Type III Fuzzy Impulsive Controller Based on PDC

I. Zamani*. M. Shafiee.*

*Electrical Engineering Department, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran (e-mail: {zamaniiman, mshafiee}@aut.ac.ir)

Abstract: This paper investigates an approach for the stabilization of continuous-time type III (Takagi-Sugeno-Kang) fuzzy system based on impulsive parallel distributed compensation (PDC). Based on the Lyapunov criterion, some conditions are derived to stabilize type III fuzzy systems. These conditions are given in term of some matrix inequalities.

Keywords: Impulsive T-S Fuzzy Systems, Parallel Distributed Compensation, Stability Analysis, Asymptotically Stability, Matrix Inequality.

1. INTRODUCTION

Among of the models of fuzzy systems, Takagi-Sugeno-Kang (T-S) model is mentioned which was presented by Tanaka et al. (1992) as a new fuzzy system after advent of Mamdani model. Because of nonlinear form of consequents in the rules of this model and applicable of many linear control theory, this model (i.e. T-S) was prospered at a high rate. Thereafter, PDC technique was presented by the same author as a new method for designing and controlling for such type of fuzzy systems.

By using impulsive differential equations, we can construct a new version of PDC to control and stabilize the fuzzy systems. Such as other control systems, performance and stability are important aspects of designing for these kinds of systems. Based on these remarks, innovative methods and relevant conditions for stability analysis of continuous impulsive fuzzy systems were found Zhang et al. (2006), Zamani et al. (2009a, b), Liu et al. (2007), Cai et al. (2008). In these papers, fuzzy impulsive controller shares the same plant rules which restrict stabilization. In this correspondence, Stability and stabilization of impulsive PDC are given.

The contributions of this paper are organized as follows: After introduction of impulsive equation, in section three, we introduce the preliminaries concepts of type III fuzzy systems. In the second part by using the concepts of impulsive equation, type III fuzzy impulsive controller based on PDC is introduced and sufficient conditions for stability analysis, asymptotically stability analysis are given. Finally, results are argued in conclusion section.

2. PRELIMINAREIS

In this section, we introduce the concepts and definitions we need to introduce T-S fuzzy impulsive controller. Consider the nonlinear system

\[
\begin{align*}
\dot{x} &= f(t,x) \\
y &= \phi(x)
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the state variable, \(y \in \mathbb{R}^m\) is the output variable, \(f(t,x)\) and \(\phi(x)\) are continuous functions in their respective domains of definition. An impulse control law of (1) is given by a sequence \(\{t_k, I_k(y(t_k))\}\), where

\[
0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots, \quad t_k \to \infty \text{ as } k \to \infty
\]

and \(I_k(y)\) is a continuous function which maps \(\mathbb{R}^m\) to \(\mathbb{R}^n\) for all \(k=1,2,\ldots\) Clearly, if the solution of (1) exists, we can rewrite (1) as follows:

\[
\begin{align*}
\dot{x} &= f(t,x), \quad t \neq t_k \\
y &= \varphi(x), \quad t \neq t_k \\
\Delta x &= I_k(y), \quad t = t_k \\
x(t_0) &= x_0, \quad k = 1,2,\ldots
\end{align*}
\]

(2)

where \(x(t_k^+) = x(t_k^-) - x(t_k^-) = \lim_{t \to t_k^+} x(t)\). We call (2) as impulsive differential system. Without loss of generality, we assume \(f(t,0) \equiv 0\) and \(\varphi(0) = 0\) so that system (2) admits a trivial solution.

3. FUZZY SYSTEMS AND IMPULSIVE EQUATIONS

The concepts and definitions we need to introduce T-S fuzzy systems and fuzzy impulsive controller are presented firstly. In the continuous case, T-S fuzzy model can be described by the following fuzzy rules:

Plant rules:

if \(p_1(t)\) is \(M_1^i\) and \(p_2(t)\) is \(M_2^i\) and \ldots and \(p_s(t)\) is \(M_s^i\)

then

\[
\begin{align*}
\dot{x}(t) &= f_i(x(t),t) + B_i l(t) \\
\dot{y}_i(t) &= \varphi_i(x)
\end{align*}
\]

\((i = 1,2,\ldots,r)\)
where \( r \) is the number of fuzzy rules, and \( p(t) \) is vector of premise variables such that \( p(t) = [p_1(t), p_2(t), \ldots, p_r(t)]^T \in \mathbb{R}^n \). \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) is state vector at time \( t \), \( n \) is the number of states variable and \( y(t) \) is output of the subsystem. \( M_j \) stands for the fuzzy set of \( j \)th antecedent variable in the \( j \)th rule. \( f_i(x(t), t) \) and \( \varphi_i(x) \) have the same mentioned properties of (2).

