Variable Structure Based Adaptive Control with Tuning Function Design for a Class of Unknown Switched Linear Systems

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Abstract: This paper considers the control problem of switched linear systems with unknown parameters and unknown switching signals. For the considered system, robust model reference adaptive control (MRAC) approach can only guarantee error boundedness under an average dwell time (ADT) condition which is implicit and is difficult to estimate. We propose a variable structure (VS) based adaptive control with tuning function design to achieve better results than the MRAC approach. Moreover, the VS control is partially extended to relative degree two case and using similar approach, it can be generalized to higher relative degree case.

Keywords: Adaptive control, variable structure control, switching theory, backstepping.

1. INTRODUCTION

Recently, theories and analysis tools for switched systems are extensively investigated and available results are increasing. It is practicable to consider the problem of controlling a switched system with only partially known information. We classify switched systems into two types. If the switching behavior can be controlled or designed, we designate this type of switched systems as switching control systems. On the other hand, if the system has switching behaviors that can not be manipulable by the designer but governed by environment or the plant, the switched system is called a non-autonomous switched system. Usually the uncontrolled switching in such system is regarded as a destabilizing term and we want to suppress the effects of switchings by the designed controllers. In this paper, a non-autonomous switched system with unknown parameters and unknown switching signals is to be dealt with and we intend to design an adaptive controller that can guarantee system stability under the conditions related to the switching frequencies. Adaptive control of unknown switched systems remains a challenging problem and relative fewer results are available nowadays. In the book Tao (2003), page 270, it reads as follows: "For a general linear system with non-small parameter variations including unknown jumping parameters at unknown time instants, new adaptive control designs are yet to be developed." Since parameters and switching signals of the switched systems considered are unknown, the controller should have strong robustness to ensure stability while maintaining performance to some extent. Recently, in Kuiper and Ioannou (2010), the authors propose a mixing control scheme that combines $H_\infty$ control theory and adaptive control. For the newly developed adaptive control schemes, their applicability to switched systems still remains questionable and is only justified by simulation demonstrations. There are some results using VS design for the non-autonomous switched systems, e.g., Lian and Zhao (2009); Cardim et al. (2009), and Wang et al. (2009), but the switching signals and plant information are assumed to be known. In this paper, we propose a VS based adaptive control scheme for the non-autonomous switched systems and show stability properties of the closed-loop system by thorough theoretic analysis stemming from switched system theories as well as other stability theories. We first discuss the stability properties of switched system with robust MRAC approach. It is shown that the stability result dependents on the switching signal and can not be derived explicitly due to the unknown parameters. Hence, we propose a VS based adaptive controller with tuning function design and the stability result is independent of the switching signals contributed by the parameterization of switching parameters and VS adaptive law. For relative degree two case, the VS design is not directly applicable and we use VS adaptive law for one parameter and robust adaptive law for the others. Signal boundedness and error convergence are guaranteed independent of the switching signals.

This paper is organized as follows. Problem formulation is given in section 2. We analyze the stability properties of switched systems with robust MRAC in section 3. In section 4, we propose a VS adaptive control with tuning function design for relative degree one and two case to resolve the difficulty of unavailable ADT condition. The conclusions are given in section 5.
2. PROBLEM FORMULATION

Consider the SISO switched linear system in observer canonical form
\[
\dot{x}_p = A_p x_p + b_p u
\]
\[
y_p = e_T^T x_p = [1, 0, \ldots, 0] x_p
\]
where \( u \in \mathbb{R} \), \( b_p \in \mathbb{R}^n \), \( x_p \in \mathbb{R}^n \),
\[
A_p = \begin{bmatrix}
-a_p^1 & 0 & \cdots & 0 \\
-a_p^2 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
-a_p^n & 0 & \cdots & 0 \\
-a_p^{n-1} & 0 & \cdots & 0 \\
-a_p^n & 0 & \cdots & 0
\end{bmatrix}, \quad b_p = \begin{bmatrix}
0 \\
\vdots \\
0 \\
1 \\
0 \\
0
\end{bmatrix}
\]
and \( \sigma : [0, \infty) \to \mathcal{P} \) is a non-zero switching signal that governs the switching sequence of the switched system. \( \mathcal{P} = \{1, 2, \ldots, p\} \) is the set of switching index. Note that in this paper, we use \( c_i = [0, 0, 1, 0, 0, \ldots, 0]^T \) to stand for the vector with zeros except for the \( i \)th component being unit.

