an inverse feedforward compensation of hysteresis is to cascade the inverse hysteresis operator \(H_x^{-1}\) with the actual hysteresis, which is represented by the hysteresis operator \(H_x\), to obtain an identity mapping between the desired actuator output \(x_d(t)\) and the actuator response \(x(t)\). The structure of inverse feedforward hysteresis compensation is shown in Fig. 2.

The inverse PI operator \(H_x^{-1}\), uses \(x_d(t)\) as input and transforms it into a control input \(v = H^{-1}(x_d)\) which produces \(x(t)\) in the hysteretic system that closely tracks \(x_d(t)\).

4. ROBUST ADAPTIVE CONTROL FOR A GENERAL NONLINEAR DYNAMIC SYSTEM

A general class of nonlinear systems with \(m\) generalized coordinates and existence of external forces could be represented as

\[
X^{(n)} = f(X, \dot{X}, ..., X^{(n-1)}) + B(X, \dot{X}, ..., X^{(n-1)})u(t) + F
\]
where \( X = [x_1, x_2, ..., x_m] \) is the vector of generalized coordinates. \( X^{(i)} = [x_1^{(i)}, x_2^{(i)}, ..., x_m^{(i)}] \) is the \( i \)th time derivative of the generalized coordinates and \( F \) is the external forces Vector.

Due to the parametric uncertainty, unmodeled dynamic and identification error, the dynamic error could be described as

\[
X^{(n)} = \left( \hat{f} + f + (\hat{B} + B)u(t) + d(t) + F \right) \quad (5)
\]

where \( \hat{f} \) and \( f \) are the known part and \( \hat{B} \) and \( B \) are the unknown part of \( f \) and \( B \), respectively. \( d(t) \) would be the vector of bounded external disturbance (Zeinali et al.(2010)). The total uncertainties could be represented as \( P(t) \). Finally, the dynamic model of the system would be such as (6).

\[
X^{(n)} = \left( \hat{f} + Bu(t) + F + P(t) \right) \quad (6)
\]

Sliding mode control designs asymptotically stable hyperplanes such that all system trajectories converge to these hyperplanes and slide along their path until approach the desired destination. The objective is accurate trajectory tracking of generalized coordinates.

For simultaneous trajectory tracking and robustness, the sliding hyperplane, tracking error and its time derivative could be selected as

\[
S = e^{(n-1)} + K_{(n-2)}e^{(n-2)} + ... + K_1e + e \quad (7)
\]

\[
e = x_d - X
\]

\[
e = \hat{x}_d - \hat{X}
\]

\( S \) is an \( n \times 1 \) vector and \( K_{(i)} \) is an \( n \times n \) diagonal positive definite gain matrix.

By substitution of (6) in the time derivative of \( S \), it could be represented as

\[
\dot{S} = X^{(n)} - x_d^{(n)} + K_{(n-2)}e^{(n-1)} + ... + K_1\hat{e} + \hat{e} \quad (8)
\]

\[
= \hat{f} + Bu(t) + F + P(t) - x_d^{(n)} + K_{(n-2)}e^{(n-1)} + ... + K_1\hat{e} + \hat{e}
\]

Therefore, the control input (9) would be proposed as

\[
u(t) = B^{-1}(\hat{f} - F - \hat{P}(t) + x_d^{(n)} - (K_{(n-2)}e^{(n-1)} + ... + K_1\hat{e} + \hat{e}) - \eta_1S - \eta_2sgn(S)) \quad (9)
\]

Where \( sgn(.) \) represents the signum function, \( \eta_1 \) and \( \eta_2 \) are \( n \times n \) diagonal positive definite gain matrices and \( \hat{P}(t) \) is the estimated perturbation that would be obtained by an adaptive approach.

By substitution of the control input (9) in the dynamic model (6), the error dynamic equation (10) would be derived.

\[
\dot{S} + \eta_1S + \eta_2sgn(S) = \hat{P}(t) \quad (10)
\]

By proper opting of the gain matrices \( \eta_1 \) and \( K_{(i)} \), the sliding hyperplane \( S \) and therefore tracking error vector \( e \) would tend to zero (Slotine (1991)).

**Theorem 1:** Utilizing the control input (9), the proposed adaptive perturbation estimation law \( \hat{P} = -\Gamma S \) could guarantee the asymptotically stability of dynamic system (6) and convergence of the estimated perturbation.

