Differential Estimators for State Observers in Vehicle Dynamics: HOSM and ALIEN

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Abstract: In this paper we compare robust differentiators to design observers to estimate the system state vector (HOSMO and ALIEN). We use for validation a driving simulator developed by Oktal. Robust partial state observers are developed for estimation using High Order Sliding Mode Observers and Algebraic approach (ALIEN∗).

Keywords: Vehicles Dynamics; High Order Sliding Modes Observers; Robust Nonlinear Observers; Robust Estimation; Differential Estimators, ALIEN Estimators.

1. INTRODUCTION

Several reasons leading car accidents can be avoided using embedded systems in recent vehicles. Such situations may appear when the vehicle is driven beyond the adherence or stability limits. However active safety systems, developed recently and installed on vehicles, improve the controllability by real-time monitoring and controlling the dynamic stability (EBS, ABS, ESP). The active safety is an important research on Intelligent Transportation Systems (ITS) technology. Nevertheless, the possibility of rectifying an unstable condition can be compromised by physical limits. Therefore, it is extremely important to detect (on time) a tendency towards instability. This has to be done without adding expensive sensors, so it requires quite robust observers.

Many research, in the previous works, deal vehicle modeling and analyzing (Ramirez Mendoza [1997]), state estimation and behavior analysis (Meizel [2001]). Recently, many studies have been performed on estimation of the frictions and contact forces between tires and road (Gustafsson [1997]). State estimation and deduction of contact forces have been considered in (Ray [1997]) using statistic prediction. Tire forces can be represented by the nonlinear (stochastic) functions of wheel slip. An estimation based on least squares method and Kalman filtering is applied for estimation of contact forces. In (Gustafsson [1997]) presented a tire/road friction estimation method based on Kalman filter to give a relevant estimates of the slope of µ versus slip (λ), that is, the relative difference in wheel velocity. The paper (Carlson [2004]) presented an estimator for longitudinal stiffness and wheel effective radius using vehicle sensors and Global Positioning System (GPS) for low values of slip. In (Carlson [2004]) an estimation based on least squares method and Kalman filtering is applied for contact forces estimation. The contact forces are assumed slow enough to permit adaptation of linearized contact interaction model and step by step estimations are developed. In (Drakunov [1992], de Wit and Tsotras [1999]) application of sliding mode control is proposed. Observers based on the sliding mode approach have been also used in Rabhi et al. [2010]. The work in (M’sirdi et al. [2004]) is focused on estimation of the tires slip, adherence, stiffness and effective radius. The tire forces are then identified after. The contribution is the robust on-line estimation of the tire effective radius, wheel slip and velocities by using only simple low cost sensors (ABS sensors) and vehicle velocity. The state estimation can be used to estimate a critical driving situation to improve the security (M’sirdi et al., M’sirdi et al. [2006b]).

Acceleration and braking maneuvers modify the wheel slip. This phenomenon could be controlled by means of its regulation while using sliding mode approach (Utkin et al., Filippov [1988]). This methods enhances the road safety leading better vehicle adherence and maneuverability but needs sensors and robust observers. The involved problems deal, in this case, with the full system observability and controllability. Then when estimation is done step by step with a procedure involving simple models, finite time converging observers and sliding mode approach to be robust and noise free (M’sirdi et al., M’sirdi et al. [2007]). The first steps avoid lack of sensors by means of finite time converging estimations. The first step may
be estimation of successive derivatives of position measurements. The robustness of differential estimators like HOSM (High Order Sliding Mode) (Levant [1993], Levant [1998], Bartolini et al. [2003]) or Algebraic or numeric derivative estimators seems to be a good challenge. The performance of these are under investigation in our work and this paper gives the first results. Observers robust to unknown inputs are efficient for estimation of road profile and the contact forces.

![Vehicle dynamics and reference frames](image)

In what follows, after brief presentation of the vehicle model structure (see M’sirdi et al. for details), we consider estimation of the partial states (velocities and accelerations of vehicle sub models). We use the Robust Differential Estimators (RDE) (Levant [1998], Sira-Ramirez [1999], M’sirdi et al. [2007]) based on super twisting algorithm (Levant [1993]) which ensures a finite time convergence to the values of the corresponding derivatives. With the RDE, we get estimations for velocities and accelerations which can be considered as sensors measurements after a finite time.

