Simultaneous Roll Damping and Course Keeping via Sliding Mode Control for a Marine Vessel in Seaway

C. Carletti * A. Gasparri ** S. Longhi * G. Ulivi **

* Dipartimento di Ingegneria Informatica, Gestionale e dell’Automazione, Università Politecnica delle Marche, Ancona, Italy (e-mail: carletti@diiga.univpm.it, longhi@univpm.it).
** Dipartimento di Informatica e Automazione, Università RomaTre, Roma, Italy (e-mail: gasparri@dia.uniroma3.it )

Abstract: In this paper the simultaneous roll reduction and course keeping problem for a surface vessel by means of active fins and rudder is addressed. This work extends the results proposed in [Carletti et al. (2010)] by considering an alternative nonlinear multivariable ship dynamics where the wave effects are modeled as forces and moments affecting the system dynamics rather than as disturbances affecting the control inputs. A theoretical analysis of the boundedness of the zero dynamics along with the design of a sliding mode control are proposed. Simulations results are provided to corroborate the theoretical analysis.

1. INTRODUCTION

Roll damping and course keeping are two important control problems in the marine engineering. Indeed, roll oscillations induced by waves significantly affect the comfort of crew and passengers and increase the possibility of cargo and on-board equipment damage. In addition, tracking a predefined, presumably optimum, path allows to minimize time and cost of the course. These problems have been deeply investigated in literature: active tanks, gyroscopes, active fins represent some of the most popular solutions for the roll damping [Perez (2005)]; PID, linear quadratic optimal control, state feedback linearization, SISO sliding mode control, nonlinear back-stepping are some of the control techniques developed for the course autopilot [Fossen (2002)]. Nonetheless, the roll damping and the course keeping should not be considered separately. In fact, when the rudder is used to turn the ship, at the same time, a roll moment is created. In the same way, a steering motion can arise from the active fins if these are located aft of the Center of Gravity (CG). This coupling between the roll and yaw dynamics is particularly relevant for slender and relatively fast monohull, e.g. motor-yachts and patrol vessels, as the one considered in this paper. The Rudder Roll Damping (RRD) technique exploits this rudder double effect to perform the roll motion damping simultaneously with the ship course control. The cost-effectiveness of this approach compared with active fins solutions has made it very popular. Unfortunately, roll damping by rudder is not a straightforward control problem due, above all, to the nonlinear coupling in the roll-yaw dynamics [Blanke and Christensen (1993)] and the unstable zero dynamics in the rudder-roll system [Fossen and Lauvdal (1994)]. Moreover, the effectiveness of RRD control in roll motion reduction has been debated [Stoustrup et al. (1995) and references therein]. The roll damping can be improved by introducing the active fins in the control system. Despite of their cost and resistance, Fins Roll Damping (FRD) is largely utilized in surface vessels due to their effectiveness in roll reduction. In the literature, several works have been proposed for the roll motion reduction by means of both fins and rudder. However, the majority of these approaches, e.g., [Roberts et al. (1997)], [Katebi (2004)] and [Koshkouei et al. (2007)], are based on a linear model to describe the ship dynamics, and therefore significant nonlinear effects, such as the instability of the zero dynamics, are neglected. Furthermore, the proposed solutions are based on independent FRD and RRD controllers, which do not consider their mutual effect on the roll angle. In [Carletti et al. (2010)] an attempt to take into account the nonlinear coupling between yaw and roll dynamics for simultaneous roll reduction and course keeping has been proposed. The twofold control objective was obtained by means of a MIMO variable structure control law using together fins and rudder, with good performances in different environments. This work represents an extension of the result proposed in [Carletti et al. (2010)]. The novelty is the adoption of a more realistic nonlinear multivariable ship dynamics where the wave effects are modeled as forces and moments affecting the system dynamics rather than as disturbances affecting the control inputs.