**Proof:** The positive definite Lyapunov function candidate

\[
V = \frac{1}{2} S^T S + \frac{1}{2} \hat{P}^T \Gamma^{-1} \hat{P} \quad (11)
\]

where \( \Gamma \) is an \( H \times H \) matrix.

Utilizing the control input (9) and proposed adaptive law, the time derivative of the function would be obtained as

\[
\dot{V} = S^T \hat{S} + \hat{P}^T \Gamma^{-1} \dot{\hat{P}}
\]

\[
= S^T (\hat{P} - \eta_1 S - \eta_2 sgn(S)) + \hat{P}^T \Gamma^{-1}(-\Gamma S)
\]

\[
= -S^T \eta_1 S - S^T \eta_2 sgn(S)
\]

Lyapunov function time derivative is negative semidefinite obviously.

Based on the Lyapunov theorem, negative definiteness of \( \dot{V} \), could guarantee the asymptotically stability of the system. It could be deduced that the proposed Lyapunov candidate function would be bounded, additionally. Obviously, \( V \) would be uniformly continuous. Based on the Barbalat lemma, \( \dot{V} \) would tend to zero as \( t \to \infty \) (Slotine (1991)). As a result it could be seen that

\[
t \to \infty \quad \dot{V} \to 0, \quad S \to 0, \quad \hat{P}(t) \to 0, \quad \hat{P}(t) \to 0, \quad \hat{P}(t) \to \text{constant}
\]

The convergence of the estimated perturbation could be guaranteed, too.

5. OBSERVER-BASED ADAPTIVE ROBUST CONTROL WITH ENVIRONMENT FORCE ESTIMATION

5.1 Sliding Mode Control for piezoelectric actuators

The control objective is accurate trajectory tracking of piezoelectric actuator and acceptable control input. Considering uncertainties in parameter identification, hysteresis model and probably unmodeled dynamics, a sliding mode controller would be designed. Therefore, the sliding hyperplane, tracking error and its time derivative could be selected as

\[
s = \hat{e} + \lambda e; \quad e = x_d - x, \quad \hat{e} = \hat{x}_d - \hat{x}
\]

where \( \lambda \) is a positive gain.

The control input would be proposed as

\[
u(t) = H_s^{-1} \left( \frac{1}{k} \left[ \left( m\ddot{x}_d + m\lambda\dot{x} + F_c - \hat{P} + cx(t) + kx(t) + \eta_1S + \eta_2 sgn(s) \right) \right] \right) \quad (12)
\]

where \( sgn(s) \) represents the signum function, \( \eta_1 \) \( \eta_2 \) are positive gains and \( \hat{P}(t) \) is the estimated perturbation represented as \( \hat{P} = -ps \). \( p \) is the adaptation gain. By substitution of the control input (12) in the dynamic model (3), the error dynamic equation (13) would be derived.

\[
\dot{s} + \eta_1s + \eta_2 sgn(s) = \hat{P}(t) \quad (13)
\]
By the condition of $|\dot{P}(t)| < \eta_2$ and proper opting of the gains $\eta_1$, $\eta_2$ and $\lambda$, the sliding hyperplane $s$ and therefore tracking error $e$ would tend to zero.

Considering the control input (12), the variable structure controller would contain both position and velocity states of the actuator. The only measurable state of the actuator is position. Although the first time derivative of the position could be obtained by the numerical differentiation, but the experimental results show that it would be too noisy. Considering high cost force sensors, a force observer would be required to estimate the external.

5.2 Full state observer with external force estimation

In this part the disturbance observer introduced in (Daly et al. (2009)) would be utilized with modification. The most problem is that the proposed observer would estimate all external disturbances contained both external forces and perturbations together.

The basic innovation is concentrated in proposing an approach estimating the perturbation separately and import it to the force estimation algorithm. As a result, the pure external forces could be estimated.

Utilizing the hysteresis inverse analytical model, the nonlinear dynamic of the actuator would be transformed to the linear model as (14).