We denote the set of switching signals by \( \mathcal{S} \). By different properties of switching signals, we can classify several sets of switching signals. A switching signal has dwell-time (DT) \( \tau_d \) if all time intervals during switchings are greater than \( \tau_d \). We denote the set of switching signals that have dwell-time \( \tau_d \) by \( \mathcal{S}_{\tau_d} \). In Hespanha and Morse (1999), the concept of dwell-time is extended to the average sense. Let \( \mathcal{N}_0(t, t_0) \) denote the number of switchings of the switching signal \( \sigma \) during the time interval \( (t_0, t) \). We call the switching signal \( \sigma \) has average dwell-time (ADT) \( \tau_0 \) if, given any time interval \( (t_0, t) \), it satisfies
\[
\mathcal{N}_0(t, t_0) - N_0 \leq \frac{t - t_0}{\tau_0},
\]
where \( N_0 \) is a given positive constant. It means that \( \mathcal{N}_0(t, t_0) \), the number of discontinuities of the switching signal \( \sigma \) in any open interval \( (t_0, t) \), is bounded above by the length of the interval normalized by ADT \( \tau_0 \) plus a chatter bound \( N_0 \). The set of switching signals that have ADT \( \tau_0 \) is denoted by \( \mathcal{S}_{\tau_0} \).

The corresponding input-output representation of switched system (1) is
\[
y_p = W_{p(t)}(s)[u] = \frac{Z_{p\sigma(t)}(s)}{\Delta_p(t)(s)}[u],
\]
where \( Z_{p\sigma}(s) = b_p^r s^n + b_{p-1}^r s^{n-1} + \cdots + b_1^r s + b_0^r \) and \( R_{p\sigma}(s) = s^n + a_1^r s^{n-1} + \cdots + a_{n-1}^r s + a_n \), \( i = 1, 2, \ldots, p \).

Here, \( a_1^i \) denote a parameter of the \( i \)th subsystem. To avoid confusion, we will use \( (z_1^i) \) to represent \( z_1 \) to the \( i \)th power. The transfer functions \( W_{p\sigma}(s) \), \( s \in \mathbb{P} \), are strictly proper and parameters of them are all unknown. Only the input and output can be measured, and we do not know when the plant switchings occur.

Given a reference signal \( y_m = W_m(s)r = \frac{Z_m(s)}{R_m(s)}r \), the control purpose is to design the output feedback control \( u \) such that all signals in the switched system are bounded and the output of switched plant tracks the reference output as well as possible, i.e., making the output error \( z_1 = y_p - y_m \) as small as possible. For this tracking problem of switched systems, we do not have the information about the switching time instants nor the knowledge of the active subsystem. The following assumptions are made :

(A1) For all the transfer functions \( W_{p\sigma}, \sigma \in \mathbb{P}, R_{p\sigma}(s) \) is of order \( n \) and \( Z_{p\sigma}(s) \) is of order \( m \), which means that \( W_{p\sigma} \) has relative degree \( N := n - m \);

(A2) The first \( N \) derivatives of the reference signal \( y_m(t) \) are known and bounded.

(A3) All the plants are completely controllable and observable, and minimum phase.

(A4) The signs of \( b_p^r \) are all positive for all \( i \in \mathbb{P} \).

(A5) All the unknown parameters lie in a known compact set, which implies that the upper bound of unknown parameters are known.