2. NONLINEAR SHIP MODEL

The control design deals with a nonlinear ship model affine in the inputs and disturbances of the form:
\[
\dot{x} = f(x) + g(x) \cdot u + d(x) \cdot w \\
y = h(x)
\]
where \( x \in \mathbb{R}^5 \) is the state vector, \( y \in \mathbb{R}^2 \) is the output and \( u \in \mathbb{R}^2 \) is the input, while \( w \in \mathbb{R}^3 \) represents the external disturbances. This mathematical model is obtained by combining a ship manoeuvring model along with the waves-induced forces/moments obtained from the seakeeping theory. This allows to have a realistic description of...
a steering vessel in seaway [Perez (2005)]. The reference vessel is a small relatively fast monohull whose principal characteristic parameters are available in [Perez et al. (2006)] and for which a suitable set of manoeuvring coefficients was published in [Blanke and Christensen (1993)]. The ship dynamics has been implemented by using the Marine System Simulator (MSS) Matlab toolbox developed by the Norwegian University of Science and Technology (NTNU)[MSS (2010)].

The ship dynamics in seaway can be described in several ways. The ship motion in seaway can be described in several ways. Moreover, the wave-induced loads can be completely described by their Power Density Spectra (PSD), Pni(ω), which is the unitary PSD of ni. The filters Hi(s) are usually represented as a 2nd order transfer function:

\[
\tau_{cs} = \tau_{rud} + \tau_{fins},
\]

where \( \tau_{cs} \) is the total vector of forces and moments acting on the ship and \( \tau_{fins} \) is the force due to the control surfaces while \( \tau_{prop} \) is the propulsion forces and moments.

The nonlinear hydrodynamic term has been modeled following the approach given in [Blanke and Christensen (1993)] and using the coefficients released therein. The control module \( \tau_{cs} \) takes into account the effects on the ship motion due to the control surfaces, namely rudder \( (\tau_{rud}) \) and active fins \( (\tau_{fins}) \). In particular, it can be detailed as follows:

\[
\tau_{cs} = \tau_{rud} + \tau_{fins}.
\]

In order to have a dynamic model affine in the inputs, the rudder mechanical angle \( \delta_{rudder} \) and the force \( F_{fins} \) have been chosen as control variables. In particular, \( F_{fins} \) represents the resulting hydrodynamic force component normal to the fins, defined as a function of the lift \( L_{fins} \) and drag force \( D_{fins} \) arising on them as \( L_{fins} \cos(\alpha) + D_{fins} \sin(\alpha) \) with \( \alpha \) the fins effective angle of attack between the foil and the incoming flow. As a result, the control module for the rudder is:

\[
\tau_{rud} \approx \left[ \begin{array}{c}
L_{rud} \\
-R_{rud} + L_{rud} \\
-LCG \cdot L_{rud}
\end{array} \right] \approx \left[ \begin{array}{c}
|\tau_{rud} \cdot \gamma_{rud}|
\end{array} \right] \cdot \delta_{rudder}
\]

where \( L_{rud} \approx \gamma_{rud} \delta_{rudder} \) is the lift force acting on the rudder, while \( R_{rud} \) and \( LCG \) are the distance between the rudder Center of Pressure \( CP \) and the ship \( CG \) along the z-axis and x-axis of the b-frame respectively. For the fins module, the control for the fins is:

\[
\tau_{fins} \approx \left[ \begin{array}{c}
-R_{fins} \\
2 \cdot \sin(\beta_{tilt})
\end{array} \right] \cdot F_{fins}
\]

where \( R_{fins} \) is the fins roll arm (the distance of the fin \( CP \) from the ship \( CG \)) and \( F_{fins} \) is the longitudinal distance between the fin \( CP \) and the ship \( CG \), while \( \beta_{tilt} \) represents the fins tilt angle. Further details can be found in [Perez (2005)].

Note that, the propulsion component \( \tau_{prop} \) will be neglected in the rest of the paper as a consequence of the constant ship cruise speed assumption \( (\nu = U) \). In fact, this implies that the propeller effect is completely compensated by the hull hydrodynamic resistance.

