Formation Adaptive Control for Nonholonomic Dynamic Agents: Regulation and Tracking

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Abstract: In this paper, a solution for the formation control of a group of nonholonomic uncertain agents is proposed. The configuration pattern is generated from a given potential function, which may also incorporate a strategy for collision avoidance. The system parametric uncertainties are compensated by a robust adaptive control algorithm named binary adaptive control which combines the good transient properties and robustness of Sliding Mode Control with the desirable steady-state properties of parameter adaptive systems delivering continuous control signals, thus avoiding chattering. First, a static formation (regulation) scenario is considered. Conditions for semi-global stability of the multi-agent system are established using Lyapunov theory for decentralized control schemes. Then, the control scheme is extended for trajectory tracking to be performed by the group of agents while maintaining a desired geometric pattern. In the case of regulation, only the relative position information with respect to neighboring agents and individual velocity are available for the control implementation for each agent. In the case of tracking, also the velocities of the neighbors are needed. Simulations are presented to validate the theoretical results.

Keywords: adaptive control, cooperative control, nonholonomic systems, multi-agent systems.

1. INTRODUCTION

Multi-agent systems have important advantages over single agent systems such as flexibility, robustness, efficiency and redundancy. In several applications, a group of agents is required to accomplish desired tasks cooperatively, maintaining a specific pattern called formation. Different approaches for formation control have been proposed in the literature, e.g., leader-follower [Desai et al., 1988], behavior-based [Balch and Arkin, 1998], virtual structure [Tan, 1996], consensus [Olfati-Saber and Murray, 2004] and artificial potential functions [Leonard and Fiorelli, 2001].

Holonomic mobile agents have been more frequently considered in the literature. However, many applications involve robots with nonholonomic constraints, e.g., unicycles. In most papers, only kinematic models for the nonholonomic agents have been considered in the literature. Mastellone et al. [2008] proposed a decentralized control scheme which achieves dynamic formation control and collision avoidance for a group of kinematic nonholonomic robots. When high performance is desirable, for example, in terms of small tracking error, the agents dynamics have to be considered particularly when there are uncertainties in their dynamic models. In [Dong and Farrel, 2009], feedback control of a group of nonholonomic dynamic systems with uncertainty is considered control, based on a consensus strategy and backstepping techniques. Collision avoidance problem is not directly addressed.

Here, we extend the formation control design based on potential function approach, proposed in [Pereira et al., 2009], to the nonholonomic case, so that the robot motion tends to follow the vector field defined by the potential function gradient while obeying the nonholonomic constraints. We do this for both the regulation case, where some specified static geometric formation of the agents is asymptotically achieved, and the tracking case, where they are required to follow some specified trajectory in desired geometric formation.

We will use $|·|$ to denote norms; $\sigma_m(·)$ and $\sigma_M(·)$ denote the minimum and maximum singular values, respectively. Given a set of vectors $x_1, x_2, \ldots, x_N$, the vector $x$ is defined as $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$.

2. PROBLEM STATEMENT

Consider a group of $N$ nonholonomic mobile agents modeled by the following equations

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = B_i(q_i)\tau_i + J_i^T(q_i)\lambda_i$$

and

$$J_i(q_i)\dot{q}_i = 0$$

where $i = 1, \ldots, N$, $q_i \in \mathbb{R}^n$ is generalized coordinate of the $i$-th agent, $M_i \in \mathbb{R}^{n \times n}$ corresponds to the inertia matrix, $C_i \in \mathbb{R}^{n \times n}$ to the matrix of Coriolis and centrifugal forces, $\lambda_i \in \mathbb{R}^m$ is the vector of constraints forces, $J_i(q_i) \in \mathbb{R}^{m \times n}$ is the matrix associated with the constraints, $B_i(q_i) \in \mathbb{R}^{m \times r}$ is the input transformation matrix, with $r = n - m$. 
\( \tau_i \in \mathbb{R}^r \) denotes the control inputs. In this work, we consider wheeled mobile robots moving on the plane. In our case, \( n = 3, r = 2, m = 1 \) and \( q_i = [x_i, y_i, \psi_i]^T \), where \( x_i \) and \( y_i \) are the coordinates in the \( x \)-axis and \( y \)-axis respectively and \( \psi_i \) is the heading angle.

We consider two particular formation problems. First, we design control laws so that all \( N \) agents converge to a desired static configuration pattern defined by given inter-agent distances (regulation case). Then we consider the problem where the group must track a reference trajectory, maintaining a desired configuration pattern (trajectory tracking case). In both situations, we assume that each agent \( i \) knows its own states and the states of its (information) neighbors, as defined by the information graph defined in the next subsection. As in [Pereira et al., 2009], in the regulation case, only the positions of the neighbors are needed.