The Algebraic Identification and Estimation Numeric method called ALIEN proposed in (Fliss [2008]) for differential estimation will be also considered (Villagra et al.) and compared to Sliding Mode Observers and High Order Sliding Mode based Robust differentiators. ALIEN algorithms also avoid the use of the system dynamic model, then robustness feature is inherent. The developed observers estimate the velocities and accelerations based on positions measurements (Villagra et al.). These observers are applied to vehicle dynamics and are compared with sliding mode observers in a realistic simulation using a 16 DoF non linear model validated for a Peugeot 406 (M’sirdi et al. [2006b]) and will be implemented to run on a driving simulator (SCaNERStudio) proposed by Oktal (www.oktal.fr).

2. VEHICLE ROAD INTERACTION MODEL

2.1 Nominal Global Dynamical Model

The dynamic equations of the vehicle dynamics have been computed using a symbolic computation software considering sixteen generalized variables: six for positions and orientations of body, four for suspensions ones, two for front wheel steering and four for wheel rotations M’sirdi et al. [2004]. The model of the vehicle is developed by assuming the car body rigid and pneumatic contact permanent (see figure (1)). The generalized coordinate vector \( \mathbf{q} \in \mathbb{R}^{16} \) is defined as in M’sirdi et al. [2007]. The vehicle motion can be described by the following passive model (see eg M’sirdi et al. for details):

\[
\tau = M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + V(q, \dot{q}) + \eta_0(t, q, \dot{q})
\]

\[
\dot{\mathbf{q}} = \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}})
\]

\[
\mathbf{v} = (K_q \mathbf{q} + K_p \dot{\mathbf{q}}) + G(q)
\]

where \( x, y, z \) represent displacements in longitudinal, lateral and vertical directions. Angles of roll, pitch and yaw are \( \theta, \varphi \) et \( \psi \) respectively. The suspension elongations are noted \( q_{3i} : (i = 1, ..., 4) \), \( \delta_i : (i = 3, 4) \) stands for the two front wheels steering angles, finally \( \varphi_i : (i = 1, ..., 4) \) are the wheels rotation angles. The vectors \( \dot{q}, \ddot{q} \in \mathbb{R}^{16} \) are velocities and corresponding accelerations respectively. The normal forces \( F_N \) (one for each wheel) are orthogonal to the road. These forces are positive if the corresponding end point of the wheel (wheel contact point) is greater than position of road \( x_{road} \) (contact with road) or zero if there is no contact with the wheel. \( F_N \) is equal to \( F_z \) only when the slope \( \beta \) and bank angle \( \gamma \) of the road are both zero.

The input torque \( \tau \) is composed by a part coming from actuators, and reaction of the road (M’sirdi et al. [2007]). In equation (2), we remark that there are inputs from driving \( \Gamma_x \) and external inputs (road reactions) given by \( J^T F \) where \( J \) is the Jacobian matrix, of dimensions 16x12, and \( F \) is the 12-dimensional input force vector acting on the wheels (longitudinal \( F_{z1} \), lateral \( F_{y1} \) and normal \( F_{y3} \), \( i = 1, ..., 4 \));

The \( M(q) \) and \( C(q, \dot{q}) \) are the inertia and the Coriolis and Centrifugal forces matrices respectively. The vector \( V(q, \dot{q}) \) summarize the suspensions and gravitation forces with \( K_v \) and \( K_p \) are respectively the damping and the stiffness matrices, \( G(q) \) is the gravity term. \( \eta_0(t, q, \dot{q}) \) represents the uncertainties and the neglected dynamics.

Model Properties The class of mechanical models we consider have several interesting properties. Equation (1) involves a passive system. Matrices \( M \) and \( C \) are such as:

1. \( M(q) \) is Symmetric Positive Definite (SPD).
2. \( N = M(q) - 2C(q, \dot{q}) \) is skewsymmetric, i.e \( \forall v \in \mathbb{R}^n \), \( v^T N v = 0 \) or \( N = -N^T \)
3. \( C(q, \dot{q}) = C(q, \dot{q}) v - \Pi(q, v) \epsilon \), avec \( \epsilon = v - \dot{q} \) et \( \Pi(q, v) = \frac{\partial}{\partial q} \{C(q, \dot{q})\} \) (see details in de Wit and Tsiotras [1999], M’sirdi et al. [2007])

In equation (2), we remark that there are control inputs from driving and external inputs coming from exchanges with environment (road reactions).