3. ROBUST MRAC OF SWITCHED SYSTEMS

In this section, we propose an output feedback model reference adaptive control (MRAC) scheme with a robust adaptive law for the considered problem. For each subsystem in system (1) with index \( i \in \mathbb{P} \), we know that there exists \( \theta_i^* = [k_1, \theta_{p_1}, \theta_{p_0}, \theta_{p_2}]^T \in \mathbb{R}^{2n} \) such that when \( u = \theta_i^T \omega \)
\[
\dot{\omega}_1 = \Lambda\omega_1 + gu
\]
\[
\dot{\omega}_2 = \Lambda\omega_2 + gy_p,
\]
where \( \Lambda \in \mathbb{R}^{(n-1) \times (n-1)} \), \( \det(sI - \Lambda) = \lambda(s) = s^{n-1} + \lambda_1 s^{n-2} + \cdots + \lambda_1 s + \lambda_0 \) is a designed monic Hurwitz polynomial that contains \( Z_m(s) \) as a factor, \( \omega_1, \omega_2 \in \mathbb{R}^{n-1}, g = [1, 0, 0, \ldots, 0]^T \in \mathbb{R}^{n-1} \), and \( \omega = [r, \omega_1, y_p, \omega_2]^T \in \mathbb{R}^{2n}, \) then \( y_p = W_{p\sigma}(s)u = W_m(s)r \). Since parameters of the plants are unknown, \( \theta_i^* \) are unknown, and we choose \( \hat{\theta} = [k_0, \theta_{p_1}^T, \theta_{p_0}, \theta_{p_2}^T]^T \) as the estimation of \( \theta_i^* \). Then, with this certainty equivalence principle controller
\[
\dot{u} = \hat{\theta}^T \omega,
\]
provided there is no switching, we have
\[
y_p = W_m(s)(r + \frac{1}{k_1}(\hat{\theta}^T \omega))
\]
where \( \hat{\theta}_i = \hat{\theta} - \theta_i^*, \quad i \in \mathbb{P} \). Now consider the case of switched plant with the switching signal \( \sigma(t) \). Let \( z_1 = y_p - y_m \), then \( \dot{u} = \hat{\theta}^T \omega \) will lead to the error equation
\[
z_1 = \frac{1}{k_0} W_m(s)(\hat{\theta}^T \omega)
\]
Let \( x_p = \begin{bmatrix} x_p^T, \omega_1, \omega_2 \end{bmatrix}^T \in \mathbb{R}^{3n-2} \), then the state space representation of the closed-loop systems would be
\[
\dot{x}_p = A_{m\sigma} x_p + B_{m\sigma} r + B_{p\sigma} (\hat{\theta}_{p\sigma}^T \omega)
\]
\[
y_p = [e_1^T, 0, 0]^T x_p = C^T x_p
\]
where
\[
C^T (sI - A_{m\sigma})^{-1} B_{m\sigma} = W_m(s), \quad \forall i \in \mathbb{P}.
\]
Note that for all \( i \in \mathbb{P}, B_{m\sigma} = k_i B_{p\sigma} \), and \( k_i^* = \frac{1}{k_1} \). From (9), the reference model can be realized by the nonminimal state space representation
\[
x_m = A_{m\sigma} x_m + B_{m\sigma} r
\]
\[
y_m = C^T x_m
\]
and if we define \( e = x_p - x_m \), then the error equation of (7) can be realized as
\[
\dot{e} = A_m e + B_p \sigma (\tilde{\theta}_\sigma^T \omega)
\]
\[z_1 = C^T e \tag{11}\]

System (11) is a switched system and switches between stable systems may lead to an unstable system. Consider the case \( N = 1 \). We can choose \( W_m(s) \) to be strictly positive real (SPR) and from MKY lemma (see Ioannou and Sun (1996)), we know that for \( i \in \mathbb{P} \), given \( Q_i > 0 \), there exists \( P_i = P_i^T > 0 \) such that \( A_m P_i + P_i A_m = -Q_i < 0 \) and \( P_i B_m = C \), or, \( P_i B_m = \frac{1}{k_i} C = \frac{k_m}{k_i} C \).

For \( N = 1 \), if the gradient adaptive law
\[
\hat{\theta} = -\text{sgn}(k_p \sigma) \Gamma z_1 \omega = -\Gamma z_1 \omega \tag{12}
\]
is applied, where \( \Gamma \) is a positive definite gain matrix, then for each subsystem, the multiple Lyapunov functions (MLFs) can be defined as
\[
V_i = e^T P_i e + \tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i, \quad i = \{1, 2, \ldots, p\}
\]
and have
\[
\dot{V}_i = -e^T Q_i e \leq 0,
\]
for each subsystem. This means that if the plant is non-switching, signal boundedness and error tracking can be derived by Lyapunov theorem and Barbalat’s lemma. However, switches between stable systems may lead to instability. Thus, we incorporate a robust adaptive law into the controller.

By (A5), we assume that for all \( i \in \mathbb{P} \), \( \theta_i^* \) belong to a convex set \( \mathcal{G} \) where
\[
\mathcal{G} = \{ \theta \in \mathbb{R}^{2n} | g(\theta) = \sigma^T \theta - M^2 \leq 0 \}
\]
for some known positive constant \( M_k \).