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = u(t) - F_e + P(t)$$  \hspace{1cm} (14)

$P(t)$ represents all perturbations and disturbances of the system except external forces. State estimation would be revised as

$$\dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 \text{sgn}(y - \hat{x}_1)$$  \hspace{1cm} (15)

$$\dot{\hat{x}}_2 = -\frac{k}{m}\hat{x}_1 - \frac{c}{m}\hat{x}_2 + \frac{1}{m}(u + \hat{P}) + \lambda_2 \text{sgn}(\hat{x}_2 - \hat{x}_2)$$

where $\lambda_1$, $\lambda_2$ are positive constant gains and $\hat{x}_2$ would be as

$$\hat{x}_2 = \hat{x}_2 + \left(\lambda_2 \text{sgn}(y - \hat{x}_1)\right)_{eq}$$  \hspace{1cm} (16)

($\cdot_{eq}$) is a lowpass filter. Defining the states estimation errors such as $e_{o1} = \hat{x} - x$ and $e_{o2} = \hat{x} - \hat{x}$, the error dynamic of the observer would be represented as (17).

$$\dot{e}_{o1} = e_{o2} + \lambda_1 \text{sgn}(y - \hat{x}_1)$$  \hspace{1cm} (17)

$$\dot{e}_{o2} = -\frac{k}{m}e_{o1} - \frac{c}{m}e_{o2} + \frac{1}{m}\hat{P} + \frac{1}{m}F_e + \lambda_2 \text{sgn}(\hat{x}_2 - \hat{x}_2)$$

Environment force could be estimated as (18).

$$\hat{P}_e = -m(\lambda_2 \text{sgn}(\hat{x}_2 - \hat{x}_2))_{eq}$$  \hspace{1cm} (18)

The stability of the observer could be proved in two parts as mentioned in (Daly et al. (2009)). But in the second part the condition $\lambda_2 > \left|\frac{k}{m}e_{o1} - \frac{c}{m}e_{o2} + \frac{1}{m}\hat{P} + \frac{1}{m}F_e\right| + \varepsilon_2$ would be revised.

5.3 Observer-Based Sliding Mode Control with adaptive perturbation estimation

The proposed sliding mode control should be reformed based on the estimated velocity state. Therefore, the sliding hyperplane and the time derivative of the tracking error would be as

$$\dot{s} = \dot{\hat{e}} + \lambda e, \quad e = x_d - \hat{x}, \quad \dot{\hat{e}} = \hat{x}_d - \hat{x}$$

Where $\hat{x}$ and $\hat{x}_d$ denote the estimated position and velocity state respectively and $\lambda$ is a positive gain.

Considering the new definition, the control input (19) would be transformed to

$$\nu(t) = H_s^{-1}\left(\frac{1}{k}(m\ddot{x}_d + m\lambda\dot{\hat{e}} + \hat{P}_e - \hat{P} + c\ddot{x}(t)) + k\ddot{x}(t) + \eta_1\ddot{s} + \eta_2\text{sgn}(\ddot{s})\right)$$

Substitution of the control input (19) in the dynamic model (3), the equation could be formed as

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = m\ddot{x}_d + m\lambda\ddot{\hat{e}} + \hat{P}_e - \hat{P} + c\ddot{x}(t) + k\ddot{x}(t) + \eta_1\ddot{s} + \eta_2\text{sgn}(\ddot{s}) - F_e$$

Adding the term $\pm m\ddot{x}$ to the first side, the error dynamic equation (21) would be derived.

$$m\dddot{s} + \eta_1\ddot{s} + \eta_2\text{sgn}(\ddot{s}) + m\dot{e}_{o2} + c(e_{o2} + k\varepsilon_{o1})$$  \hspace{1cm} (21)

Finally, utilizing the estimation error dynamic of the observer (17) and the external force estimation (18), the error dynamic equation would be revised as

$$m\dddot{s} + \eta_1\ddot{s} + \eta_2\text{sgn}(\ddot{s}) = \hat{P}$$  \hspace{1cm} (22)

By the condition of $|\hat{P}(t)| < \eta_2$ and correct selection of the gains $\eta_1$, $\eta_2$ and $\lambda$, the sliding hyperplane $\ddot{s}$ and therefore tracking error $e$ would tend to zero.

In the presence of estimated velocity state and external disturbances, an adaptive approach would be proposed for perturbation estimation.

Theorem 2: Utilizing the control input (19) and proposed adaptive perturbation estimation as $\hat{P}(t) = -k\ddot{s}$, the asymptotically stability of dynamic system in the presence of the estimated state and external forces would be guaranteed.