2.2 Disturbance Modeling

The ship motion in seaway can be described in several ways [Perez and Blanke (2002)]. A common framework for the design of control algorithms is based on the linear waves theory and assumes the sea elevation at a certain location to be represented by means of a zero mean Gaussian stochastic process. As a consequence, the excitation loads due to the regular \( (i.e., \text{random}, \text{const.}) \) and irregular \( (\text{random}, \text{both in time and space}) \) sea are described as summation of several regular waves responses. Moreover, the wave-induced loads can be completely described by their Power Density Spectra (PSD), and thus characterized in the frequency domain. As a result, the wave effect on the ship dynamics is described as an additive disturbance input \( w \) in the nonlinear model of eq. (1). Let us assume \( w = [w_1 w_2 w_3]^T \) to be the 3DOF vector representing the 1st order wave forces and moments acting on the vessel. In the Laplace domain, each component \( w_i \) can be considered as the output of a white Gaussian noise \( \eta_i \), filtered by a linear filter \( H_i(s) \):

\[
w_i(s) = H_i(s) \cdot \eta_i(s), \quad i = 1, 2, 3.
\]

Indeed, the PSD for \( w_i(s) \) is:

\[
P_{w_i}(\omega) = |H_i(\omega)|^2 P_{\eta_i}(\omega), \quad i = 1, 2, 3.
\]

with \( P_{\eta_i}(\omega) \) the unitary PSD of \( \eta_i \). The filters \( H_i(s) \) are usually represented as a 2nd order transfer function:
with $\lambda_{wi}$ the damping coefficient, $\omega_{wi}$ the dominating wave frequency in the encounter frequency domain and $\sigma_i$ a constant describing the wave intensity. For each DOF, the filter parameters have been tuned by comparing the output motion due to the wave disturbance simulated by the shaping filters with the output obtained by the motion RAOs, available in [Perez (2005)].

In particular, as the ship moves ($U \neq 0$), the encounter frequency $\omega_{wi}$ is considered and computed as:

$$\omega_{wi} = \omega_{oi} - \frac{\sigma_i U}{g} \cos \chi$$

with $\omega_{oi}$ the modal frequency of $P_{wi}(\omega)$. Moreover, the constant $\sigma_i$ is computed as:

$$\sigma_i = \sqrt{\max(S_{wi}(\omega))}$$

and $\lambda_{wi}$ chosen such that $P_{wi}(\omega)$ fits $S_{wi}(\omega)$ using least-square criterion. Further details can be found in [Fossen (2002)].

### 2.3 State Space Representation

In order to obtain the ship dynamics state space representation given in eq. (1), the 3DOF manoeuvring model described in eq. (2) and the wave disturbance described in eq. (9) have been combined. In addition, the roll angle $\phi$ and the yaw angle $\psi$ have been considered as part of the state as well. Therefore, the state vector $x$ can be defined as follows:

$$x = \begin{bmatrix} v & p & r & \phi & \psi \end{bmatrix}^T \in \mathbb{R}^5, \quad (10)$$

where $\hat{\psi} = \psi - \psi_d$, with $\psi_d$ the desired yaw angle. At this point, the nonlinear ship dynamics in the augmented state space can be written as follows:

$$M \cdot \dot{x} + \hat{C}(x) \cdot x = T_{hyd}(x) + T_{ca} \cdot u + T_w \cdot w \quad (11)$$

where the input is $u = [\delta_{rud} \ N_{fins}]^T$, while the matrices $M$, $C$, $T_{ca}$, $T_w$ and the vector $T_{hyd}$ are defined as follows:

$$M = \begin{bmatrix} M_{RB} + M_A & 0 \\ 0 & I_2 \end{bmatrix}, \quad (12)$$

$$\hat{C}(x) = \begin{bmatrix} \hat{C}_{RB} & 0 \\ E(x) & 0 \end{bmatrix}, \quad E(x) = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -\cos \phi \end{bmatrix}$$

$$T_{hyd}(x) = \begin{bmatrix} \tau_{hyd}^* \\ \tau_{fins}^* \end{bmatrix}, \quad T_{ca} = \begin{bmatrix} \tau_{rud}^* \tau_{fins}^* \end{bmatrix}, \quad T_w = \begin{bmatrix} I_3 \\ 0_{2x3} \end{bmatrix}$$

with $\theta$ opportune completion matrices, $I_3$ a $k \times k$ identity matrix, $\tau_{hyd}^*$ the nonlinear hydrodynamic term in the manoeuvring model given in eq. (3) where the terms proportional to the accelerations are not considered as they have been included in the added mass matrix $M_A$, and finally $\tau_{rud}^*$, $\tau_{fins}^*$ the coefficients vectors multiplying $\delta_{rud}$ in eq. (5) and $N_{fins}$ in eq. (6), respectively. Finally, the multivariable nonlinear ship model can be written as in (1), that is repeated here for sake of clarity:

$$\dot{x} = f(x) + g(x) \cdot u + d(x) \cdot w$$

$$y = h(x) \quad (13)$$

where $x \in \mathbb{R}^5$ is the state vector, $u = [\delta_{rud} \ N_{fins}]^T \in \mathbb{R}^2$, $w = [w_1 \ w_2 \ w_3]^T \in \mathbb{R}^3$, $h(x) = [\phi \ \psi]^T \in \mathbb{R}^2$ and:

$$f(x) = -M^{-1} \cdot C(x) \cdot x + M^{-1} \cdot T_{hyd}(x)$$

$$g(x) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = M^{-1} \cdot T_{ca}$$

$$d(x) = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = M^{-1} \cdot T_w \quad (14)$$

### 3. ZERO DYNAMICS ANALYSIS

The zero dynamics describes the “internal behavior” of a system when input and initial conditions are chosen to force the output to zero [Isidori (1995)]. The zero dynamics plays an important role in the control system design. For instance, the presence of an unstable zero dynamics prevents the application of important control techniques, such as feedback linearization, and significantly affects the achievable performances, for example by imposing limitation on the maximum control gain.

#### 3.1 Normal Form Derivation

In order to analyze the multivariable nonlinear ship model, the eq. (11) is transformed to normal form. Let $\{r_1, \ldots, r_n\}$ be the vector of relative degree at a point $x^0$, i.e., the number of time the i-th output $y_i$ must be differentiated to have at least one component of the input vector $u$ explicitly appearing [Isidori (1995)]. Let us now recall the Lie derivative, i.e., the derivative of $\lambda(\cdot)$ along $f(\cdot)$, as $L_f \lambda(x) = \sum_i \frac{\partial \lambda}{\partial x^i}$. It can be shown that such a multivariable nonlinear ship model has relative degree $\{r_1, r_2\} = \{2, 2\}$ at $x^0 = [0, 0, 0, 0, 0]^T$. In fact, the Lie derivatives with respect to the input $u$ and the disturbance $w$ are respectively:

$$L_{\partial u} h_i = 0 \quad i \in \{1, 2\}, \quad j \in \{1, 2\} \quad (15)$$

and:

$$L_{\partial w} h_i = 0 \quad i \in \{1, 2, 3\}, \quad j \in \{1, 2\} \quad (16)$$

Furthermore the matrix $A(x)$ defined as:

$$A(x) = \begin{bmatrix} L_{\partial x_1} h_1(x) & L_{\partial x_2} h_1(x) & L_{\partial x_3} h_1(x) \\ L_{\partial x_1} h_2(x) & L_{\partial x_2} h_2(x) & L_{\partial x_3} h_2(x) \\ L_{\partial x_1} h_3(x) & L_{\partial x_2} h_3(x) & L_{\partial x_3} h_3(x) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cos(x_4) \\ a_{21} & a_{22} & \cos(x_4) \\ a_{31} & a_{32} & \cos(x_4) \end{bmatrix} \quad (17)$$

has full rank if the following holds:

$$a_{11} a_{22} - a_{12} a_{21} \neq 0, \quad \cos(x_4) \neq 0 \quad (19)$$

It should be noticed that while the condition given by eq. (18) is related to the vessel parameters, the second condition given by eq. (19) is always satisfied in practice as $\cos(x_4) = 0$ implies that $x_4 = \frac{\pi}{2} + k \pi$, $k \in \mathbb{N}$. Indeed, this is an inadmissible operational condition, being $x_4$ the roll angle. Thus, the system has relative degree $\{2, 2\}$ for any state $x^0 = [x_1, x_2, x_3, x_4, x_5]^T$ with $x_4 \neq \frac{\pi}{2} + k \pi$, $k \in \mathbb{N}$.

Therefore, according to [Isidori (1995)], the nonlinear system describing the vessel can be transformed to normal form by applying the following local coordinates transformation $z(x) = [z_1(x), z_2(x), z_3(x), z_4(x), z_5(x)]^T$ at $x^0$.