2.2 Coupled sub models

The model (1) is made of 5 sub-models corresponding respectively to chassis translations and rotations, Suspensions elongations, wheel steering and wheel rotations, corresponding to vectors \( q_1, q_2, q_3, q_4 \) and \( q_5 \) respectively.

\[
q_1^T = [x, y, z]; q_2^T = [\theta_x, \theta_y, \theta_z]; q_3^T = [q_{31}, q_{32}, q_{33}, q_{34}]
\]

\[
q_4^T = [\delta_3, \delta_4]; q_5^T = [\varphi_1, \varphi_2, \varphi_3, \varphi_4]
\]
\[ \Gamma_e = \begin{bmatrix} \mathbf{Q}_{31} & \mathbf{Q}_{32} & \mathbf{Q}_{34} & \Gamma_{e4} & \Gamma_{e5} \end{bmatrix}^T \]  
\[ V(q, \dot{q}) = \begin{bmatrix} V_1 \ V_2 \ V_3 \ V_4 \ V_5 \end{bmatrix}^T \]  
\[ \eta_i(t, x_1, x_2) = \begin{bmatrix} \eta_1 \ \eta_2 \ \eta_3 \ \eta_4 \ \eta_5 \end{bmatrix}^T \]  
\[ \mathbf{q}^T = \begin{bmatrix} q_1^T \ q_2^T \ q_3^T \ q_4^T \ q_5^T \end{bmatrix} \]  
\[ F_T = J_1^T F \quad F_R = J_2^T F \quad F_s = J_s^T F \]  
Where \( \mathbf{Q}_{ij} \) is the \( i \times j \) null matrix. The 5 sub-models are (M'sirdi et al. [2007]):

\[ \begin{align*} 
\Sigma_{11} : & \quad \dot{q}_1 = f_1(\dot{q}_2, \ddot{q}_2, F_T, V_1) + \eta_1^c \\
\Sigma_{12} : & \quad \dot{q}_2 = f_2(\dot{q}_1, \ddot{q}_1, F_R, V_2) + \eta_2^c \\
\Sigma_{2} : & \quad \dot{q}_3 = f_3(q_3, \dot{q}_3, F_s, V_3) + \eta_3^c \\
\Sigma_{31} : & \quad \dot{q}_4 = f_4(q_5, \dot{q}_4, V_4) + \eta_4^c \\
\Sigma_{32} : & \quad \dot{q}_5 = f_5(q_4, \dot{q}_5, \Gamma_{e5}) + \eta_5^c 
\end{align*} \]

The Subsystems \( \Sigma_{11} \) and \( \Sigma_{12} \) are associated to describe the chassis dynamics noted as \( \Sigma_1 \). Subsystems \( \Sigma_{31} \) and \( \Sigma_{32} \) are used to handle wheel dynamics, noted \( \Sigma_3 \).

Approximations are made when neglecting the coupling terms \( \eta_i^c \) due to connections with the other subsystems. We can verify that these \( \eta_i^c \) are bounded such as \( |\eta_i^c| < k_i \forall t, i = (1, ..., 5) \). Where \( M_{ij} \) and \( \mathbf{C}_{ij} \) \((i, j = 1, ..., 5)\) are the components of matrices \( M(q) \) and \( C(q, \dot{q}) \) respectively as detailed in (M'sirdi et al. [2007]).

Dynamics of the chassis: The dynamic of chassis translations and rotations \( \Sigma_{11} \) and \( \Sigma_{12} \) is described by:

\[ \Sigma_1 : \quad \begin{bmatrix} F_T \\ F_R \end{bmatrix} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + C_1 \dot{q}_2 + V_1 + \mu_1 \]

Where \( M_1 = (M_{11} M_{12}) \), \( C_1 = \begin{bmatrix} C_{11} \ C_{12} \end{bmatrix} \), \( \dot{J}_T = \begin{bmatrix} J_1^T \\ J_2^T \end{bmatrix} \),