We design the robust adaptive law as
\[
\dot{\hat{\theta}} = \text{Pr}( - \Gamma z_1 \omega - \beta(\hat{\theta} - c^*)) \tag{14}
\]
where \( \beta \) is a positive constant, \( c^* \) is an arbitrarily chosen constant vector and \( \text{Pr} \) is the projection operator defined by
\[
\text{Pr}(x) := \begin{cases} 
  x & \text{if } x \in \mathcal{G}^0 \text{ or } x \in \partial \mathcal{G} \\
  (I - \nabla g \nabla g^T) x & \text{otherwise}
\end{cases} \tag{15}
\]
Here, \( \mathcal{G}^0 \) is the interior of \( \mathcal{G} \), \( \partial \mathcal{G} \) is the boundary of \( \mathcal{G} \) and \( \nabla g \in \mathbb{R}^{2n} \) is the gradient of function \( g \). Note that the projection adaptive law has an additional term compared to the adaptive law without projection. However, in stability analysis, this additional term will produce a semi-negative term in the time derivative of Lyapunov function to be defined below (see Ioannou and Sun (1996)). Hence, the projection will not change the boundedness of \( V \) of the adaptive system and we will neglect this additional term coming from projection in the stability analysis in the following derivation.

**Theorem 1.** For switched system (1) which satisfies (A1)-(A5) and \( N = 1 \), using the controller (6), (14), if the switching signal \( \sigma \in \mathcal{S}_\sigma \) where \( \tau_m > \frac{\ln M}{m} \), for some positive constant \( m \) and \( M \), then all signals of the system are bounded and the output error will be bounded in a set depends on \( \beta, c^* \) and the unknown parameters. \( \square \)

To proof Theorem 1, we will use the following result from Vu et al. (2007).

**Lemma 2.** Consider the switched system \( \dot{x} = f_i(x, t, u) \). If there exist continuously differentiable function \( V_i, i \in \mathbb{P} \), class \( \mathcal{K}_\infty \) functions \( \bar{a}_1, \bar{a}_2, \bar{\gamma} \), and numbers \( m > 0, M \geq 1 \), such that for all \( x \in \mathbb{R}^n, u \in \mathbb{R}^l \) and for all \( i, j \in \mathbb{P} \), we have
\[
\bar{a}_1(|x|) \leq V_i \leq \bar{a}_2(|x|)
\]
\[
\frac{\partial V_i}{\partial x} f_i(x, u) \leq -m V_i(x) + \bar{\gamma}(|u|)
\]
\[
V_i(x) \leq M V_j(x)
\]
If \( \sigma \) is a switching signal having ADT \( \tau_m > \frac{\ln M}{m} \), then the switched system is input-to-state stable (ISS), that is, there exist \( \alpha, \gamma, \in \mathcal{K}_\infty \) and \( \beta \in \mathcal{K} \) such that
\[
\alpha(|x(t)|) \leq \beta(|x(t_0)|, t) + \gamma(|u_i|)
\]
\( \square \)

The proof of Theorem 1 is as follows.

**Proof.** Uniformly boundedness of the estimation parameter \( \theta \) is guaranteed by the projection adaptive law (14). The closed-loop system is a switched system, thus we consider the MLFs \( V_i = e^T P_i e + \tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i \), where \( i = \{1, 2, \ldots, p\} \).

Then,
\[
\dot{V}_i = -e^T Q_i e - 2\beta \tilde{\theta}_i^T (\tilde{\theta}_i^* - c^*)
\]
\[
\leq -e^T Q_i e - 2\beta \tilde{\theta}_i^T (\tilde{\theta}_i^* - c^*)
\]
\[
\leq -m V_i + \bar{c}
\]
where \( \bar{c} \geq 2|\beta \tilde{\theta}_i^T (\tilde{\theta}_i^* - c^*)| \) is a bounded constant. Moreover, from definition of the MLFs we know that there exists \( M \geq 1 \) such that \( V_i \leq M V_j \), \( \forall i, j \in \mathbb{P} \). Hence from Lemma 2, we can conclude that the system is ISS if the switching signal has ADT \( \tau_m > \frac{\ln M}{m} \). More precisely, there exist class \( \mathcal{K} \) functions \( \alpha, \bar{\gamma} \) and a class \( \mathcal{K} \) function \( \beta \) such that \( \alpha(|x, \theta|) \leq \beta(|x, \theta|), t + \gamma(|u|) \).

