Robust Recursive Kalman Filtering for Attitude Estimation*

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Abstract: This paper proposes a new attitude reference system based on a new robust recursive Kalman filter (RRKF) for determination of a vehicle attitude. The attitude estimate is performed based on the combination of signals of low-cost magnetometers, rate gyros and accelerometers considering a rigid body model formulated in terms of quaternions. Experimental results are presented based on a comparative study among the RRKF, the standard extended Kalman filter and an $H_\infty$ filter.

Keywords: RRKF, Attitude algorithms, inertial navigation, inertial measurement units, extended Kalman filters, robust estimation.

1. INTRODUCTION

The rigid body attitude estimation is essential to guarantee the stability of autonomous vehicles (Farrel (2008)). This kind of estimate is generally solved fusing the signals of inertial measurement units (IMUs). When we have accurate sensors, IMUs based on Kalman filters have provided excellent results to obtain the attitude estimate and the bias of the sensors, see for instance Farrel and Barth (1998), Farrel (2008), and Rogers (2003). For some important applications, IMUs composed of low-cost inertial sensors have been used in the development of some specific inertial navigation systems, see Bijkier and Steyn (2008); Jung and Tsiotras (2007); Castellanos et al. (2005). Small unmanned aerial vehicles, whose costs are not expensive, are examples where these navigation systems have been used.

The low-cost of these inertial sensors means, in general, the increase of uncertainties and noises, see Xing and Gebre-Egziabher (2008) and references therein. These uncertainties violate the central premise of the Kalman filter, in which the underlying state-space model is accurate (Sayed (2001)). This kind of problem motivates the use of robust estimation methods to limit the performance degradation of standard optimal filters.

Few works have been proposed in the literature to deal with attitude robust estimation. Recently, Abdelkrim et al. (2008) proposed an $H_\infty$ filter based on Simon (2006) for unmanned aerial vehicle localization, in which only simulations were presented. Nevertheless, there exist recursive state-space estimation methods developed for situations in which the parameters of the underlying linear model are subject to uncertainties (Bianco et al. (2008), Ishihara and Terra (2008), Ishihara et al. (2006), and Sayed (2001)). These methods have been applied to practical problems and the results have shown the performance improvement of the estimates, if compared with the standard extended Kalman filter. See for instance Dominguez et al. (2006), where the authors used this class of filter to fingertip tracking in human-machines interfaces.

Taking into account this scenario, this paper applies the robust recursive Kalman filter to estimate rigid body attitude. It is based on penalty game approach and it assumes that there exist uncertainties in all parameter matrices of the model. This is an interesting advantage if compared with the filters proposed in Ishihara et al. (2006) and Sayed (2001). Some details of the deduction of this filter, the stability and convergence proofs, are shown in a companion paper also presented in the 18th IFAC World Congress, Terra et al. (2011). A comparative study among this new robust filter, the standard extended Kalman filter, and the $H_\infty$ filter proposed in Abdelkrim et al. (2008) is performed based on experimental results.

This paper is organized as follows. Section 2 presents the sensor models we are dealing with. Section 3 presents the kinematic model of a rigid body using quaternions. Section 4 presents the attitude state-space form for a rigid body. Section 5 presents the standard extended Kalman filter, an $H_\infty$ filter, and the robust recursive Kalman filter we proposed. Section 6 presents the result of the comparative study performed, and Section 7 provides some conclusive remarks.

2. SENSOR MODELS

In this section we present the inertial sensors models we are considering to perform the attitude estimates. They are modeled considering that all measurements are subject to uncertainties. In this context, angular velocity, static acceleration, and magnetometer measurement of a rigid body will be used to define a kinematic model, also subject to uncertainties. The actual rigid body angular velocity $\omega$ is described as

$$\omega = \omega_k + \delta\omega_k - b_k - \eta_k,$$

where $\omega_k$ is the measured angular velocity from gyros, $\delta\omega_k$ is an uncertain term of $\omega_k$, $b_k$ is the gyro bias and $\eta_k$ is...
the Gaussian white noise of the gyro measurements. The gyro bias $b_g$ is defined in terms of a Gauss-Markov process
\[ \dot{b}_g = -\frac{1}{\tau_g} b_g + \eta_{b_g}, \] (2)
where $\eta_{b_g}$ is the Gaussian white noise of the gyro biases and $\tau_g$ is the correlation time of this process. The actual rigid body acceleration $a$ is given by:
\[ a = a_a + \delta a_a - \eta_a, \] (3)
where $a_a$ is the accelerometer measurement, $\delta a_a$ is an uncertain term of $a_a$, and $\eta_a$ is a Gaussian white noise of the accelerometer measurements. The magnetometer measurement $m_m$ is modeled as:
\[ \dot{m}_m + \delta m_m = m + m_b + \eta_m, \] (4)
where $\delta m_m$ is an uncertain term of $m_m$, $m$ is the Earth magnetic field vector, $m_b$ is the magnetic field vector generated by the vehicle and $\eta_m$ is the Gaussian white noise. It is assumed that the magnetometer is isolated from the vehicle magnetic field, so that the $m_m$ term can be assumed to be zero.