In particular, the following choice is possible for the first $r_1 + r_2 = 4$ variables:
\[ \xi_1 = \begin{pmatrix} \xi_1^1 \\ \xi_2^1 \end{pmatrix} = \begin{pmatrix} z_1(x) \\ z_2(x) \end{pmatrix} = \begin{pmatrix} h_1(x) \\ L_1 h_1(x) \end{pmatrix} \] (20)

\[ \xi_2 = \begin{pmatrix} \xi_1^2 \\ \xi_2^2 \end{pmatrix} = \begin{pmatrix} z_3(x) \\ z_4(x) \end{pmatrix} = \begin{pmatrix} h_2(x) \\ L_2 h_2(x) \end{pmatrix} \] (21)

while the remaining \((n-r_1-r_2=1)\) variable transformation can be chosen arbitrarily:

\[ \eta = z_5(x) \] (22)

so that the Jacobian matrix describing the local coordinates transformation \(z(x)\) is nonsingular at \(x^0\). Furthermore, since \(\mathcal{G} = \text{span}(g)\) is an involutive set, \(z_5(x)\) can be chosen so that [Isidori (1995)]:

\[ L_{y_1} z_5(x) = 0 \quad i \in \{1,2\}. \] (23)

with \(g = [g_1 \ g_2]\).

Note that, the constraint given by eq. (23) can be satisfied by assuming:

\[ \eta = z_5(x) = \alpha + \beta \eta + \gamma \tau \] (24)

with:

\[ [\alpha, \beta, \gamma, 0, 0] \in \ker(g^T). \] (25)

In particular, since \(g^T\) has the following structure:

\[ g^T = \begin{pmatrix} g_11 & g_12 & g_{13} & 0 & 0 \\ g_21 & g_22 & g_{23} & 0 & 0 \end{pmatrix} \] (26)

a base for the null space is:

\[ \ker(g^T) = \begin{pmatrix} 0 & 0 & g_{12}g_{23} - g_13g_{22} \\ 0 & 0 & g_{12}g_{21} - g_{11}g_{22} \\ 0 & 0 & g_{12}g_{13} - g_13g_{12} \\ 0 & 0 & g_{11}g_{22} - g_{12}g_{21} \\ 1 & 0 & 0 \end{pmatrix} \] (27)

Therefore, by choosing:

\[ \begin{align*}
\alpha &= \frac{g_{12}g_{21} - g_13g_{12}}{g_{12}g_{21} - g_{11}g_{21}} \\
\beta &= \frac{g_{12}g_{23} - g_13g_{22}}{g_{12}g_{21} - g_{11}g_{21}} \\
\gamma &= \frac{g_{11}g_{22} - g_{12}g_{21}}{1}
\end{align*} \] (28)

the Lie Derivatives turn out to be:

\[ \begin{align*}
L_{g_1} z_5(x) &= \alpha g_11 + \beta g_12 + \gamma g_{13}, \\
L_{g_2} z_5(x) &= \alpha g_21 + \beta g_22 + \gamma g_{23}
\end{align*} \] (29, 30)

which can be re-written in a matrix form as follows:

\[ \begin{pmatrix} g_{11} & g_{12} & g_{13} & 0 & 0 \\ g_{21} & g_{22} & g_{23} & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \] (31)

being the vector \([\alpha \beta \gamma 0 0]^T\) an element of the null space base for \(g^T\).

As a result, the ship dynamics of eq. (1) can be written in normal form as follows:

\[ \begin{align*}
\xi_1^1 &= \xi_2^1 \\
\xi_2^1 &= F_1(\xi, \eta) + \sum_{j=1}^{2} G_{1j} \cdot u_j + \sum_{j=1}^{3} D_{1j} \cdot w_j \\
\xi_2^2 &= F_2(\xi, \eta) + \sum_{j=1}^{2} G_{2j} \cdot u_j + \sum_{j=1}^{3} D_{2j} \cdot w_j \\
\eta &= q(\xi, \eta) + n(\xi, \eta) \cdot w
\end{align*} \] (32)

where \(F = R^{2 \times 1}\) with \(F_1 = L_{T_{h_{1}}} h_{1}\), \(G \in R^{2 \times 2}\) with \(G_{ij} = L_{A_{ij}} L_{T_{h_{1}}} h_{1}\) and \(D \in R^{2 \times 3}\) with \(D_{ij} = L_{A_{ij}} L_{T_{h_{1}}} h_{1}\).