By the singleton fuzzifier, product inference and the center average defuzzifier, the final output of the fuzzy systems can be represented as:

\[
\hat{x}(t) = \sum_{i=1}^{r} \left( \mu_i(p(t)) f_i(x(t), t) + B_i \varphi_i(x) \right) \tag{3}
\]

where \( \omega_i(p(t)) = \prod_{j=1}^{s} \alpha_{M_j}^i(p(t)) \) and \( \sum_{i=1}^{r} \omega_i(p(t)) \neq 0 \) for all \( t \geq 0 \). \( \alpha_{M_j}^i(p(t)) \) is grade of \( p(t) \) by \( M_j^i \) and

\[
\left( \frac{\omega_i(p(t))}{\sum_{i=1}^{r} \omega_i(p(t))} \right) := \mu_i(p(t))
\]

Obviously it holds: \( 0 \leq \mu_i(p(t)) \leq 1 \) for all \( i = 1, 2, \ldots, r \) and \( \sum_{i=1}^{r} \mu_i(p(t)) = 1 \).

In the case of impulsive PDC for fuzzy systems, the consequents of fuzzy rules are presented by convenient dynamic systems and a fuzzy impulsive controller is designed as Fig. 1.

![Fuzzy Impulsive Controller Based on PDC](image)

**Fig. 1, Fuzzy Impulsive Controller Based on PDC**

By substituting \( u(t) \), we obtain the following formulation of the closed loop models:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(p(t)) f_i(x(t), t) + B_i \varphi_i(x(t)) + B_i \left( \sum_{k=1}^{\infty} \delta(t - \tau_k) I_{jk}(x) \right)
\]

However, system (4) implies that

\[
\text{x}(t) = \text{x}(t + h) - \text{x}(t - h)
\]

\[
= \int_{t-h}^{t-h} \left( \sum_{i=1}^{r} \mu_i(p(t)) f_i(x(t), t) \right) dt
\]

\[
+ B_i \left( \sum_{k=1}^{\infty} \delta(t - \tau_k) (I_{jk}(x)) \right)
\]

(5)

where, \( h > 0 \) is sufficient small. As \( h \to 0^+ \), one obtains

\[
\text{x}(t_{k}^+) - \text{x}(t_{k}^-) = \sum_{i=1}^{r} \mu_i(p(t)) f_i(x(t), t) B_i I_{jk}(x)
\]

(6)

To apply the results of impulsive differential equations, system is rewritten as

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(p(t)) f_i(x(t), t), \quad t \neq t_k
\]

\[
\Delta x = I_k(x) = \sum_{i=1}^{r} \mu_i(p(t)) B_i I_{jk}(x), \quad t = t_k
\]

(7)

In the continuation of this paper, we consider \( f_i(x(t), t) \) and \( I_{jk}(x) \) as follow

\[
f_i(x(t), t) = A_i x(t) + \alpha_i \varphi_i
\]

in which

\[
\alpha_i = \begin{cases} 0, & \text{if } i \text{ is a rule that contains the origin} \\ 1, & \text{otherwise} \end{cases}
\]

and \( I_{jk}(x(t)) = D_{jk} x(t) \).

**4. STABILITY ANALYSIS**

By using the analysis of fuzzy affine systems Zhu et al. (2006), and similar Zamani et al. (2009), the rules sets are divided to \( I_0 \) and \( I_1 \). \( I_0 \) contains the rules which include the origin and \( I_0 \) rules does not contain the origin. For \( I_1 \) the following condition is satisfied.

\[
I_1 = \{i: 1 - x_i^T Q x_i < 0, 1 < l \leq r\}
\]

\[
I_0 = \{1, 2, \ldots, r\} \setminus I_1
\]

(8)

More details have been given in Zhu et al. (2006).

Theorem 1: The origin of the fuzzy impulsive controller (7) is stable if there are positive matrices \( P \), scalars \( \mu_k, v_k, \gamma, \Delta \tau_k, \tau_i \) and matrices \( B_i \) such that the following conditions are satisfied.

\[
\mu_k + \nu_k \leq 0, \quad \nu_k \geq -1
\]

(9.a)
$$\left( \Omega_{ij} - \frac{1}{2} \right) \leq 0 \quad (9.b)$$

$$\left( \Omega_{ij} \right) \leq 0 \quad i < j \quad (9.c)$$

For $i \in I_o$

$$\psi_i < 0 \quad (9.d)$$

For $i \in I_i$

$$\left( \psi_i - \tau_i Q_i \right) \leq 0 \quad (9.e)$$

where $i, j = 1, 2, \ldots, r, \ \Omega_i = 1 - x_i^T Q_i x_i, \ \psi_i = A_i^T P + P A_i - \frac{1}{2} \Delta P \Delta_i P$ and $\Omega_{ij} = \frac{P B_i D_{ij} + D_{ij}^T B_i^T P - \nu_k P}{\Delta_{ij}}.$