As usual, it is convenient to convert (1)-(2) into a suitable form, considering that the vector \( \tilde{g}_i \) can be written as

\[
\tilde{g}_i = R_i(q_i)\nu_i \tag{3}
\]

where \( R_i(q_i) \in \mathbb{R}^{n \times r} \) spans the null space of \( J_i(q_i) \) defined as

\[
R_i(q_i) = \begin{bmatrix} \cos(\psi_i) & 0 \\ \sin(\psi_i) & 0 \end{bmatrix} \tag{4}
\]

and \( \nu_i = [u_i, w_i]^T \) and \( u_i, w_i \) are the linear and angular velocities, respectively. Using the fact that \( J_i(q_i)R_i(q_i) = 0 \), the mathematical model represented by the equations (1)-(2) can be expressed by the following equation

\[
\dot{M}_i(q_i)\nu_i + C_i(q_i, \nu_i)\nu_i = B_i(q_i)\tau_i \tag{5}
\]

where \( C_i(q_i, \nu_i) = R_i^T(q_i)M_i(q_i)\dot{R}_i(q_i) + R_i^T(q_i)C_i(q_i)\dot{R}_i(q_i) \) and \( M_i(q_i) = R_i^T(q_i)M_i(q_i)R_i(q_i) \). \( B_i(q_i) \in \mathbb{R}^{r \times r} \) is assumed invertible and defined by \( \dot{B}_i(q_i) = R_i^T(q_i)B_i(q_i) \).

As is well known, the following properties hold [Dong and Farrel, 2009]:

- \( \dot{M}_i \) satisfies \( \dot{h}_{mi} |v| \leq v^TM_i(q_i)v \leq \dot{h}_{M_i} |v| \), with positive constant \( \dot{h}_{mi} \) and \( \dot{h}_{M_i}; \forall v \in \mathbb{R}^n \);
- \( (\dot{M}_i - 2\dot{C}_i) \) can be chosen skew-symmetric;
- there is a parametric vector \( a_i \) that satisfies \( M_i(q_i)\dot{\nu}_i + C_i(q_i, \nu_i)\dot{\nu}_i = Y_i(q_i, q_i, \nu, \dot{\nu}, \dot{\nu}) \) \( \tag{6} \)

where \( a_i \) are unknown parameter vectors while \( Y_i(q_i, q_i, \nu, \dot{\nu}, \dot{\nu}) \) is a matrix of known functions.

The transformed system (3)-(5) describes the motion of the original system (1)-(2).

2.1 Information topology

The information topology of the group of \( N \) agents can be described by a graph \( \mathcal{G} := (V, E) \), named information graph, where \( V := \{v_1, \ldots, v_N\} \) is a set of nodes, which represent the agents, \( E \subseteq V \times V \) is the set of edges. The set of indices corresponding to the information neighbors of agent \( i \) is denoted \( \mathcal{N}_i \) and is defined as

\[
\mathcal{N}_i := \{j | e_{ij} = (v_i, v_j) \in E\}. \tag{7}
\]

In this work, we assume that the information topologies are represented by strongly connected graphs, i.e., there exists at least one directed path between any two nodes in both directions. However, as in [Mastellone et al., 2008], for total collision avoidance, we have to assume that any two vehicles close enough can communicate their state to each other so that a repulsion force can be produced to avoid collision. Such information exchange is not required when the vehicles are not close.

3. FORMATION CONTROL - REGULATION CASE

In this section, we derive a control scheme so that the \( N \) agents reach a desired geometric pattern. To this end, we consider that each member of the group moves in the direction of the descending gradient of artificial potential function (APF), which is specified by the designer to generate interaction rules among the group members.

Definition 1 - For the planar case, the robot position \( z_i \in \mathbb{R}^2 \) is given by \( z_i = h^Tq_i = [x_i, y_i]^T \), where, clearly \( h^T = [1, 0] \).

Definition 2 - A potential function \( J_{ij}(z_{ij}) \) is a twice (continuously) differentiable, nonnegative function of the distances \( |z_{ij}| = |z_i - z_j| \) between agents \( i \) and \( j \), such that \( J_{ij} \) attains its unique minimum when the agents are located at a desired distance \( d_{ij} \).