### 3.2 Stability Analysis

Let us now investigate the properties of the zero dynamics \(\eta = g(0, \eta) + n(0, \eta) \cdot w\), which can be detailed as follows:

\[ \eta = -a_1 |\eta| |\eta| - b_1 |\theta_1| + c_1 \theta_2 + \gamma \tau \] (33)

where \(a, b, a_1, b_1, c_1, \gamma \in \mathbb{R}\) all non-negative real constants. In order to investigate the stability of the zero dynamics, let us consider the following Lyapunov candidate \(V(\eta) = \frac{1}{2} \eta^2\). In addition, let us assume the disturbances to be bounded \(|w_0(t)| \leq W_i\) with \(i \in \{1,2,3\}\). For the sake of the analysis, it should be noticed that the dynamics described in eq. (33) has a forcing term, \(\alpha \eta_1 + \beta \eta_2 + \gamma \tau\), which prevents the system to have an equilibrium point at the origin \(x^0\). Nevertheless, the Lyapunov analysis can still be used, as explained in [Khalil (2002)], to prove the boundedness of the solution of the state equation. To this end, let us consider the derivative of the chosen Lyapunov candidate \(V(\eta) = \frac{1}{2} \eta^2\):

\[ V(\eta) = \dot{V}(\eta) = -a_1 |\eta| |\eta| - b_1 |\theta_1| + c_1 \theta_2 + \gamma \tau \] (34)

where \(\dot{\theta} = \dot{\alpha} + \dot{\beta} + \gamma \tau\). In particular, it can be shown that:

\[ V(\eta) > 0 \quad \text{i.e.} \quad |\eta| < |h(\dot{\eta})| \]

\[ V(\eta) < 0 \quad \text{i.e.} \quad |\eta| > |h(\dot{\eta})| \] (35)

where:

\[ h(\dot{\eta}) = \frac{b + \sqrt{b^2 + 4 a |\dot{\eta}|}}{2a} \] (36)

Therefore, by assuming \(c > \frac{h(\dot{\eta})}{2}\), any state starting in the set \(\{V(x) \leq c\}\) will remain therefor in all future time since \(V\) is negative definite on the bound \(V = c\). As a result, the solutions of the state system are uniformly bounded.

### 4. CONTROL DESIGN

#### 4.1 Sliding Mode Control

In this section, the design of a sliding mode control is discussed. The choice of a robust control technique is motivated by the fact that modeling simplifications, ship parameters variability and their coupling effects between roll and sway-yaw dynamics can significantly affect the overall performance of the control systems. In addition, it should be noticed that the boundedness of the zero dynamics, discussed in Section 3.2, does not introduce any particular limitation to the application of such a control technique.

The control problem is to drive the roll angle \(\psi\) to zero and the yaw angle \(\psi\) to a specific value \(\psi_0\) or, equivalently, to drive the variable \(\varphi = \psi - \psi_0\) to zero. The control inputs are the rudder angle \(\delta_{rudd}\) and the normal force \(N_{\text{fins}}\) arising on the fins. The choice of \(N_{\text{fins}}\) instead of the fins angle \(\theta_{\text{fins}}\) is due to the constrain of affine input required by the sliding mode control technique.

Let us consider the multivariable nonlinear ship model given in eq. (32) and let us assume the disturbance \(w\) to be bounded by a known bound \(W = [W_1 \ W_2 \ W_3]\), i.e., \(|w_0| < W_1, i \in \{1,2,3\}\). Furthermore, let us consider \(\Delta F\) and \(\Delta G\) to be modeling uncertainties of \(F\) and \(G\) respectively with known bounds, as described in the following equations:

\[ \Delta F = F - \tilde{F} \quad \Delta G = G - \tilde{G} \] (37)

with \(\tilde{F}\) and \(\tilde{G}\) the nominal models.