Proof:

Let $V(x) = x^T P x$. By using theorem A.1 (has been given in appendix) and since $V(x)$ is continuously differentiable then,

$$D^+ V(x) - \frac{H_k}{\Delta_t} x^T P x = x^T P x + x^T P x - \frac{H_k}{\Delta t} x^T P x$$

$$= \sum_{i=1}^r \mu_i (p(t)) x^T A_i^T P x + \sum_{i=1}^r \mu_i (p(t)) x^T P A_i x$$

$$+ \sum_{i=1}^r \mu_i (p(t)) e_i^T P x + \sum_{i=1}^r \mu_i (p(t)) x^T P e_i - \frac{1}{\Delta t} x^T P x \quad (10)$$

By noticing that $\sum_{i=1}^r \mu_i (p(t)) = 1$ and if $i \in I_o, \ \sigma_i = 0$, so

$$\sum_{i=1}^r \mu_i (p(t)) x^T \left( A_i^T P + P A_i - \frac{1}{\Delta t} \right) x \leq 0 \quad (11)$$

If $\psi_i \leq 0$ then above inequality hold. If $i \in I_i, \ \sigma_i \neq 0$, so

$$D^+ V(x) - \frac{H_k}{\Delta t} x^T P x = x^T P x + x^T P x - \frac{1}{\Delta t} x^T P x$$

$$= \sum_{i=1}^r \mu_i (p(t)) x^T A_i^T P x + \sum_{i=1}^r \mu_i (p(t)) x^T P A_i x$$

$$+ \sum_{i=1}^r \mu_i (p(t)) e_i^T P x + \sum_{i=1}^r \mu_i (p(t)) x^T P e_i$$

$$+ \sum_{i=1}^r \mu_i (p(t)) \tau_i \left( 1 - x_i^T Q_i x_i \right) + x_i^T Q_i x_i - x_i^T Q_i x_i - \frac{1}{\Delta t} x^T P x \quad (12)$$

where

$$\Pi = \left( x^T \right) (\psi_i - \tau_i Q_i) \left( \psi_i - \tau_i Q_i \right) \quad (9.e)$$

Therefore, if $\psi_i - \tau_i Q_i < 0$ the above inequality will be satisfied. To satisfy condition (ii) of theorem A.1 we can get

$$V(\tau_i, x + I_k) = (x + I_k)^T P (x + I_k)$$

$$= x^T P x + x^T P I_k + I_k^T P x + I_k^T P I_k$$

$$= x^T P x + x^T P \sum_{i=1}^r \mu_i (p(t)) \mu_i (p(t)) (B_i I_j k(x))$$

$$+ \left( \sum_{i=1}^r \sum_{j=1}^r \mu_i (p(t)) \mu_i (p(t)) \left( B_i I_j k(x) \right)^T \right) P x$$

$$+ \sum_{i=1}^r \mu_i (p(t)) \mu_i (p(t)) \left( B_i I_j k(x) \right)^T$$

$$- x^T P x - v_k x^T P x \quad (13)$$

Therefore, if the following inequality holds then (13) will be satisfied.

$$x^T P \sum_{i=1}^r \mu_i (p(t)) \mu_i (p(t)) (B_i D_{i,k}) x$$

$$+ \left( \sum_{i=1}^r \sum_{j=1}^r \mu_i (p(t)) \mu_i (p(t)) (B_i D_{i,k}) x \right)^T P x - v_k x^T P x$$

$$+ \sum_{i=1}^r \mu_i (p(t)) \mu_i (p(t)) (B_i D_{i,k}) x$$

$$x \left( \sum_{i=1}^r \sum_{j=1}^r \mu_i (p(t)) \mu_i (p(t)) (B_i D_{i,k}) x \right) \leq 0 \quad (14)$$
Using Schur complement and \( \Omega_{ijk} = P B_j D_{jk} + D_{jk}^T B_i^T P - v_i k P \), we can get

\[
\begin{align*}
\left( \sum_{i,j} \mu_i(p(t)) \mu_j(p(t)) x^T(\Omega_{ijk}) x \right) \\
\left( \sum_{i,j} \mu_i(p(t)) \mu_j(p(t))(B_i D_{jk} x) \right)^T \leq 0
\end{align*}
\]

We can rewrite above inequality as

\[
\sum_{i,j} \mu_i(p(t))^2 \left( x^T(\Omega_{ijk}) x \right) \leq 0
\]

So, if (17) holds then (16) will be negative.