Let us define the vector of inter-agent relative positions (or inter-agent position errors) as the vector \( \delta z \) given by the following set:

\[
\{z_{ij} | i = 1, \ldots, N - 1, j \in N_i, j > i\}. \tag{8}
\]

Then, the class of potential functions \( J(\delta z) \) is defined by

\[
J(\delta z) = \sum_{i \in N_i, j > i} J_{ij}(z_{ij}). \tag{9}
\]

Since each agent only can move in the space of directions allowed by the nonholonomic constraints, the following kinematic strategy is proposed: 1) the descending gradient vector field is used as reference for the robot heading; 2) projection of \( -\nabla z_i \) onto the current heading direction of agent is used to control its linear velocity.

Thus, the desired agent velocities are given by

\[
\nu_{di} = [u_{di}, \ w_{di}]^T. \tag{10}
\]

The term \( u_{di} \) is defined as

\[
u_{di} = -k_{wi}(\nabla z_i)J^T R_{ii} \tag{11}\]

where \( k_{wi} > 0 \) and \( R_{ii} = [\cos(\psi_i), \sin(\psi_i)]^T \). Thus, the term \( -\nabla z_i J^T R_{ii} \) represents the scalar projection (or scalar component) of the vector \( -\nabla z_i \) onto the direction of the unit vector \( R_{ii} \).

The term \( w_{di} \) is defined as

\[
w_{di} = -k_{wi}(\psi_i - \psi_{di}) \tag{12}\]

where \( k_{wi} > 0 \) and \( \psi_{di} \) is the direction of descending gradient of \( J \), expressed by

\[
\psi_{di} = \tan(2 \frac{\partial J}{\partial y_i} - \frac{\partial J}{\partial x_i}). \tag{13}\]

Since one cannot assign instantaneously the agent velocities as in the kinematic case, let us define the auxiliary error \( s_i \) as

\[
s_i = \nu_i - \nu_{di}. \tag{14}\]

The next step consists in designing the control signal \( \tau_i \) such that the auxiliary error \( s_i \) tends to zero in spite of the system uncertainties so that the desired kinematics \( \nu_i - \nu_{di} \) holds asymptotically.

Using (14) and its derivative and considering (5), one gets

\[
\dot{M}_i s_i + \dot{C}_i s_i = B_i \tau_i - \dot{M}_i \nu_{di} - \dot{C}_i \nu_{di}. \tag{15}\]
Then, consider the linear parametrization $Y_i \theta^*_i = \bar{C}_i \nu_{di}$, where $Y_i$ is a regressor matrix composed of known functions of $\tilde{q}$ and $\theta^*_i \in \mathbb{R}^{m_i}$ is a parameter vector for the $i$-th agent.

Now, equation (15) can be written as
\[
\dot{M}_i \dot{s}_i + \bar{C}_i s_i = B_i \tau_i - Y_i \theta^*_i - \dot{M}_i \nu_{di},
\]
(16)
The following control law is proposed
\[
\tau_i = B_i^{-1} (Y_i \theta_i - K_{Di} s_i)
\]
where $K_{Di}$ is symmetric positive definite and $\theta_i = [\theta_1, ..., \theta_m]^T$ is an adaptive parameter vector. Introducing the parameter mismatch $\tilde{\theta}_i = \theta_i - \theta^*_i$, one can rewrite (16) as
\[
\dot{M}_i \dot{s}_i + \bar{C}_i s_i = Y_i \tilde{\theta}_i - K_{Di} s_i - \dot{M}_i \nu_{di}.
\]
(18)

Based on binary adaptive control, as introduced in Hsu and Costa [1990] to design robust Model Reference Adaptive Control law for linear plants (B-MRAC), the following adaptation law is proposed
\[
\dot{\tilde{\theta}}_i = -\sigma \tilde{\theta}_i - \Gamma_i Y_i^T s_i; \quad \Gamma_i^T = \Gamma_i > 0.
\]
(19)

The $\sigma$-factor, also called projection factor, is defined as:
\[
\sigma = \left\{ \begin{array}{l}
0 \quad \text{if } |\tilde{\theta}_i| < M_{\tilde{\theta}_i} \text{ or } \sigma_{eq} < 0 \\
0 \quad \text{if } |\tilde{\theta}_i| \geq M_{\tilde{\theta}_i} \text{ and } \sigma_{eq} \geq 0
\end{array} \right.
\]
(20)

where $\sigma_{eq} = -\tilde{\theta}_i^T \Gamma_i Y_i^T s_i/|\tilde{\theta}_i|^2$ and $M_{\tilde{\theta}_i} > |\theta^*_i|$.

Repeatedly, this allows absence of loss of generality assume $\Gamma_i = \gamma_i I$. The following corollary explains the rational behind binary adaptation.