Since \( \bar{c} \) is bounded, we can conclude that all the signals in the switched systems are bounded, if the ADT condition is satisfied. \( \square \)

Using the proposed robust adaptive law for the switched systems, signals boundedness can be guaranteed by the ADT condition. However, since parameters are unknown, \( M, \) and ADT \( \tau_m \) are difficult to derive. Moreover, performance of tracking error can not be assured. These concerns make the robust MRAC approach impractical for switched systems.

4. VS ADAPTIVE CONTROL WITH TUNING FUNCTION DESIGN

The result of robust MRAC approach for the considered problem depends on the unknown ADT condition which is difficult to be obtained due to the unknown parameters. We propose a VS based adaptive control scheme with the tuning function design method to improve the result.
4.1 VS Based Tuning Function Design with Relative Degree One

For switched systems (1), the adaptive backstepping control using tuning function is appealing since the unknown parameters are parameterized in the input channel. Details of tuning function designed is introduced in Kristić et al. (1995). Now, we design a VS adaptive backstepping controller using tuning function design to the switched linear systems with relative degree one. First, an adaptive observer using K-filters is used to estimate the state. Rewrite system (1) as

\[
\dot{x}_p = Ax_p + F(y_p, u)^T \theta^\sigma \\
y_p = C^T x_p,
\]

where

\[
A = \begin{bmatrix} 0 & I_{n-1} \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \quad \theta^\sigma = \begin{bmatrix} b_m^\sigma \\ a_1^\sigma \\ \vdots \\ a_m^\sigma \end{bmatrix},
\]

and

\[
F(y_p, u)^T = \begin{bmatrix} 0_{(N-1)\times(n+m+1)} \\ I_{(m+1)\times(n+m+1)} \end{bmatrix} \begin{bmatrix} -I_{n\times n} y_p \end{bmatrix}_{n\times(n+m+1)}.
\]

Choose a vector \( k = [k_1, \ldots, k_m]^T \) such that \( A_0 = A - k e^T \) is Hurwitz. Then there exists \( P = P^T > 0 \) such that

\[
A_0^T P + P A_0 = -I.
\]

Define the following filters

\[
\dot{\xi} = A_0 \xi + ky \\
\dot{\Omega}^T = A_0 \Omega^T + F(y_p, u)^T,
\]

and let the state estimation be

\[
\hat{x}_\epsilon = \xi + \hat{\Omega}^T \theta^\sigma,
\]

then the state estimation error \( \epsilon = x_p - \hat{x}_\epsilon \) will vanish exponentially with \( \hat{\epsilon} = A_0 \hat{\epsilon} + \epsilon \)

\[
x_p = \hat{x}_\epsilon + \epsilon = \xi + \hat{\Omega}^T \theta^\sigma + \epsilon.
\]

From the special structure of \( F(y_p, u)^T \), we can denote \( \Omega^T = [v_1, \ldots, v_m, \Xi] \), where \( v_j \in \mathbb{R}^n, j = 0, 1, \ldots, m \), are column vectors that can be obtained from a filter of input u, and \( \Xi \) can be obtained from a filter of output \( y_p \). Implementation of K-filters are summarized in the following (Kristić et al. (1995)):

\[
\hat{\eta} = A_0 \eta + e_n y, \quad \hat{\lambda} = A_0 \lambda + e_n u \\
\hat{\Omega}^T = [v_1, \ldots, v_m, \Xi], \quad v_j = (A_0)^j \lambda, \quad j = 0, \ldots, m \\
\Xi = -[(A_0)^{n-1} \eta, \ldots, A_0 \eta, \eta], \quad \xi = - (A_0)^n \eta
\]

Suppose that relative degree of \( W_{pi} \) is one, \( \forall i \in \mathbb{P} \), that is, \( N = 1 \). Then, \( \dot{x}_i = x_2 - a_1^\sigma y_p + b_m^\sigma u = x_2 - y_p e_1^\sigma a^\sigma + b_m^\sigma u. \)

By (21), \( x_2 \) can be represented by

\[
x_2 = \xi_2 + [v_{m,2}, v_{m-1,2}, \ldots, v_{0,2}, \Xi(2)] \theta^\sigma + \epsilon_2.
\]

Here the subindex 2 in the right hand side denotes the second component of a column vector. For example, \( v_{m,2} \) denotes the second component of \( v_m \).