Then, the following two sliding surfaces can be defined:

\[ s_1 = \xi_1^1 + \lambda_1 \xi_1^1 = 0, \quad \lambda_1 > 0 \]

\[ s_2 = \xi_2^1 + \lambda_2 \xi_2^1 = 0, \quad \lambda_2 > 0 \] (38)

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and the Lyapunov candidate $V(s) = \frac{1}{2} s^T s$ with $s = [s_1 \ s_2]^T$. At this point, the following control law is considered:

$$
\begin{bmatrix}
u_1 \\
u_2 
\end{bmatrix} = \mathcal{G}^{-1} \begin{bmatrix}
\hat{F}_1(\xi, \eta) - \lambda_1 \xi_1 - k_1(\xi, \eta) \text{sign}(s_1) \\
\hat{F}_2(\xi, \eta) - \lambda_2 \xi_2 - k_2(\xi, \eta) \text{sign}(s_2) 
\end{bmatrix}
$$

(39)

where the invertibility of $\mathcal{G}$ is a consequence of the ship dynamic model, and $k = [k_1 \ k_2]^T$ is a gain vector required to guarantee the negative definiteness of $V(s)$. As far as the implementation of the control law is concerned, the function $\text{sign}(s)$ has been replaced by the continuos function $\tanh(\epsilon \cdot s)$, $\epsilon \in \mathbb{R}$. This allows to significantly reduce the chattering arising from the sliding mode control without affecting the stability analysis.

### 4.2 Case Study

In order to prove the effectiveness of the proposed control technique a case study is described. The ship parameters used for the nonlinear ship model described in eq. (1) can be found in [Perez (2005)].

Limitations in magnitude and rate for the control variables due to the control surfaces hydraulic machinery are introduced within the simulation model. This has been achieved by using the Simulink blocks implementing the hydroservos model proposed in [Amerongen (1982)]. In particular, the rudder angle saturation is set to $20^\circ$ while the rudder speed is limited to $20\text{deg/s}$. The saturations values for $N_\text{rudder}$ and $N_\text{rudder}$ are computed by considering a fins maximum angle of $28.8\text{deg}$ (fins static stall angle) and a fins maximum angle rate equal to $23\text{deg/s}$, that is a maximum fins force of $10^7 N$ and a maximum fins force rate equal to $2 \cdot 10^5 N/s$, respectively.

Furthermore, the proposed control system has been tested with the ship cruise speed equal to $7m/s$, and against different sailing conditions. In particular, a sea state 3 with $H_s = 0.3m$ and $T = 6s$ and a sea state 5 with $H_s = 2.5m$ and $T = 7.5s$, where $H_s$ is the significant wave height and $T$ is the average wave period, have been simulated. In addition, for each sea state, quartering, beam and bow seas have been considered.

As far as the effectiveness of the integrated control system for roll damping is concerned, the percentage Reduction Statistic of Roll (RSR), defined as $RSR = 100 \left(1 - \frac{\chi_e}{\chi_e}\right)$, is considered, where the subscripts $c$ and $u$ stand for controlled and uncontrolled respectively, and $S$ is the Root Mean Square (RMS) of the roll signal.

Figure 2 describes the roll and yaw angles for a simulation with the following operative condition: sea state 5 with an encounter angle $\chi_e = 130\text{deg}$. In particular, the sliding mode control is not activated the first $300s$ and is turned on after $300s$ for the rest of the simulation. The yaw reference angle changes from $0\text{deg}$ to $15\text{deg}$ at $t = 650s$. It should be noticed that the roll motion is significantly reduced while the yaw angle is promptly regulated to the desired reference after the sliding mode control is activated.

Figure 3 and Figure 4 describe the control inputs (rudder on the left, fins on the right) efforts in magnitude and velocity respectively. It can be noticed that the rudder angle and the fins force are significantly below the saturation thresholds imposed by their corresponding mechanical systems ($20\text{deg}$ for the rudder angle, $10^7 N$ for the fins force). Furthermore, the same holds for the rudder angle rate (saturation value: $20\text{deg/s}$) and fins rate (saturation value: $2 \cdot 10^5 N/s$). Indeed, this is extremely important for the life of the mechanical system.

Table 1 and Table 2 give a synoptic overview of the results obtained for the two sea states with respect to different encounter angles $\chi_e$, namely $30\text{deg}$, $90\text{deg}$ and $130\text{deg}$. In particular, it can be noticed that a substantial reduction of the roll damping is achieved for both slight and rough sea conditions. Again, it is worthy to notice that the control inputs required to obtain these performances never reach the saturation limits, both in amplitude and rate. The following controller parameters have been used for the simulations: $(\lambda_1, \lambda_2) = \{0.3, 0.3\}$, $(k_1, k_2) = \{0.2, 0.004\}$ and $(\epsilon_1, \epsilon_2) = \{50, 50\}$.