**Proposition 1.** Consider the system represented by the equations (14)-(16), with control law (17) and adaptation law (19)-(20) and suppose $\theta_i(0) \leq M_{\tilde{\theta}_i}$ with a constant $M_{\tilde{\theta}_i} \leq |\theta^*_i|$. Then: 1) $|\tilde{\theta}_i(t)| \leq M_{\tilde{\theta}_i}$, $\forall t$; 2) $|s_i(t)|$ tends exponentially fast to a residual value of order $O(\gamma_i^{-1/2})$.

Proof: see [Pereira et al., 2009].

The following theorem establishes the semi-global stability property for the regulation case.

**Theorem 1.** Consider the system (3)-(5), (17), (19), (20) which has the state vector $\eta = [q^T \quad \nu^T \quad \theta^T]^T$. Assume $\theta_i(0) \in B_{\theta_i} = [\theta_i; |\theta_i| \leq M_{\theta_i}]$, with constant $M_{\theta_i} \geq |\theta^*_i|$ and consider the invariant equilibrium set
\[
\Omega_\nu = \left\{ \eta: \nu = 0; \quad \nabla_\nu J = 0; \quad \tilde{\theta} = 0 \right\}.
\]
Then, starting from a set $\mathcal{D}_0$ defined by $V_A(s, \theta, \dot{\theta}, \psi) \leq V_{A0}$, with $V_A$ as defined in Appendix A, with constant $V_{A0} > 0$ large enough to comply with the condition on $\theta(0)$, but otherwise arbitrary (for semi-global stability), the following holds: (a) the system trajectories tend asymptotically to the equilibrium set $\Omega_\nu$; (b) all closed loop signals are uniformly bounded and the multi-agent system tends asymptotically to some constant formation.

**Proof:** see Appendix A

**Remark 1.** Note that in general the local minima of $J$ are isolated extrema. Thus, from the Lyapunov analysis presented above, it follows that, if the groups starts sufficiently close to a desired formation, convergence to it could be guaranteed. Also note that $z$ and $q$ are linearly related by $z_i = [x_i \ y_i]^T$.

4. DYNAMIC FORMATION CASE

In this section, the group is required to track a reference trajectory, maintaining a geometric pattern as in [Mastellone et al., 2008]. In this problem, we design a binary adaptive controller for each agent, so that $|z_i - z_j| \rightarrow d_{ij}$ and $(q(t) - q_i(t)) \rightarrow 0$, as $t \rightarrow \infty$, where $q_i(t) = [x_i(t) \ y_i(t) \ \psi_i(t)]^T$ is the generalized coordinate of the $i$-th virtual leader robot defined by following expression
\[
q_i(t) = q_{0i}(t) + R_{v} q_{0v}.
\]
(21)

We assume $\Gamma_o \not\in B_{\theta_o} = [\theta_o; |\theta_o| \leq M_{\theta_o}]$ and $M_{\theta_o} > |\theta^*_o|$ and the $\gamma_o$ is constant matrix that defines a desired orientation $\psi_o$ of the geometric formation, expressed by
\[
R_{o} = \begin{bmatrix}
\cos(\psi_o) & -\sin(\psi_o) & 0 \\
\sin(\psi_o) & \cos(\psi_o) & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]
(23)

The constant vector $q_{0i} = [x_{0i} \ y_{0i} \ \psi_{0i}]^T$ is defined according to the desired formation geometric pattern. Therefore, $\dot{\psi}_{ri} = \dot{\psi}_{o} \ \forall ri \in \mathcal{N}_o$, $u_{ri} = u_{sri}$, $w_{ri} = w_{sri}$.

To perform the tracking objective, for each agent, the desired velocity for tracking, denoted by $\nu_{ri}$, is added to the one for formation, denoted by $\nu_{ri}$, thus generalizing what was proposed for a single nonholonomic vehicle, as in [Fukao et al., 2000]. To this end, we use the auxiliary error function (14) for each agent, as in in the regulation case, redefining the ”desired values” for $\nu_{di}$ as follows
\[
\nu_{di} = \nu_{f_i} + \nu_{ri}
\]
(24)

where $\nu_{f_i} = [u_{f_i} \ w_{f_i}]^T$ refers to the geometric formation pattern and $\nu_{ri} = [u_{ri} \ w_{ri}]^T$ refers to trajectory tracking, with
\[
u_{f_i} = -k_{iws} (\nabla_z J)^T \mathcal{R}_{li} \quad \nu_{ri} = k_{iws} (\nabla_z J)^T \mathcal{R}_{ri} + k_{iws} \psi_r - \psi_i \quad \nu_{ri} = -k_{iws} (\nabla_z J)^T \mathcal{R}_{ri} + k_{iws} \psi_r - \psi_i \quad \psi_r = \psi_{ri} = \psi_{ri}
\]
(25)

where $k_{iws}$ and $k_{iws}$ are positive constants, $\mathcal{R}_{li} = [\cos(\psi_i) \ \sin(\psi_i) \ \sin(\psi_i) \ \cos(\psi_i)]^T$, $\mathcal{R}_{ri} = [\cos(\psi_i) \ \sin(\psi_i) \ \sin(\psi_i) \ \cos(\psi_i)]^T$. $\mathcal{N}_o$ (velocities of neighbors are required).