Thus, we have

\[
\dot{x}_1 = \xi_2 + [v_{m,2}, v_{m-1,2}, \ldots, v_{0,2}, \Xi(2)] \theta^\sigma + \epsilon_2 + b_m^\sigma u \\
= \xi_2 + v^T \theta^\sigma + \epsilon_2 + b_m^\sigma u,
\]

where \( v = [v_{m,2}, v_{m-1,2}, \ldots, v_{0,2}, \Xi(2)] \theta^\sigma \).

Define the output error \( z_1 = y_p - y_m \), then

\[
\dot{\hat{z}}_1 = \xi_2 + v^T \theta^\sigma + \epsilon_2 + b_m^\sigma u - \hat{y}_m.
\]

Design the control law as

\[
u = \gamma \alpha_1, \quad \gamma > 0, \quad \gamma > 1/|b_m^\sigma|,
\]

where \( \gamma \) is the estimation of \( \gamma^\sigma = 1/\|b_m^\sigma\| \) and \( \alpha_1 = -c_1 z_1 - d_1 z_1 - \gamma \hat{\theta} + \hat{y}_m \), where \( v_a = \|[v_1, \ldots, v_{n+m+1}]^T \| \).

The estimate parameters are decided by the variable structure adaptive laws similar to that in Queiroz et al. (2008):

\[
\dot{\theta}_i = \text{sgn}(z_i) \hat{\theta}_i, \quad \hat{\theta}_i > |\theta_i^\sigma|,
\]

\[
\gamma = -\gamma \text{sgn}(b_m^\sigma) \text{sgn}(\alpha_1 z_1), \quad \gamma > 1/|b_m^\sigma|,
\]

for \( i = 1, 2, \ldots, n + m + 1, \) and \( \sigma(t) \in \mathbb{P} \). Here \( v_1 \) stands for the i\th component of the regressor vector \( v \). Now we can discuss the stability properties of the closed-loop system. Consider a Lyapunov function \( V_1 = \frac{1}{2}(z_1^2) + \frac{1}{2\sigma^2 c_1 T} \epsilon \), then the time derivative of \( V_1 \) is

\[
\dot{V}_1 = -c_1 (z_1^2) + (v^T \theta^\sigma - v_a^T \theta^\sigma) z_1 - b_m^\sigma \gamma^\sigma \alpha_1 z_1 - \frac{1}{4d_1} \epsilon^T \epsilon
\]

\[
- d_1 z_1^2 + z_1 \epsilon_2 - \frac{1}{4d_1} \epsilon^T \epsilon
\]

\[
\leq -c_1 (z_1^2) + (v^T \theta^\sigma - v_a^T \theta^\sigma) z_1 - b_m^\sigma \gamma^\sigma \alpha_1 z_1 - \frac{1}{4d_1} \epsilon^T \epsilon
\]

\[-d_1 z_1 + \frac{1}{4d_1} \epsilon^T \epsilon
\]

\[
- \left( b_m^\sigma \gamma^\sigma \alpha_1 z_1 + |\alpha_1| \right) - \frac{1}{4d_1} \epsilon^T \epsilon
\]

\[
\leq -c_1 (z_1^2) - \frac{1}{4d_1} \epsilon^T \epsilon
\]
Theorem 3. For switched system (1) which satisfies (A1)-(A5) with $N = 1$ and using the VS based adaptive tuning function design controller (26), (27), (28), all signals in the closed loop system are bounded and the tracking error $z_1$ will converge to zero independent of the switching signals.

Remark 1. For relative degree one case, given any non-zero switching signals $\sigma$, the output will converge to the reference output exponentially and all signals in the switched system are bounded. This good property is attributed to the parametrization of the K-filter and the VS adaptive law. The parametrization in tuning function scheme leads the switched system to a non-switched one with a step change in the input channel. VS adaptive law is insensitive to parameter jumps compared with traditional integral type adaptive law. Thus, the stability properties are derived independent of the switching signal. In Queiroz et al. (2008), the output feedback VS adaptive backstepping control with tuning function design is also proposed for non-switched systems.

4.2 VS Based Tuning Function Design with Relative Degree Two

We extend the VS design to the case with $N = 2$ and choose $v_m$ as the virtual control input for backstepping. Assume that

- (A6) $b_m^{(t)} \equiv b_m = \frac{1}{\gamma}$ is a non-switching parameter.