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{ and } \mu_1 = -\begin{bmatrix} M_{11} & 0 \\ M_{13} & M_{22} \end{bmatrix} \begin{bmatrix} \eta_1^c \\ \eta_2^c \end{bmatrix} \]

are respectively a Positive Definite reduced inertia matrix, a reduced matrix of Coriolis and Centrifugal forces, a reduced Jacobian matrix, the vector of suspension and gravitation forces associated to both subsystems and the coupling terms due to dynamics of the other subsystems \( \Sigma_2 \) and \( \Sigma_3 \). By choosing \( x_1 = (q_1, \dot{q}_2) \) and \( x_2 = (\dot{q}_1, \ddot{q}_2) \), the equivalent state space representation can be written:

\[ \begin{align*} 
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= M_{1}^{-1}(J_{11}^T F - C_1 x_{12} - V_1 - \mu_1) \\
y_1 &= h(x_{11}, x_{12}) 
\end{align*} \]

Suspensions Dynamics \( \Sigma_2 \): This dynamics is given by:

\[ \Sigma_2 : \quad F_s = M_3 \dot{q}_3 + \bar{C}_3 \dot{q}_3 + V_3 + \mu_2 \]

Where \( M_{3,3} \) is the SPD inertia matrix associated to subsystem \( \Sigma_2 \) and the coupling term \( \eta_3^c \) is given by equation (??). Let \((x_1, x_2) = (q_3, \dot{q}_3)\) and where \( \mu_2 = -M_{33} \eta_3^c \), the state space representation of \( \Sigma_2 \) is then:

\[ \begin{align*} 
\dot{x}_{21} &= x_{22} \\
\dot{x}_{22} &= M_{3}^{-1}(J_{33}^T F - \bar{C}_3 x_{22} - V_3 - \mu_2) \\
y_2 &= h(x_{22}) 
\end{align*} \]

Wheel dynamics \( \Sigma_3 \): The wheel dynamics are given by the wheels steering of the two front wheels \( \Sigma_{31} \) and the wheels rotations of the four wheels \( \Sigma_{32} \).

\[ \begin{align*} 
\dot{x}_1 &= q \\
\dot{x}_2 &= \bar{\epsilon}(x_1, x_2) + \xi(x_1, x_2) 
\end{align*} \]
where the nominal part of the system dynamics is represented by the function $f(x_1, x_2)$ containing some known nominal functions, while uncertainties and neglected dynamics are in $\mu$. The unknown input components are

$$\xi = M^{-1}J^T F_i$$

(18)

The system (16), understood in Filippov’s sense (Filippov [1988]) is assumed such that the functions $f(.)$ and the perturbation $\xi(.) + \mu_i$ are Lebesgue-measurable and uniformly bounded in any compact region of the state space.

### 3.2 HOSM Observer with unknown input estimation

We assume $x_1$ available for measurement. In order to estimate the state vector $x$ and to deduce the unknown inputs vector $U = F_i$, we propose the (super-twisting) Second Order Sliding Mode observer (Davila and Fridman [2009]; M’sirdi et al. [2006]; Rabhi et al. [2010]):

$$\dot{x}_1 = \ddot{x}_2 + z_1$$

$$\dot{x}_2 = f(t,x_1, \dot{x}_2) + z_2$$

where $\dot{x}_1$ and $\dot{x}_2$ are the state estimations, $\alpha$ and $\lambda$ are the observer gains and the correction variables $z_1$ and $z_2$ are calculated by the super-twisting algorithm

$$z_1 = \lambda|x_1 - \dot{x}_1|^{1/2} \text{sign}(x_1 - \dot{x}_1)$$

$$z_2 = \alpha \text{sign}(x_1 - \dot{x}_1)$$

(20)

The at the instant moment we take $\dot{x}_1(t = 0) = x_1(t = 0)$ and $\dot{x}_2(t = 0) = 0$ to ensure observer convergence. It is important to note that in a first step, input effects on the dynamic are rejected by the proposed observer like a perturbation. Taking $\dot{x}_1 = x_1 - \dot{x}_1$ and $\dot{x}_2 = x_2 - \dot{x}_2$ we obtain the equations for the estimation error dynamics

$$\dot{\tilde{x}}_1 = \tilde{x}_2 - \lambda|\tilde{x}_1|^{1/2} \text{sign}(\tilde{x}_1)$$

$$\dot{\tilde{x}}_2 = f(t,x_1, \dot{x}_2) - \alpha \text{sign}(\tilde{x}_1)$$

(21)