The control law is defined as
\[
\tau_i = B_i^{-1} (Y_i \theta_i - K_{Di} s_i).
\]
(32)

Then, we obtain the following closed-loop equations
\[
\dot{M}_i \dot{s}_i + \bar{C}_i s_i = Y_i \theta_i - K_{Di} s_i.
\]
(33)

The following theorem establishes the semi-global stability result for the trajectory tracking case.
Theorem 2. Consider the system (3)-(5), (14), (19), (20), (24) and (32). Assume $\vartheta_i(0) \in B_{\vartheta i} = \{\vartheta_i : |\vartheta_i| \leq M_{\vartheta i}\}$ with constant $M_{\vartheta i} \geq |\vartheta_i^*|$ and $q_{ai}(t) \in C^2$. Then, starting from a set $D_1$, defined similarly as $D_0$, from Theorem 1, by changing $V_A$ by $V_B$, as defined in Appendix 2, and the positive constant $V_{A0}$ by $V_{B0}$ (arbitrarily large), and choosing the gains $k_{v1}, k_{v2}, k_{w1}, k_{w2}$ of order $O(\alpha^{-1})$, where $\alpha$ is a small parameter, the following holds: (a) $(q_i(t) - q_i^*) \rightarrow O(\alpha)$, as $t \rightarrow \infty$; (b) all close loop signals are uniformly bounded and the multi-agent system tends asymptotically close to some constant formation corresponding to $|\nabla_x J| = O(\alpha)$.

Proof: see Appendix B.

5. SIMULATIONS

In this section, simulation results are presented to illustrate the proposed cooperative control design and performance of theoretical results. Only the more difficult problem of trajectory tracking case will be discussed. Simulations show three uncertain nonholonomic agents in circular trajectories as we can see in the Figure 1. The dynamics of each agent is described by suitable form described by the equations (3) and (5), so that

$$\mathbf{M}_{Ri} \dot{\nu}_i + \mathbf{C}_{Ri}(\dot{q}_i) \nu_i = \tau_i \quad (34)$$

$$\dot{q}_i = R_i(q_i) \nu_i \quad (35)$$

where $\nu_i = [\nu_{1i}, \nu_{2i}]^T$ with $\nu_{1i}$ and $\nu_{2i}$ being the wheels angular velocities, $q_i = [x_i, y_i, \psi_i]^T$ is a generalized coordinate, $z_i = [x_i, y_i]^T$ is the planar position, $\tau_i = [\tau_{1i}, \tau_{2i}]^T$ being $\tau_{1i}$ and $\tau_{2i}$ the control torques applied to the robot wheels, $\mathbf{M}_{Ri} \in \mathbb{R}^{2 \times 2}$ is a constant inertia matrix, $\mathbf{C}_{Ri}(\dot{q}_i) \in \mathbb{R}^{2 \times 2}$ is the centripetal and Coriolis matrix, defined as

$$\mathbf{M}_{Ri} = \begin{bmatrix} m_{11} & -m_{12} \\ -m_{12} & m_{11} \end{bmatrix}, \mathbf{C}_{Ri} = \begin{bmatrix} 0 & c_i \dot{\psi} \\ -c_i \dot{\psi} & 0 \end{bmatrix}. \quad (36)$$

The nominal values are the following: $m_{11} = 22.02$, $m_{12} = -0.86$, $c_i = 7.94$. We have used the potential function based on [Gazi, 2005] and it has the form of the equation (9), where $J_{ij}$ is defined by the formula

$$J_{ij}(z_{ij}) = \frac{a_{ij}}{2} |z_{ij}|^2 + \frac{b_{ij}c_{ij}}{2} \exp \left(-\frac{|z_{ij}|^2}{c_{ij}}\right)$$

where $a_{ij}$ are the attraction constants and $b_{ij}$ is the repulsion constant. The parameter $c_{ij} = d_{ij}^2 / \log(b_{ij}/a_{ij})$. The constant $d_{ij}$ gives the desired inter-vehicule distances. The initial velocities were set equal to zero. For the potential function, the parameters were chosen as $a_{ij} = 0.01$, $b_{ij} = 10$, $d_{ij} = 50$. The adaptive parameter were initialized at the nominal values as $\theta_i(0) = 3$ (true values are 7.94) and the binary control was used with $M_{\theta} = 1.2 |\theta_i^*|$ and adaptation gain $\gamma_1 = 2$. Adaptive control allows precise trajectory tracking so that $(x_i - x_f)$, $(y_i - y_f)$ and $(\psi_i - \psi_f)$ tend to small residuals, as shown in Figure 2 and 3. Inter-agent distances closely tend values to desired ones, as shown in Figure 4.