<table>
<thead>
<tr>
<th>Sea State</th>
<th>$\chi_e = 30^\circ$</th>
<th>$\chi_e = 90^\circ$</th>
<th>$\chi_e = 130^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSR(\phi)</td>
<td>92.8895</td>
<td>98.4939</td>
<td>93.1738</td>
</tr>
<tr>
<td>RSR(\phi)</td>
<td>95.8686</td>
<td>98.2842</td>
<td>98.7056</td>
</tr>
</tbody>
</table>

Table 1. Control system performances for the vessel sailing with $U = 7m/s$ in slight waves.

<table>
<thead>
<tr>
<th>Sea State</th>
<th>$\chi_e = 30^\circ$</th>
<th>$\chi_e = 90^\circ$</th>
<th>$\chi_e = 130^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSR(\phi)</td>
<td>92.5472</td>
<td>96.5104</td>
<td>96.5854</td>
</tr>
<tr>
<td>RSR(\phi)</td>
<td>95.6688</td>
<td>96.4392</td>
<td>95.6200</td>
</tr>
</tbody>
</table>

Table 2. Control system performances for the vessel sailing with $U = 7m/s$ in rough waves.

As far as a comparative analysis is concerned, to the best of the authors knowledge, all the contributions available in the literature rely on independent FRD and RRD controllers under the assumption of decoupled roll and yaw dynamics. In this work, on the one hand the decoupling assumption between roll and yaw dynamics is released, on the other hand, an integrated fins-rudder control law which takes into account the mutual interaction of the two actuators over each one of those dynamics is designed. Nevertheless, the control system proposed in Lauvval and Fossen (1997) and Koshkouei et al. (2007) can be considered for comparison. In particular, compared to these works, our approach turns out to be more effective in terms of roll reduction and control efforts.

Finally, Figure 5 describes the evolution of the zero dynamics. According to the theoretical analysis, the zero dynamics $\dot{\eta} = q(0, \eta)$ is bounded within $\pm 8$. Note that, the peak at time $t = 650s$ is due to the change of the yaw angle reference from $0\text{deg}$ to $15\text{deg}$.

### 5. CONCLUSIONS

In this paper, the problem of simultaneous roll reduction and course keeping for a marine vessel by means of active fins and rudder has been addressed. This work extends the results proposed in [Carletti et al. (2010)] by considering a more realistic nonlinear multivariable ship dynamics where the wave-effects are modeled as forces and moments affecting the system dynamics rather than as disturbances affecting the control inputs. A theoretical analysis of the boundedness of the zero dynamics along with the design of a sliding mode control have been proposed. Simulations carried out by exploiting the Marine System Simulator (MSS) have shown the effectiveness of the proposed control system.
Fig. 2. Roll and Yaw angles obtained during a 1000 seconds simulation in sea state 5 - $\chi_c = 130$deg conditions. The controller action starts at $t = 300s$. The reference yaw angle is changed from 0deg to 15deg at $t = 650s$.

Fig. 3. Rudder and fins efforts during the simulations in sea state 5 - $\chi_c = 130$deg conditions. The controller action starts at $t = 300s$. The reference yaw angle is changed from 0deg to 15deg at $t = 650s$. The values for the rudder angle $\delta$ and the fins force $N_{\text{fins}}$ do not ever achieve their saturation values, i.e. 20deg and $10^5N$ respectively, and a good gap from those is kept.

Fig. 4. Rudder and fins force rate during the simulations in sea state 5 - $\chi_c = 130$deg conditions. The values for the rudder angle rate $\dot{\delta}$ and the fins force rate $\dot{N}_{\text{fins}}$ do not ever achieve their saturation values, i.e. $20deg/s$ and $2 \cdot 10^5N/s$ respectively.

Future work will be focused on an experimental validation of the proposed control system to validate its effectiveness in a real environment.

Fig. 5. Zero Dynamics evolution. Note that, the zero dynamics is bounded within $\pm 8$. The peak at time $t = 650$ is a consequence of the yaw reference angle change.

REFERENCES