Let us recall that

$$F(t,x_1, x_2, \dot{x}_2) = f(t,x_1, x_2) - f(t,x_1, \dot{x}_2) + \xi(t,x_1, x_2)$$

Consider $\nu_{\text{max}}$ and $x_{\text{max}}$ as defined such that $\forall t \in \mathbb{R}^+ \forall x_1, x_2 \leq \nu_{\text{max}}$ and $x_1 \leq x_{\text{max}}$. In our case, the system states are bounded, then the existence of a constant bound $f^+$ is ensured such that the following condition holds for any possible $t$, $x_1$, $x_2$ and $|\dot{x}| \leq 2 \nu_{\text{max}}$.

$$|F(t,x_1, x_2, \dot{x}_2)| < f^+$$

(22)

Remark: The state boundedness is true, because mechanical systems are passive and BIBS stable, and the control action input $u$ is bounded. The maximal possible acceleration in the system is a priori known and it coincides with the bound $f^+$. In order to define the bound $f^+$ let us consider the system physical properties. We have:

$$\frac{m I}{\lambda} \leq M \leq \frac{m L}{\epsilon}$$

where $m$ is the minimal eigenvalues and $\lambda$ the maximal one and $\tau$ and $k$ are positive constants. Then we obtain

$$\max(M^{-1}) = \frac{m I}{\lambda}$$

and $f^+$ can be written as

$$f^+ = \frac{1}{m} (\nu_{\text{max}}^2 + \epsilon x_{\text{max}})$$

(23)

Let $\alpha$ and $\lambda$ satisfy the following inequalities, where $p$ is some chosen constant, $0 < p < 1$

$$\lambda > \sqrt{\frac{2}{\alpha - f^+}}$$

(24)

The convergence of this observer ensures that in finite time $\forall t > 0$ then $\dot{z}_2 = 0$ holds after some finite time $T$

$$\dot{z}_2 = f(x_1, x_2) - f(x_1, \dot{x}_2) + \xi - z_2 = 0$$

(25)

### 3.3 Input Force estimation

Let us take a low pass filtering of $z_2$ which is defined in equation 19 and 20, then we obtain in the mean average:

$$\dot{\tilde{z}}_2 = \dot{\tilde{z}}_2 + \xi - z_2$$

(26)

It was assumed that the term $z_2$ changes at a high (infinite) frequency. However, in reality, various imperfections make the state oscillate in some vicinity of the intersection and components of $z_2$ are switched at finite frequency, this oscillations have high and slow frequency components. The high frequency term $z_2$ is filtered out and the motion in the sliding mode is determined by the slow components (Utkin et al. [1999], Filippov [1988]). The equivalent control is close to the slow component and may be derived by low pass filtering. The filter time constant should be sufficiently small to preserve the slow components undistorted but large enough to eliminate the high frequency component. Thus the conditions $\tau \to 0$ where $\tau$ is the filter time constant, and $\delta / \tau \to 0$, where $\delta$ is the sampling interval may be filter out the high frequency components.

### 4. ALIEN OBSERVERS

#### 4.1 Presentation of the Algebraic Approach

Here we recall the estimation approach for vehicle velocities at its center of gravity presented in (Villagra et al. and Fliess [2008]). It is an algebraic estimation techniques (ALIEN $^1$). It uses only acceleration equations with respect to a rotating frame. $V_y$, $\psi$, $\gamma_x$ and $\gamma_y$ are respectively the lateral velocity, the yaw velocity, the longitudinal acceleration, the lateral acceleration.

$$\gamma_x(t) = V_{x}(t) + \psi(t)V_y(t)$$

(27)

$$\gamma_y(t) = \dot{V}_{y}(t) - \psi(t)V_x(t)$$

The longitudinal and lateral velocities ($V_x$, $V_y$) cannot be simultaneously estimated from equations (27) if values $V_{x0}$ and $V_{y0}$ at initial time $t_0$ are known. By means of diagnosis tools, the velocities ($V_x$, $V_y$) can be written ;

$$V_{x}(t) = R_{x}(t) + G_{x}(t)$$

$$V_{y}(t) = R_{y}(t) + G_{y}(t)$$

(28)