The role that uncertainties $\delta \omega$, $\delta a_a$, and $\delta m_m$ play for each filtering approach considered in this paper will be described in Section 5. In the following section we define the kinematic model used to estimate the attitude of a rigid body.

3. KINEMATIC MODEL

Quaternions have been used in a vast quantity of references to describe the movement of a rigid body, see for instance Kuipers (1998) and references therein. An interesting aspect of this approach is that it is possible obtain the measurements of a rigid body attitude free of singularities.

By convention, the quaternion is represented as:
\[ q = \cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} r, \quad q \in \mathbb{T}, \] (5)
where $r$ is a unit vector, $\alpha$ is a rotation around this axis and $\mathbb{T} = \{ q \mid q^T q = 1, q = [q_0 \, q^T]$, $q_0 \in \mathbb{R}, \, q \in \mathbb{R}^{3 \times 1} \}$. The product of quaternions is represented by $\otimes$ and is given by:
\[ a \otimes b = \begin{bmatrix} a_0 b_0 - a \cdot b \\ a_0 b + b_0 a + a \times b \end{bmatrix}. \] (6)

Concerning attitude, quaternion represents a rotation of inertial navigation frame, $\mathcal{I}$, to body frame, $\mathcal{B}$, Figure 1.

If $s$ is a vector expressed in the inertial navigation frame $\mathcal{I}$, then its coordinates in $\mathcal{B}$ are given by:
\[ b = q \otimes s \otimes q^{-1}, \] (7)
where $b = [0 \, b^T]^T$, $s = [0 \, s^T]^T$, and $q^{-1} = [q_0, -q^T]^T$. Then, (7) can be rewritten as:
\[ b = R(q) s, \] (8)
where $R(q)$ is the rotation matrix of Euler angles $(\psi, \theta, \phi)$, with $(zyx)$ convention, defined as
\[ R(q) = (q_0^2 - q^T q)I_{3 \times 3} + 2(q q^T - q_0 q^T + q^T q_0). \] (9)
The quaternion derivative can be written as:
\[ \dot{q} = \frac{1}{2} q \otimes \begin{bmatrix} 0 \\ \omega \end{bmatrix} = \frac{1}{2} \Omega(\omega)q = \frac{1}{2} \Xi(\omega), \] (10)
where
\[ \Omega(\omega) = \begin{bmatrix} 0 & -\omega^T \\ \omega & -\omega \end{bmatrix}, \quad \Xi(q) = \begin{bmatrix} -q^T \\ I_{3 \times 3} q_0 + [q^*] \end{bmatrix}, \]
where $\omega$ is the angular velocity vector of body frame, $[\omega^*]$ and $[q^*]$ are representations of the cross product
\[ \mathcal{E}^* = \begin{bmatrix} 0 & -E_3 & E_2 \\ -E_2 & E_3 & 0 \end{bmatrix}, \]
and $I_{3 \times 3}$ is the identity matrix. Substituting (1) into (10), it follows that:
\[ \dot{q} = \frac{1}{2} \Xi(q)(\omega_k + \delta \omega_k - b_g) - \frac{1}{2} \Xi(q) \eta_k, \] (12)
\[ = \frac{1}{2} \Omega(\omega_k)q + \frac{1}{2} \delta \Omega(\delta \omega_k) q - \frac{1}{2} \Xi(q) b_k - \frac{1}{2} \Xi(q) \eta_k. \] (13)
Considering that $\Xi(q)$ has the uncertain term $\delta \Xi(\delta q)$, due to the influence of sensors’ uncertainties, Equation (13) can be written as
\[ \dot{q} = \frac{1}{2} \Omega(\omega_k)q + \frac{1}{2} \delta \Omega(\delta \omega_k) q - \frac{1}{2} \Xi(q) b_k - \frac{1}{2} \Xi(q) \eta_k \]
\[ - \frac{1}{2} \delta \Xi(\delta q) b_k - \frac{1}{2} \Xi(\delta q) \eta_k. \] (14)

In the following section measurement and kinematic equations aforementioned will be written in a unified way in terms of state and output equations.