Proof: Choosing the positive definite Lyapunov function candidate $V = \frac{1}{2}(\dddot{s}^2 + \frac{1}{2k}\dddot{\hat{P}}^2)$, by utilizing control input (19) and proposed perturbation estimation, its time derivative would be obtained as

$$\dot{V} = \dddot{s}\dddot{s} + \frac{1}{p}\dddot{\hat{P}}$$  \hspace{1cm} (23)

$$= \dddot{s}\left(\dddot{\hat{P}}(t) - \eta_1\dddot{s} - \eta_2\text{sgn}(|s|)\right) + \frac{1}{p}\dddot{\hat{P}}(-k\ddot{s})$$

$$= -\eta_1\dddot{s}^2 - \eta_2|s|$$

The time derivative of the proposed Lyapunov function candidate is negative definite. Therefore, asymptotic stability of the system would be guaranteed.
A \text{sat}(\cdot) \text{ function} \text{ is} \text{ used} \text{ instead} \text{ of} \text{ a} \text{ \text{sgn}(\cdot) \text{ function} \text{ in} \text{ the} \ \text{control} \text{ law}. \text{ Therefore,} \text{ the} \text{ problem} \text{ of} \text{ chattering} \text{ would} \text{ be} \text{ eliminated.} \text{ Fig. 3} \text{ depicts} \text{ the} \text{ overall} \text{ block} \text{ diagram} \text{ of} \text{ the} \text{ proposed} \text{ observer-based} \text{ sliding} \text{ mode} \text{ control} \text{ with} \text{ adaptive} \text{ perturbation} \text{ estimation} \text{ and} \text{ external} \text{ force} \text{ estimation.}

Fig. 3. The block diagram of closed loop system

6. EXPERIMENTAL RESULTS

The proposed controller was verified experimentally. The experimental setup contains a PI 611.1S piezoelectric actuator contact with a loadcell as an environment. There is an Amplifier/Driver for deriving the actuator. The piezoelectric actuator is shown in Fig. 4.

Fig. 4. The Piezoelectric actuator contacting with an environment

Matlab/Simulink software is utilized for implementation of the control approach. A dSPACE 1104 data acquisition board is used for data capturing.

Designing the control scheme in Simulink, the control input would be sent to the actuator driver by a D/A. The amplifier would amplify the actuator output. Utilizing two A/Ds, the actuator output and the external force would be transferred to the Simulink again.

The multi frequency desired trajectory would be \( x_d = 37 + 12\sin(10t) + 8\sin(8t) + 6\sin(9t) \) in micrometer. Since the actuator contacts with the environment primarily, a periodic external force is exerted with such an input. Such as mentioned there are dual hysteresis effects.

One is in the deriving the piezoelectric between input voltage and actuator output. Utilizing the modified PI model, the hysteresis was identified. Fig. 5 depicts the normalized effect of hysteresis compensation by inverse PI model even in the presence of environment force.

The identified linear model parameters, controller gains and observer gains would be selected as in Table 1.

Table 1. Controller Parameters

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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>( m ) \text{ (Kg)}</td>
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<td>( \lambda )</td>
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</tr>
<tr>
<td>( c ) \text{ (N.s/m)}</td>
<td>1e4</td>
<td>( \varepsilon )</td>
<td>1</td>
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<tr>
<td>( k ) \text{ (N/m)}</td>
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<td>( \eta_1 )</td>
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<td>( \lambda_2 )</td>
<td>2800</td>
<td>( p )</td>
<td>.2</td>
</tr>
</tbody>
</table>

By implementation of the controller with the selected parameters, despite the presence of the uncertainties, Fig. 6 represents the accurate trajectory tracking and tracking error, respectively.

Fig 6. The actuator position closely follows the desired trajectory.
The proposed approach could estimate the actuator output precisely as shown in Fig. 7.

![Fig 7. Precise estimation of the actuator position.](image)

Fig. 7. Precise estimation of the actuator position.

Fig. 8 represents the multi frequency environment force estimation by the proposed approach.

![Fig 8. The external force estimation.](image)

Fig 8. The external force estimation.

Finally, Fig. 9 depicts that the control input would be bounded.

![Fig 9. The actuator position closely follows the desired trajectory.](image)

Fig 9. The actuator position closely follows the desired trajectory.

7. CONCLUSIONS

Dynamic model of piezoelectric actuators contains the hysteresis nonlinearities effect and external forces coupled with a second-order linear dynamic. The generalized Prandtl-Ishlinskii model would identify and compensate the hysteresis effect. Considering to the position as an only measurable state, a sliding-observer estimates full states of the actuator. Proposing an adaptive perturbation estimation approach, the pure external force is estimated instead of expensive force sensors. Base on the estimated states and environment force, a sliding mode control has was proposed and implemented on the actuator to achieve the appropriate trajectory tracking. Experimental results demonstrate that the proposed controller based on the estimated states is achieved in trajectory tracking and external force estimation.

REFERENCES