Now

$$\hat{z}_1 = \xi_2 + b_m v_m + \varphi^T \theta^T \sigma + \epsilon_2 - \gamma y_m,$$

where $\varphi = [0, v_m, \ldots, v_m]$. Let $z_2 = v_{m,2} - \gamma y_m - \alpha_1$ and choose

$$\alpha_1 = \gamma \hat{z}_1 = (\gamma - \gamma) \alpha_1$$

with $\hat{z}_1 = -c_1 z_1 - d_1 z_1 - \xi_2 - \varphi^T \theta$. Then the system becomes

$$\dot{z}_1 = -c_1 z_1 - d_1 z_1 + \epsilon_2 - b_m (\gamma y_m - \alpha_1) \hat{z}_1 + \varphi^T \theta^T \sigma + b_m z_2$$

Choose

$$\dot{V}_1 = \frac{1}{2} \gamma z_1^2 + \frac{1}{2} \varphi^T \Gamma^{-1} \varphi + \frac{b_m}{\gamma} \gamma z_1^2 + \frac{1}{2} \epsilon_2^T \epsilon,$$

where $V_1$ is a non-switching time interval, then

$$\dot{V}_1 = z_1 \gamma \dot{z}_1 - \varphi^T \Gamma^{-1} \varphi - \frac{b_m}{\gamma} \gamma z_1^2 - \frac{1}{2} \epsilon_2^T \epsilon$$

and $\gamma$ is a specified constant vector. Note that $sgn(b_m) = 1$ and $-c_1 z_1^2 + \gamma z_1 - \varphi^T \theta^T \sigma \leq 0$. Moreover, $\dot{\theta} - \theta_{\epsilon} = -\varphi^T \theta - \theta_{\epsilon}$.

where $\theta_{\epsilon}$ is a specified constant vector. Note that $sgn(b_m) = 1$ and $-c_1 z_1^2 + \gamma z_1 - \varphi^T \theta^T \sigma \leq 0$. Moreover, $\dot{\theta} - \theta_{\epsilon} = -\varphi^T \theta - \theta_{\epsilon}$.

where $\dot{\theta} = \gamma [z_1 + (\gamma - \gamma^*) + \phi(\gamma)]$,

where $\delta_1 > 0$ can be chosen arbitrarily, $\gamma^* \leq \gamma$ is a positive constant which plays the role as $c^*$ in (14), and $|\phi(\gamma)| < c_3$ is a bounded smooth function to make $\dot{\gamma}$ continuously differentiable at $|z_1| = \delta_1$, for example, we can use functions like tanh() to construct $\phi$. When $|z_1| \geq \delta_1$, we design

$$\dot{\gamma} = -\gamma \gamma [z_1 + (\gamma - \gamma^*) + \phi(\gamma)]$$

(32)

Then, when $|z_1| < \delta_1$, we have

$$\dot{V}_1 = -c_1 (z_1)^2 - b_m z_1 z_2 - b_m (|\gamma| z_1 - \gamma) + \varphi^T \Gamma^{-1} \varphi - \frac{1}{4} \gamma^2 \epsilon_2^T \epsilon$$

$$= -c_1 (z_1)^2 - b_m z_1 z_2 + c_1$$

(33)

where $c_1 > 4|b_m|^2 + |z_2|^2, |b_m (\gamma - \gamma^*) + \gamma| > 0$ is a positive constant.

When $|z_1| > \delta_1$, the VS adaptive law of $\dot{\gamma}$ is active and $\gamma = 0$. It can be derived that

$$\dot{V}_1 = -c_1 (z_1)^2 + b_m z_1 z_2 + \varphi^T \Gamma^{-1} \varphi - \frac{1}{4} \gamma^2 \epsilon_2^T \epsilon$$

(33)

The ideal for the design of $\dot{\gamma}$ is to use a smooth function $\gamma$ to approximate $\gamma$ such that backstepping can be proceeded for higher relative degree case. The design of $\theta$ will be discussed latter.

Let $z_2 = v_{m,2} - \gamma y_m - \alpha_1$ and from definition we know that $\gamma = v_{m,2} - k_2 v_{m,1} + u$, $u$ is a function of $y, \eta, \gamma, \dot{\theta}$.