6. CONCLUSIONS

We have proposed a framework to design static (regulation) or dynamic (tracking) formation control for a group of uncertain nonholonomic dynamic agents. Artificial potential functions were used to avoid collisions and to generate a desired geometric pattern with prescribed inter-agent distances and binary adaptive control was used to cope with the agent uncertainties. Control laws were designed which guarantee semi-global stability and, either asymptotic static formation, or arbitrarily precise dynamic formation. The control laws for regulation only require each agent to know its own velocity and the position differ-
Appendix A. PROOF OF THEOREM 1

Consider the following candidate Lyapunov function

$$V_A(s, \tilde{s}, \tilde{\psi}) = V_1 + \beta W_1$$ (A.1)

where $\beta$ is a positive constant and

$$V_1 = \sum_{i=1}^{N} \left( \frac{1}{2} s_i^T M_i s_i + \frac{1}{2\gamma} \bar{\theta}_i^T \bar{\theta}_i \right)$$ (A.2)

where the elements of $\bar{z}$ is the vector of position differences between neighbors (i.e. $z_{ij}$), $\psi$ is the vector of heading errors (i.e., $\psi_i - \psi_{\alpha_i}$) and $W_1$ is given by

$$W_1 = J(z) + \alpha \Pi(\psi - \psi_d)$$ (A.3)

where $\alpha$ is a positive constant and $J(z)$ is defined as in section (3) and $\Pi(\psi - \psi_d) = \sum_{i=1}^{N} \frac{1}{2} (\psi_i - \psi_{\alpha_i})^2$. Using the skew-symmetry property, the time-derivative of (A.2) is given by

$$\dot{V}_1 = \sum_{i=1}^{N} \left[ -s_i^T K_i \psi_i - \frac{\sigma}{\gamma} [\bar{\theta}_i + \bar{\theta}_i^T] \bar{\theta}_i \right]$$ (A.4)

Note that the second term within brackets is non-positive (see Hsu and Costa [1990]). Now, it can be shown that the last term of (A.4) is expressed by

$$\dot{w}_d = K_d H \nu$$ (A.5)

where $H$ depends on the gradient and on the Hessian of the potential function $J$. It is norm bounded by a constant $\mathcal{D}_0$. The derivative of (12) is $\dot{w}_{d1} = k_{\nu} (w_{d1} - \psi_{d1})$, where $\psi_{d1}$ is defined by (as shown in Gouvea et al., 2010)

$$\dot{\psi}_{d1} = L_{11} \sum_{j=1}^{N} L_{21j} u_{j1}$$ (A.6)

$$L_{11} = \frac{1}{\gamma} \left( \frac{\partial^2 V_i}{\partial x_i \partial x_i} \right)^2 + \frac{\gamma}{\sigma} \left( \frac{\partial^2 V_i}{\partial y_i \partial y_i} \right)^2$$ (A.7)

$$L_{21j} = \frac{\partial V_i}{\partial x_i} \left( \frac{\partial^2 V_i}{\partial y_j \partial y_j} \sin(\psi_j) + \frac{\partial^2 V_i}{\partial y_j \partial x_j} \cos(\psi_j) \right)$$

$$- \frac{\partial V_i}{\partial y_j} \left( \frac{\partial^2 V_i}{\partial x_i \partial y_j} \cos(\psi_j) + \frac{\partial^2 V_i}{\partial x_i \partial x_j} \sin(\psi_j) \right)$$ (A.8)

Defining $\dot{\psi}_d = [\dot{\psi}_{d1} \ \dot{\psi}_{d2} \ldots \ \dot{\psi}_{dN}]$, we obtain

$$\dot{\psi}_d = L_A u$$ (A.9)

where

$$L_A = \begin{bmatrix}
L_{11} & L_{12} & \cdots & L_{1N} \\
L_{21} & L_{22} & \cdots & L_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
L_{N1} & L_{N2} & \cdots & L_{NN}
\end{bmatrix}$$ (A.10)