4. ATTITUDE STATE-SPACE MODEL

In this section the gyro bias (2), the accelerometer measurement (3), the magnetometer measurement (4), and the quaternion equation (14) are considered in the state-space form when the system is subject to uncertainties
\[ \dot{x} = (A + \delta A)x + (B + \delta B)\eta, \]
\[ z = h(x) + \delta h(x) + v, \] (15)
where
\[ A = \begin{bmatrix} -\frac{1}{2} \Omega(\omega_k) - \frac{1}{2} \Xi(q) \\ 0_{3 \times 4} - \frac{1}{2} \tau_g I_{3 \times 3} \end{bmatrix}, \]
\[ B = \begin{bmatrix} -\frac{1}{2} \Xi(q) 0_{4 \times 3} \\ 0_{3 \times 3} I_{3 \times 3} \end{bmatrix}, \]
\[ \delta A = \begin{bmatrix} -\frac{1}{2} \delta \Omega(\delta \omega_k) - \frac{1}{2} \delta \Xi(\delta q) \\ 0_{3 \times 4} \end{bmatrix}, \]
\[ \delta B = \begin{bmatrix} -\frac{1}{2} \delta \Xi(\delta q) 0_{4 \times 3} \\ 0_{3 \times 3} 0_{3 \times 3} \end{bmatrix}, \]
\[ \eta = \begin{bmatrix} \eta_q \eta_b \end{bmatrix}^T \in \mathbb{R}^{6 \times 1} \] is a zero mean Gaussian process with covariance matrix $Q$, $x = [q^T \, b^T]^T \in \mathbb{R}^{7 \times 1}$ is the state that describes the attitude of the system, $h(x) = $ Fig. 1. Attitude reference system.
$m^T a^T T \in \mathbb{R}^{6 \times 1}$ is the measured output, $\delta h(x) = [-\delta m^T m - \delta a^T a]^T \in \mathbb{R}^{6 \times 1}$, $v = [\eta^T_m \eta^T_a]^T \in \mathbb{R}^{6 \times 1}$ is a zero mean Gaussian process with covariance matrix $R$, $m \in \mathbb{R}^{3 \times 1}$ is given by
\[ m = R(q)m_e = \left[ m_e(q_0^2 + q_1^2 - q_2^2 - q_3^2) ight], \quad (16) \]
where $m_e = [m_e \ 0 \ 0]^T$ and $m_e$ is the magnitude of the Earth magnetic field, and $\alpha \in \mathbb{R}^{3 \times 1}$ is calculated as
\[ a = R(q)g_e = \left[ 2g_e(q_3 - q_0 q_2) ight. \left. \ 2g_e(q_2 q_3 + q_0 q_1) \right]^T, \quad (17) \]
where $g_e = [0 \ 0 \ g_e]^T$ and $g_e$ is the Earth gravity constant.

The extended Kalman filter and the robust filters presented in the next section are based on discrete-time systems. In this sense (15) is discretized considering a sample time $T$,
\[ x_{k+1} = (F_k + \tilde{F}_k)x_k + (G_k + \tilde{G}_k)\eta_k, \]
\[ z_k = (H_k + \tilde{H}_k)x_k + \nu_k, \quad (18) \]
where $F_k \simeq I + AT$, $\tilde{F}_k \simeq \delta AT$, $G_k \simeq BT^T$, $\tilde{G}_k \simeq \delta BT^T$, $H_k = \partial h(x)/\partial x$, $\tilde{H}_k = \partial \delta h(x)/\partial x$.

The next section presents the filtering approaches proposed in this paper in order to estimate the attitude of a rigid body.

5. EXTENDED KALMAN AND ROBUST FILTERS

This section presents the extended Kalman filter, the $H_{\infty}$ filter proposed in Simon (2006) and robust recursive Kalman filter (RRKF) proposed in Terra et al. (2011) to estimate the attitude of the rigid body subject to uncertainties in the parameter matrices. It is also considered that the system is subject to bounded disturbances. In order to present a comparative study among these filters, it is important to emphasize the main properties of each filter that define their performance. The extended Kalman filter does not take into account the uncertainty matrices of the system (15). The $H_{\infty}$ filter presents a level of robustness in order to reject disturbances, which depends on a parameter $\gamma$ that must be adjusted off-line. The filter was not deduced to consider, in an exact way, the uncertainties of (15). The RRKF proposed is defined to reject disturbances and the uncertainties of the parameter matrices of (15).