Then

$$\dot{z}_2 = \hat{z}_2 = \gamma y_m - \gamma y_m - \alpha_1$$

Choose

$$\dot{y} = \gamma y_m - \gamma y_m - \alpha_1$$

(35)

where $a_2$ is to be designed, then

$$\dot{z}_2 = \gamma y_m - \gamma y_m - \alpha_1$$

Now consider $V_2 = \frac{1}{2}(z_2^2) + \frac{1}{2} \epsilon_2^T \epsilon$. For non-switching time interval, we have

$$\dot{V}_2 = \gamma z_2 \hat{x}_2 + \varphi^T \theta^T \sigma + \epsilon_2 - \gamma z_2 \hat{x}_2$$

(36)
Note that $-z_2 \frac{\partial \alpha_1}{\partial y} \epsilon - \frac{1}{4d_2} (\epsilon_2)^2 = d_2 \left( \frac{\partial \alpha_1}{\partial y} \right)^2 (z_2)^2 - d_2 (z_2 \frac{\partial \alpha_1}{\partial y} + \frac{1}{2d_2} \epsilon_2)^2$. Let $\tau_2 = \tau_1 - \frac{\partial \alpha_1}{\partial y} z_2$ and choose $\alpha_2 = - \beta_m z_1 + \beta_2 z_2 - d_2 \left( \frac{\partial \alpha_1}{\partial y} \right)^2 z_2 + \frac{\partial \alpha_1}{\partial \theta} \tau_2$. (37)

Time derivative of $V_2$ during non-switching time interval satisfies

$$
V_2 \leq -c_1 (z_1)^2 - c_2 (z_2)^2 - |b_m| (\gamma t)^2 - \frac{1}{4d_1} \epsilon^T \epsilon + c_7 + \hat{\theta}^T \hat{\theta} + \gamma t \frac{\partial \alpha_1}{\partial \theta} \Gamma \tau_2 - \hat{\theta}^T \hat{\theta}.
$$

(38)

Design

$$
\dot{\hat{\theta}} = \Gamma \tau_2.
$$

(39)

Then, for non-switching time interval

$$
\dot{V}_2 \leq -c_1 (z_1)^2 - c_2 (z_2)^2 - |b_m| (\gamma t)^2 - \frac{1}{4d_1} \epsilon^T \epsilon + \hat{\theta}^T \hat{\theta} + \gamma t \frac{\partial \alpha_1}{\partial \theta} \Gamma \tau_2 - \hat{\theta}^T \hat{\theta}.
$$

(40)

Proof. From analysis in (40), we can see that the Lyapunov function is exponentially convergent to a set proportional to $c_1$ and $c_2$. Hence, boundedness of $z_1$, $z_2$, $\hat{\theta}$, and $\gamma t$ are guaranteed during non-switching time interval. When parameter switches, there will be some jump in $V_2$ due to the differences between $\hat{\theta}^{\sigma}(t)$ and $\hat{\theta}^{\sigma}(t^-)$ at the switching time instant $t$. However, since the difference is a bounded quantity, the switching index set $P$ is a finite integer set, and the Lyapunov function is exponential convergent except for the switching time instant, the overall Lyapunov function $V = V_1 + V_2$ will satisfy

$$
V(t) \leq e^{-\gamma t} V(0) + C
$$

for some bounded constant $C$. For the rest of the states $x_3, ..., x_n$, boundedness can be shown by similar argument as in Theorem 3. Thus, we can conclude that all signals are bounded and the output error will converge to a residue set which is related to $c_1$, $c_2$, and the unknown parameters $\theta^\sigma$.

Remark 2. For $N = 2$, the resultant performance is not as good as that in $N = 1$ since we do not incorporate VS adaptive law in $\dot{\theta}$. The reason why not use the VS type $\dot{\theta}$ is that it will cause discontinuities in $z_2$. So we only design the VS adaptive law for $\dot{\gamma}$. Note that $z_2$ is continuous under since $\gamma$ and $\dot{\theta}$ are continuous. If a VS type $\dot{\theta}$ can be designed with which the discontinuous problem is resolved, the result will be much improved.

5. CONCLUSION

In this paper, we design robust adaptive controllers for a class of unknown switched systems and provide theoretical analysis for the stability properties. We also propose a VS based design with tuning function to obtain output error convergence for unknown switched systems. Compared with the robust MRAC approach, the proposed VS design has the advantage that the stability result is independent of the dwell time of the switching signals. From the analysis, we can see that VS type adaptive controller has good ability to suppress parameter switching. However, for backstepping approach, VS control is not always achievable. Design the adaptive controller for more general class of switched systems is the main direction of our future works.

REFERENCES