The derivative of $w_{d1}$ can be expressed as $\dot{w}_{d1} = k_{\nu} (w_{d1} - L_{11} \psi_{d1}) - L_{12} \psi_{d2} \cdots - L_{1N} \psi_{dN}$. In the vector form, i.e., $\dot{w}_d = [\dot{w}_{d1} \ \dot{w}_{d2} \ldots \ \dot{w}_{dN}]^T$, we have

$$\dot{w}_d = K_w \left[ I \otimes [0 \ 1] - L_A \otimes [1 \ 0] \right] v$$ (A.11)
where $K_w = \text{diag}(k_{w1}, k_{w2}, \ldots, k_{wN})$. In this way, $\nu_d = [\dot{u}_d \ \dot{w}_d]^T$ is written as
\[
\nu_d = K \nu
\] (A.12)
where
\[
K = \begin{bmatrix} K_w & 0 \\ 0 & K_w \end{bmatrix}, \quad G = \begin{bmatrix} I & 0 \\ H & I \end{bmatrix} T_d
\] (A.13)
and $T_d$ is a transformation matrix such that $\nu_d = T_d [u_d^T \ w_d^T]^T$.

Thus, the time-derivative of (A.4) can be rewritten as
\[
\dot{V}_1 = -s^T K D s - s^T M \dot{K} G v
\] (A.14)
where $s = [\nu^T \ \bar{v}_d]$. As $H, L_A$ is norm bounded by a constant in the set $D_0$. The derivative of $W_1$ is
\[
W_1 = \sum_{i=1}^{N} [\nabla \dot{u}_i V_i]^T \dot{z}_i + \alpha \left( \frac{\partial}{\partial \psi_i} \psi_i - \frac{\partial}{\partial \psi_i} \psi_{di} \right)
\] (A.15)
Defining $\dot{z}_i = R \dot{u}_i, \ \psi_i = w_i$, and $s_i = [s_{u_i} \ s_{w_i}]^T$, which represent the linear and angular velocity partition of $s$, respectively, and using the equations (14), (11), and (12),
\[
W_1 = \sum_{i=1}^{N} \left[ -s_{u_i} u_i + s_{w_i} w_i + \frac{\partial s_{u_i}}{\partial \nu_i} + \frac{\partial s_{w_i}}{\partial \nu_i} \right] \nu_i
\] (A.16)
This can be expressed as
\[
\dot{W}_1 = -u^T K_w - u^T K_w - s_w - w^T K_w - w^T L_u u
\] (A.17)
where $s_w = [s_{u1} \ldots s_{uN}]^T$ and $w = [w_1 \ldots w_N]^T$. Considering that $\nu = [\nu^T \ \bar{v}_d]^T$, $s = [s_{u} \ s_{w}]^T$, we obtain
\[
\dot{W}_1 = -\nu^T M(\alpha) \nu + s^T A(\alpha) \nu
\] (A.18)
with $M(\alpha)$ defined as
\[
M(\alpha) = T^T \begin{bmatrix} K_w^{-1} & -\frac{1}{2} \alpha L_A \\ -\frac{1}{2} \alpha L_A & K_w^{-1} \end{bmatrix}, \quad T = T^T \begin{bmatrix} K_w^{-1} & 0 \\ -\alpha L_A & \alpha K_w^{-1} \end{bmatrix}
\] (A.19)
The permutation matrix $T$ is defined such that $\nu = T^{-1} [u^T \ w^T]^T$. From a Schur complement argument, it follows that $\exists \alpha$ for any constant norm-bound of $L_A$ such that $M(\alpha)$ is positive definite. Now, the time-derivative of (A.1) can be rewritten as
\[
\dot{V}_A \leq -s^T K D s - s^T M \dot{K} G v + \alpha^T \beta^T A(\alpha) \nu
\] (A.20)
where $K_D = \text{diag} \{K_{D1}, K_{D2}, \ldots, K_{DN}\}$. This is rewritten as
\[
\dot{V}_A \leq -s^T \nu^T \begin{bmatrix} K_D & -\frac{1}{2} H_1(\beta) \\ -\frac{1}{2} H_1(\beta) & \beta^T M(\alpha) \end{bmatrix} \begin{bmatrix} s \\ \nu \end{bmatrix}
\] (A.21)
where $H_1(\beta) = [M \dot{K} G \ -\beta A(\alpha)]$. For $\dot{V}_A \leq 0$, the Schur complement $S_1$ should satisfy
\[
S_1 = \beta M(\alpha) - \frac{1}{4} H_1(\beta) K_D^{-1} H_1(\beta) > 0
\] (A.22)
which holds if
\[
\sigma_m(K_D) > \frac{\sigma_T^2}{4} H_1(\beta) \frac{\sigma_M^2(M(\alpha))}{\sigma_m(M)}
\] (A.23)
where the last term was determined as in [Perreira et al., 2009] and is independent of $\beta$. Then, we can conclude that $V_A$ can be negative semi-definite so that $V_A$ is uniformly bounded $\forall t$. Therefore, with $K_D$ large enough, the set $D_0$ is invariant (so that the assumed uniform bounds of $L_A$ and $H$ hold) and by Barbalat’s theorem $s(t), \nu(t) \to 0$ as $t \to \infty$. From equation (3), we have $\dot{q}_i \to 0$ and each $q_i$ tends to a constant (technically speaking, the latter conclusion requires an additional radially unbounded term, say, $J_1(z_1)$ in the potential function, see [Perreira et al., 2009]). Moreover, the equilibrium set $\Omega_e$ is reached asymptotically. This proves the theorem.