5.1 Extended Kalman filter

The classical extended Kalman filter (EKF) (Brown and Hwang (1997)) can be implemented following the following algorithm
\[ \hat{x}_{k+1|k} = F_k\hat{x}_k, \]
\[ P_{k+1|k} = F_kP_{k|k}F_k^T + G_kQ_kG_k^T, \]
\[ \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}[x_k - h(\hat{x}_{k+1|k})], \]
\[ K_{k+1} = P_{k+1|k}[H_k + P_{k+1|k}K_{k+1}]^{-1}. \]

5.2 $H_{\infty}$ filter

The objective of the $H_{\infty}$ filter is to estimate an auxiliary output, defined by an appropriate algebraic combination of the states, given by:
\[ y_k = L_kx_k, \quad (19) \]
where $L_k$ is a matrix defined by the designer in order to obtain $x_{k+1}$. The known cost function to be minimized is given by:
\[ J = \sum_{k=0}^{N-1} \|y_k - \hat{y}_k\|_{S_k}^2 + \sum_{k=0}^{N-1} \|W_k\|_{Q_k}^2 + \|V_k\|_{R_k}^2, \quad (20) \]
where $P_0, Q_k, R_k$, and $S_k$ are symmetric, positive definite matrices. The cost function can be made to be lower than $1/\gamma$ (a designer-specified bound) with the following estimation strategy:
\[ \hat{x}_k = L_k^T S_k L_k, \]
\[ K_k = P_k[I - \gamma S_k P_k + H_k^T R_k^{-1} H_k P_k]^{-1} H_k R_k^{-1}, \]
\[ \hat{x}_{k+1} = F_k\hat{x}_k + F_k K_k(\hat{x}_k - h(\hat{x}_k)), \]
\[ P_{k+1} = F_k P_k[I - \gamma S_k P_k + H_k^T R_k^{-1} H_k P_k]^{-1} F_k^T + G_k Q_k G_k^T, \]
guaranteeing that the following condition holds for each time step $k$,
\[ P_{k+1} - \gamma S_k + H_k^T R_k^{-1} H_k > 0. \quad (21) \]

5.3 Robust recursive Kalman filter

The robust recursive Kalman filter proposed in this section is based on (18) whose uncertainties $\tilde{F}_k$, $\tilde{G}_k$, and $\tilde{H}_k$ are considered structured in the following way
\[ \begin{bmatrix} \delta F_k & \delta G_k \\ \delta H_k \end{bmatrix} = \begin{bmatrix} M_{1k} & 0 \\ 0 & M_{2k} \end{bmatrix} \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} \begin{bmatrix} N_{F_k} & N_{G_k} \\ N_{H_k} & 0 \end{bmatrix}, \quad (22) \]
where $\Delta_1$ and $\Delta_2$ are random terms whose norms are given by $||\Delta|| < 1$. The uncertainties (22) allow restricting the sources of distortion by selecting appropriate entries of $\{M_{1k}, M_{2k}\}$ and $\{N_{F_k}, N_{G_k}, N_{H_k}\}$. This filter attempts to minimize the estimation error in the worst possible case created by the bounded uncertainties $\delta F_k$, $\delta G_k$, and $\delta H_k$. The robust recursive Kalman filter estimates and the corresponding recursive Ricatti equation are given by (23), (24), respectively, and
\[ \begin{bmatrix} \hat{x}_{k+1|k} \\ F_k \hat{x}_k \end{bmatrix} = \begin{bmatrix} F_k \\ H_k \end{bmatrix}, \quad \begin{bmatrix} \eta_k \\ \bar{\eta}_k \end{bmatrix} = \begin{bmatrix} [0 \ 0 \ 1] \\ [-I \ 0] \end{bmatrix}, \quad \begin{bmatrix} b_k \\ \bar{b}_k \end{bmatrix} = \begin{bmatrix} [0 \ 0 \ 0] \\ [0 \ 0 \ 0] \end{bmatrix}, \quad \begin{bmatrix} R_k \\ \bar{R}_k \end{bmatrix} = \begin{bmatrix} [M_{1k} \ 0] \\ [0 \ M_{2k}] \end{bmatrix}, \quad \lambda_k = (1 + \alpha t) ||M_k^T M_k|| (\alpha t > 0). \quad (25) \]

Remark 1. We can consider $\mu \to \infty$ and, in consequence, $\lambda_k \to \infty$ (Terra et al. (2011)). Note that this filter depends
on the inverses of these variables which go to zero. In virtue that we are considering in this paper an extended version of this filter to deal with nonlinear systems, μ can be considered as great as possible. The robust nature, the stability and convergence of this filter remains valid in this case.