Where ($R_x$, $R_y$) and ($G_x$, $G_y$) are respectively the ideal and the disturbance terms

- $R_x = r\omega_t$ : $- \omega_t$ is the static wheel radius,
- $\omega_t = \frac{1}{4} \sum_{i=1}^4 \omega_i$ is the mean rotation speed of the 4 wheels,
- $R_y = -L_1 \dot{\psi}$ : $L_1$ is the Kart front wheelbase
- $\psi$ is the yaw velocity

By using the two equations, (27) and (28), we obtain the following expressions of $(\dot{R}_x, \dot{R}_y)$:

$$\dot{R}_x = -\dot{\psi}G_{y} - \dot{G}_{x} + \dot{\psi}G_{y} + \gamma_x$$

$$\dot{R}_y = \dot{\psi}R_{x} - \dot{G}_{y} + \psi G_{x} + \gamma_y$$

(29)

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$^1$ ALIEN: Algèbre pour Identification et Estimation Numériques.
we can then write $\dot{G}_x$ and $\dot{G}_y$ by the following form:

$$\begin{cases}
\dot{G}_x = -\psi G_y - L_1 \dot{\psi}^2 - r\dot{\omega}t + \gamma_x \\
\dot{G}_y = \dot{\psi} G_x - L_1 \ddot{\psi}^2 - \dot{\psi}r\dot{\omega}t + \cdots - V_{45} - \mu_3) - z_{32}
\end{cases} \tag{34}$$

Where

$$G_x(t_0) = 0 ; \ G_y(t_0) = 0 \tag{31}$$

4.2 Algebraic Approach

Two algorithms for velocities estimation are proposed from the equations (27) to (31), and applied in same time.

Algorithm 1 (Estimation of $V_x$): To estimate the longitudinal velocity, we need that the yaw rate $\psi(t)$, longitudinal and lateral acceleration ($\gamma_x(t)$ and $\gamma_y(t)$), 4 wheel’s rotation speed $\omega_i(t)$ and lateral velocity estimation $V_y(t)$.

if $|\dot{G}_x(t)| < \epsilon_1$ then $\dot{V}_x(t_i) = r\omega_i(t)$  
else $\dot{V}_x(t_i) = \dot{V}_x(t_{i-1}) + \int_{t_i}^{t} (\gamma_x + \dot{\psi} V_y(t)) dt$

Endif

Algorithm 2 (Estimation of $V_y$): To estimate the lateral velocity, we suppose available the yaw rate $\psi(t)$, longitudinal and lateral acceleration ($\gamma_x(t)$ and $\gamma_y(t)$), 4 wheel’s rotation speed $\omega_i(t)$ and the longitudinal velocity estimation $V_x(t)$.

if $|\dot{G}_y(t)| < \epsilon_2$ then $\dot{V}_y(t_i) = -L_1 \dot{\psi}$  
else $\dot{V}_y(t_i) = \dot{V}_y(t_{i-1}) + \int_{t_i}^{t} (\gamma_y - \dot{\psi} V_x(t)) dt$

Endif

As first result of the comparative analysis, we can say that the ALIEN algorithms are quite easy to use but give only velocity estimates robust versus noise and modeling uncertainties. HOSM give interesting finite time convergence for the state components and a good convergence for force estimation.

5. COMPARATIVE SIMULATIONS AND EXPERIMENTAL RESULTS

In this section, we give some simulation results obtained by the simulator (SimK106N)(M’sirdi et al. [2007]) and (M’Sirdi et al.) developed in the laboratory in order to test and validate the proposed observers and partial state estimators. The system state evolution and forces are computed by use of a car simulation in Matlab - Simulink.