**Appendix B. PROOF OF THEOREM 2**

Consider the following Lyapunov function
\[
V_2 = V_3 + \alpha W_3
\] (B.1)
where $\alpha$ is a nonnegative constant and $V_2$ is defined as follows
\[
V_2 = \sum_{i=1}^{N} \left( \frac{1}{\gamma_i} \ | \dot{M}_i | s_i + \frac{1}{2} | \dot{\beta}_i | \dot{\beta}_i \right)
\] (B.2)
and $W_2$ is defined by
\[
W_2 = J(z) + \sum_{i=1}^{N} \left( \frac{1}{2} | \dot{z}_i - \dot{z}_i |^2 | R_{ji} |^2 \right) + \frac{1}{2} \left( \frac{1}{2} | \dot{z}_i - \dot{z}_i |^2 | R_{ni} |^2 \right)
\] (B.3)
Note that the tracking terms (three first terms under the summation) are as in [Fukao et al., 2000]. The time-derivative of $V_2$ satisfies
\[
V_2 \leq -s^T K D s
\] (B.4)
For calculating the derivative of $W_2$, let us define $e_i = (\nabla \dot{z}_j)^T R_{ji}$, $e_{\psi_i} = (\psi_i - \dot{\psi}_i)$, $d_{\psi_i} = (\psi_i - \dot{\psi}_i)$, $e_{z_i} = (\dot{z}_i - \dot{z}_i)^T R_{ii}$, $e_{z_i} = (\dot{z}_i - \dot{z}_i)^T R_{ni}$, $e_{\psi_i} = \sin(\psi_i - \psi_i)$, $e_{\psi_i} = u_{\psi_i} \cos(\psi_i - \psi_i)$. It can be shown that $W_2$ satisfies
\[
W_2 \leq -e^T K_K e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e - e_{\psi_d} e
\] (B.5)
where $K_K = \text{diag}(k_{K1}, k_{K2})$, $K_D = \text{diag}(k_{D1}, k_{D2})$, and $C = \text{diag}(\cos(\psi_i - \psi_i))$.

After some algebraic manipulation, we obtain
\[
W_3 \leq -e^T K D e - e^T L s + e^T \dot{E} v_{\psi_i}
\] (B.6)
where $e = [e_1^T \ e_{\psi_d}^T \ e_{\psi_d}^T]^T$, $s = [s_{K1}^T \ s_{K2}^T]$ and
\[
K = \begin{bmatrix} K_0 & 0 & 0 \\ 0 & K_0 & 0 \\ 0 & 0 & K_3 \end{bmatrix}, \quad L = \begin{bmatrix} I_0 & 1 \\ 1 & 0 \\ 0 & 0 \\ K_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \dot{T}, \quad \dot{E} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\] (B.7)
where
\[
A = \begin{bmatrix} K_D & -\frac{1}{2} H_1(\beta) \alpha \beta^T M(\alpha) \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 \\ E \end{bmatrix}
\] (B.8)
The symmetric matrix $A$ is positive definite if the Schur complement is positive definite, i.e.,
\[
A_4 = \begin{bmatrix} 0 \\ E \end{bmatrix}
\] (B.9)
where
\[
s_2 = \alpha R - \frac{1}{4} \sigma_T^2 L K_D^{-1} \sigma_M(\alpha)
\] (B.10)
which holds for a sufficiently small. Defining $\eta_4 = [s^T e^T]^T$, we can conclude if $K$ is chosen large enough so that $\sigma_m(K)$ is of order $O(\eta^2)$, it follows that the error norm $|\eta_4|$, and thus $\nabla \dot{z}_j$, will tend to a residual set of order $O(\alpha)$ and thus can be made arbitrarily small. Finally, all signals of the system are uniformly bounded. The semi-global stability is proved similarly as in Theorem 1, using the set $D_1$. ■