6. EXPERIMENTAL RESULTS

The experimental results were obtained through a six-degree of freedom inertial measurement unit (IMU) produced by the SparkFun Electronics, shown in Figure 2. This IMU is composed of a 3-axial accelerometer, a 3-axial gyroscope, and a 3-axial magnetometer. In order to calibrate this IMU, the noise parameters of the model presented in Section 2 were identified based on the approach proposed in Xing and Gebre-Egziabher (2008). These parameters are shown in Table 1.

Table 1. Noise parameters of the sensors.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{x}^{2}$</th>
<th>$\sigma_{y}^{2}$</th>
<th>$\sigma_{z}^{2}$</th>
<th>$\tau_{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyro</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>0.1105 (°/s)$^2$</td>
<td>12.6653 (°/s)$^2$</td>
<td>1759.07 s</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>0.0760 (°/s)$^2$</td>
<td>14.7890 (°/s)$^2$</td>
<td>1621.95 s</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>0.1170 (°/s)$^2$</td>
<td>0.5600 (°/s)$^2$</td>
<td>1093.75 s</td>
<td></td>
</tr>
<tr>
<td>Accel.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>0.0003 (m/s$^2$)$^2$</td>
<td>$x$</td>
<td>0.000095 G$^2$</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>0.0100 (m/s$^2$)$^2$</td>
<td>$y$</td>
<td>0.000098 G$^2$</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>0.0050 (m/s$^2$)$^2$</td>
<td>$z$</td>
<td>0.000098 G$^2$</td>
<td></td>
</tr>
</tbody>
</table>

The reference angle for the comparative study performed was obtained through an encoder fixed to the IMU, shown in Figure 2. The sensor output data were obtained through a standard serial port RS-232 with a frequency of 25 Hz. The filters were run in MATLAB®.

Throughout the time period of operation, the system maintains a flag indicating true (i.e.,1) if the vehicle is considered to be non-accelerating and false (i.e.,0) otherwise, see Farrel (2008) for more details. At time $t$, based on the IMU data, this flag is defined to be true when the logical condition

$$\mu(t) < \beta_{\mu} \text{ and } (\|\mathbf{u}_k\| < \beta_{\mathbf{u}}) \text{ and } (\mu_f(t) < \beta_f)$$

(26)

is true. Signal $\mu(t)$ is defined by $\mu(t) = \|a_v - g_e\|$, and parameters $\beta_{\mu}$, $\beta_{g}$, and $\beta_f$ were adjusted as 2, 4, and 1.2, respectively.

When the vehicle is accelerating, only the magnetometer signals are used in the measured output. A block diagram of the implemented attitude system is shown in Figure 3.

![Fig. 2. Inertial measurement unit with encoder.](image)

![Fig. 3. Block diagram of the attitude system.](image)

6.1 Extended Kalman Filter

The weighting matrices, Q and R, were chosen as

$$Q_k = \begin{bmatrix} Q_{x} & 0_{3 \times 3} \\ 0_{3 \times 3} & Q_{bg} \end{bmatrix}, R = \begin{bmatrix} R_{m} & 0_{3 \times 3} \\ 0_{3 \times 3} & R_{a} \end{bmatrix},$$

$$Q_{x} = \text{diag}(\sigma_{x}^{2}, \sigma_{y}^{2}, \sigma_{z}^{2}), Q_{bg} = \text{diag}(\sigma_{bg,x}^{2}, \sigma_{bg,y}^{2}, \sigma_{bg,z}^{2}),$$

$$R_{m} = \text{diag}(\sigma_{m,x}^{2}, \sigma_{m,y}^{2}, \sigma_{m,z}^{2}), R_{a} = \text{diag}(\sigma_{a,x}^{2}, \sigma_{a,y}^{2}, \sigma_{a,z}^{2}).$$

The values of these variances are presented in Table 1.
6.2 $\mathcal{H}_\infty$ filter

The weighting matrices, $Q$ and $R$, are the same matrices used in the EKF filter and $\gamma = 0.1$.