\[
\sum_{i,j} \mu_i(p(t)) \mu_j(p(t)) \left( x^T(\Omega_{ijk} + \Omega_{lik}) x \right) \leq 0
\]

By using Schur complement, if the following inequality holds

\[
\Omega_{ijk} \leq 0
\]

Also, the following must be hold.

\[
\left( x^T(\Omega_{ijk} + \Omega_{lik}) x \right) \leq 0
\]

By using Schur complement again, if the following holds then above inequality will be satisfied.

\[
\Omega_{ijk} + \Omega_{lik} \leq 0
\]

Example: Consider the s fuzzy systems by the following IF-THEN rules

IF-THEN rules

Rule 1:

IF \( x_1 \) is \( M_1(x_1) \). THEN

\[
\begin{align*}
\dot{x}_1(t) &= \begin{pmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & -1 & -3 \end{pmatrix} x_1(t) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(t),
\end{align*}
\]

Rule 2:

IF \( x_1 \) is \( M_2(x_1) \). THEN

\[
\begin{align*}
\dot{x}_1(t) &= \begin{pmatrix} 0 & 1 & 0 \\ 9.8 \sin(t) & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} x_1(t) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(t).
\end{align*}
\]

To examine the validity, we apply the following parameters which have been obtained by Matlab LMI toolbox.

\[
\begin{align*}
\mu_k &= 0.9891, v_k = -0.9892, \Delta \tau_k = 0.05
\end{align*}
\]

\[
D_{lk} = [ -0.1211 - 0.2389 - 0.0051 ]; k = 1,2,\ldots,
\]

\[
P = \begin{bmatrix} 0.0587 & 0.0330 & -0.0002 \\ 0.0330 & 0.0819 & 0.0014 \\ -0.0002 & 0.0014 & 0.0528 \end{bmatrix}
\]

Fig. 2 shows the response of the proposed systems with initial condition.

6. CONCLUSIONS

Stability analysis and systematic design of accepted performance are two important issues of T-S fuzzy systems design.

In this paper, a new approach based on Lyapunov stability is proposed to stabilize the fuzzy systems by impulsive PDC. Under some sufficient conditions, a nonlinear state feedback controller (impulsive PDC) is developed to stabilize the fuzzy system instead of parallel distributed compensation (PDC) design.

REFERENCES


Appendix

The following theorem by Liu et al. (2006), gives sufficient conditions for various stability criteria for (2).

**Theorem A.1:** Assume that

(i). \( V \in \Sigma \), there exist \( \mu_k \in \mathbb{R} \) and \( c_k \in K \) such that

\[
D^+ V(t, x) \leq \frac{\mu_k}{\Delta \tau_k} c_k (V(t, x)), \quad (t, x) \in (\tau_{k-1}, \tau_k) \times s(p)
\]

(ii). There exist \( \nu_k \in \mathbb{R} \) and \( d_k \in K \) such that

\[
V(\tau_k^+, x + I_k(x)) \leq V(\tau_k, x) + \mu_k d_k (V(\tau_k, x)), \quad x \in s(p)
\]

(iii). \( \mu_k + \nu_k \leq 0 \), for \( s \in (0, \rho) \), \( c_k(s) \leq d_k(s) \) if \( \nu_k < 0 \) and \( d_k \leq c_k \) if \( \mu_k < 0 \).

Then the system is stable. In addition to all above condition, suppose further that \( V(t, x) \) is decreasing and

(iv). For any \( \eta > 0 \), there exist a \( \sigma > 0 \) such that

\[
s + \| \nu_k \| \| d_k(s) \| < \eta, \quad \forall s \in (0, \sigma), \quad \forall k = 1, 2, \ldots
\]

Then the system is uniformly stable. Also, if (i), (ii) and (iii) and following condition hold

\[
\sum_{k=1}^{\infty} (\mu_k + \nu_k) e_k(\beta) = -\infty, \quad \forall \beta > 0
\]

where \( e_k(s) = \max\{c_k(s), d_k(s)\} \), then system is asymptotically stable. Finally, if (i), (ii), (iii), (iv) and there exist positive integer \( N \) such that

\[
\sum_{k=q+1}^{q+N} (\mu_k + \nu_k) I_k(\beta) < -C, \quad \forall q \geq 0, \quad \beta, \alpha > 0
\]

where \( I_k(s) = \max\{c_k(s), d_k(s)\} \), and the sequence \( \{\Delta \tau_k\} \) is bounded for any \( \beta, \alpha > 0 \), \( s(\rho) = \{x \in \mathbb{R}^n; \|x\| < \rho\} \). Then, system is uniformly asymptotically stable. Class \( K \Sigma \), right-hand generalized derivative , upper-right-hand Dini derivative \( D^+ (\cdot) \) and decreasing function are given in 0 Liu et al. (2006).