5.1 High Order Sliding mode Observers

Observer for the chassis Dynamics $\Sigma_1$ : Using the state space representation (11), we propose the following sliding mode observer giving the state estimates and the correction variables $z_{11}$ and $z_{12}$ where $\hat{F}_1 = J_1^{T} \hat{F}$:

$$\begin{cases}
\dot{\hat{x}}_{11} = \hat{x}_{12} - z_{11} \\
\dot{\hat{x}}_{12} = M_1^{-1}(J_1^{T} \hat{F}_1 - C_1 \hat{x}_{12} - V_{12} - \mu_1) - z_{12} \\
z_{11} = A_{11} |\hat{x}_{11} - x_{11}|^{2} \text{sign}(\hat{x}_{11} - x_{11}) \\
z_{12} = A_{12} \text{sign}(\hat{x}_{11} - x_{11})
\end{cases} \tag{32}$$

- $A_{11}$ and $A_{12}$ are observer gains to be adjusted for convergence. $\hat{F}_1$ is estimation of the forces.

The simulation results are obtained for a driving with sinusoidal steering command of 20 degrees amplitude. This figure shows also longitudinal and lateral displacements plus longitudinal velocity. Figures (2) show results of the HOSM Observers for (a) Lateral and vertical motions; (b) Pitch, Roll and Yaw of Chassis.

Observer for Suspensions Dynamics $\Sigma_2$ : We assume that the wheels are always in contact with the ground. The proposed observer for each wheel suspension is:

$$\begin{cases}
\dot{\hat{x}}_{21} = \hat{x}_{22} - z_{21} \\
\dot{\hat{x}}_{22} = M_3^{-1}(J_3^{T} \hat{F} - C_3 \hat{x}_{22} - V_3 - \mu_2) - z_{22}
\end{cases} \tag{33}$$

where $\hat{F}_2 = J_3^{T} \hat{F}$.

- $z_{21} = A_{21} |\hat{x}_{21} - x_{21}|^{2} \text{sign}(\hat{x}_{21} - x_{21})$
- $z_{22} = A_{22} \text{sign}(\hat{x}_{22} - x_{22})$
- $A_{21}$ and $A_{22}$ are observer gains to be adjusted for convergence. $\hat{F}_2$ is an a priori estimation of the forces.

Observer for Wheels Dynamics $\Sigma_3$ : By using the state space representation for the subsystem $\Sigma_3$, the proposed observer, for each wheel, is as follows:

$$\begin{cases}
\dot{\hat{x}}_{31} = \hat{x}_{32} - z_{31} \\
\dot{\hat{x}}_{32} = M_3^{-1}(J_4^{T} \hat{F} - C_4 \hat{x}_{32} - V_4 - \mu_3) - z_{32}
\end{cases} \tag{34}$$

where:

- $z_{31} = A_{31} |\hat{x}_{31} - x_{31}|^{2} \text{sign}(\hat{x}_{31} - x_{31})$
- $z_{32} = A_{32} \text{sign}(\hat{x}_{32} - x_{32})$
• $\Lambda_{31}$, $\Lambda_{32}$ and $\Lambda_{33}$ are observer gains to be adjusted for convergence, $\hat{F}$ is an a priori estimation of the forces. The results are satisfactory for the HOSM Observers.

5.2 Algebraic Approach

With the same steering angle, the car simulation and the observer give us the estimation. Figures (3c) and (3d), of the longitudinal and lateral velocity ($V_x(t)$ and $V_y(t)$) given by the algebraic approach ($\epsilon_1 = 0.15$ and $\epsilon_2 = 0.8$). The Figures (3a) and (3b) show the evolution of $\dot{G}_x$ and $\dot{G}_y$ with the time.

Fig. 3. Results of the ALIEN Approach

6. CONCLUSION

In this paper, we have compared some efficient and robust observers allowing to estimate partial states of the system. The performance is due to use of differential estimators based on equations independant from the system sub models and insensitive to noise. HOSM Observers are less sensitive to initial conditions and provide also estimates of input forces. They are also extendible to estimate more state components and have finite time convergence. The robustness of the sliding mode observer with respect to uncertainties on model parameters is an important feature which has been emphasized in simulation too. All these observers are illustrated by simulation results to show effectiveness of their performance and robustness. The advantage of HOSM is that it is less sensitive to initial conditions and it can provide full state observation and estimation of input forces. The simulations used the SimK106N developed and validated previously by our staff. This simulator is available for testing and research validation (msiridlink@free.fr). The proposed partial state estimators can be used to estimate the vehicle dynamic state and then to retrieve the unknown inputs. In the final version of this paper results got with the driving simulator of Oktal will be presented.

REFERENCES