6.3 Robust recursive Kalman filter

The weighting matrices, $Q$ and $R$, are also the same defined for the EKF and the remaining matrices were considered as:

\[
N_{F_k} = 10^2 [f_1 \cdots f_7], \\
N_{G_k} = 10^2 [g_1 \cdots g_3 0 0 0], \\
N_{H_k} = [h_1 \cdots h_7], \\
M_{1k} = 10^{-3} [1 1 1 1 0 0 0]^T, \\
M_{2k} = 10^{-3} [1 1 1 1 1 1]^T, \\
\mu = 450000, \; \alpha_f = 1,
\]

where

\[
\begin{align*}
    f_l &= \frac{\sum_{i=1}^{4} |F_k(i,l)|}{4} \text{ with } l = 1, 2, \ldots, 7, \\
    g_l &= \frac{\sum_{i=1}^{4} |G_k(i,l)|}{4} \text{ with } l = 1, 2, 3, \\
    h_l &= \frac{\sum_{i=1}^{6} |H_k(i,l)|}{6} \text{ with } l = 1, 2, \ldots, 7, \text{ and} \\
    T_k &= F_k - I_{7\times7}.
\end{align*}
\]

6.4 Comparative study

A comparative study among the EKF, the $\mathcal{H}_\infty$ filter, and the robust recursive Kalman filter proposed, was performed based on the mean of 10 consecutive data samples of each Euler angles, roll ($\phi$), pitch ($\theta$), and yaw ($\psi$). For each sample, the accuracy of the sensor measurements depends on the temperature, uncertainties and disturbances were added to the gyroscope signals to simulate static and abrupt changes of temperature in the sensor as:

\[
\omega = (1 + c)\omega_k + d_s - b_s,
\]

where $c = -0.1, -0.05, 0, 0.05, 0.1$, $d_s = \begin{bmatrix} d_{s_x} & d_{s_y} & d_{s_z} \end{bmatrix}^T$, shown in Figure 4, and

\[
d_{s_x, s_z} = 5e^{-(t-t_0)^3} \sin(1.3\pi t),
\]

where $t_d = 10s$ for $d_{s_x}$, $t_d = 20s$ for $d_{s_y}$, and $t_d = 30s$ for $d_{s_z}$. The values of parameter $c$ were chosen based on the datasheet of the gyroscope IDG-300 that composes the IMU.

The $L_2$ norms of the angle estimate errors were used to compare the performance of the respective filters,

\[
L_2[\hat{\rho}(t)] = \left( \frac{1}{(t_f - t_0)} \int_{t_0}^{t_f} \|\hat{\rho}(t)\|^2_2 dt \right)^{\frac{1}{2}},
\]

where $\|\cdot\|_2$ is the Euclidean norm, $\hat{\rho}(t)$ is the angle estimate error at time $t$, $t_0 = 0$, and $t_f = 120$ s is the experimental time. Table 2 shows the performance of the robust filters over the extended Kalman filter based on (30).

As expected, the robust recursive Kalman filter presented better results than the extended Kalman filter and the $\mathcal{H}_\infty$ filter. The robust recursive Kalman filter improved almost 8% the performance of the extended Kalman filter (EKF). Figures 5a, 5b, and 5c show the attitude estimates obtained from the filters considering the coordinate system of Figure 1. It is interesting to observe in these figures that the estimate of the angles deteriorates when the effect of the disturbances (29) takes place. We can see more details of this effect at Figure 6. They show a zoom of the estimates at the interval (29s to 31s), when the estimate of the angles are under the effect of the disturbance $d_{s_z}$.

Remark 2. The robust filters Sayed (2001) and Ishihara et al. (2006) were not considered in this comparative study because they do not consider the uncertain terms $\delta H$ and $\delta G$, respectively, in the system (18).

7. CONCLUSION

This paper developed an attitude reference system based on a new robust recursive Kalman filter for determination of a vehicle attitude based on experimental results. This filter attempts to limit the effects of the model uncertainties and disturbances on the Euler angles estimates. It is based on penalty game approach and it assumes that there exist uncertainties in all parameter matrices of the model. A comparative study was performed in order to show the advantages of the proposed approach. The overall performance of the robust recursive Kalman filter, if compared with the EKF, was almost 8%.
Fig. 5. (a) Roll $\phi$, (b) Pitch $\theta$, and (c) Yaw $\psi$ estimates for $c = 1$.

Fig. 6. (a) Roll $\phi$, (b) Pitch $\theta$, and (c) Yaw $\psi$ estimates at the interval (29s to 31s) for $c = 1$.

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