Unknown Inputs Functional Observers
Designs For Descriptor Systems with
Constant Time Delay

M. Ezzine ∗ M. Darouach ∗∗ H. Souley Ali ∗∗ H. Messaoud ∗∗∗

∗ Ecole Nationale d’Ingénieurs de Monastir, Avenue Ibn El Jazhar,
5019 Monastir, Tunisie and with CRAN Nancy Université, IUT de
Longuy, 186 Rue de Lorraine, 54400 Cosnes et Romain, France
(e-mail: montassar.ezzine@iut-longuy.uhp-nancy.fr).

∗∗ CRAN Nancy Université, IUT de Longuy, 186 Rue de Lorraine,
54400 Cosnes et Romain, France (e-mails: (darouach,
souley}@iut-longuy.uhp-nancy.fr)

∗∗∗ Ecole Nationale d’Ingénieurs de Monastir, Avenue Ibn El Jazhar,
5019 Monastir, Tunisie, (e-mail: hassani.messaoud@enim.rnu.tn)

Abstract: In this paper, both time and frequency domain new designs of Unknown Inputs Functional Observers (UIFO) for a class of descriptor systems with a constant time delay are presented. The order of this unknown input observers is equal to the dimension of the vector to be estimated. The time procedure design is based on Lyapunov-Krasovskii stability theory where, after given the existence condition of such observers, the optimal gain implemented in the functional observer with internal delay design is obtained in terms of linear matrix inequalities (LMIs). A design algorithm of UIFO is proposed ; The frequency procedure design is derived from time domain results by applying the factorization approach, where we define some useful Matrix Fraction Descriptions (MFDs). The effectiveness of the proposed approach is illustrated by a numerical example.

Keywords: Filtering techniques, Time-frequency representation , Time-delay, H-infinity optimization.

1. INTRODUCTION

Descriptor systems can be viewed as a generalization for standard systems. In fact, importance of such systems comes from their ability to describe the systems for which standard state-space representations are not applicable. Therefore, the observers design for descriptor systems are of considerable interest.

Time delay systems are commonly encountered in various engineering systems, such as chemical processes, long transmission lines in pneumatic and hydraulic systems (see Niculescu, [2001], Malek and Jamshidi, [1987], Hale and Lunel, [1993]). The time delay usually results in unsatisfactory performance and is frequently a source of instability, so control of time-delay systems is practically important. This control is often realized with the assumption that the entire state vector is available through output measurement. Since this is not generally true in the practice, it is necessary to design observers which produce an estimate of this state vector.

Observer design theory for time-delay systems has been most widely considered in the last decade and several design methods have been proposed (reducing transformation technique, coordinate change approach, LMI method, ... Pearson and Fiagbedzi, [1989], Hou and al. [2002], Darouach. [2001], Darouach. [2007], Fu and al. [2004]). The observers for systems with unknown inputs are of great interest in the failure detection and the control of systems in presence of disturbances (see Darouach and Zasadinski. [1994]). However, little research has been focused on design of UIFO for descriptor systems with time-delay.

In frequency domain, there is less literature in state estimation compared to that of time domain (see Sename. [1997], Yao and Zhang. [1996], [?], Ding and al. [1994], Ding and al. [1990], Hippe and Wurmlacher. [1990], Hippe. [1989], Hippe. [1991], Markez. [2003]), although it is the basis for most analysis performed on control systems.

Motivated by these facts, a new time and frequency domain methods for the functional observers design for time delay singular systems with unknown inputs is presented. The time domain procedure is based on Lyapunov stability theory, where we give the existence condition of such observers and the gain implemented in the design is obtained using (LMIs). The frequency domain procedure design is derived from time domain results, where we propose some suitable Matrix Fractions Descriptions (MFDs) and mainly, applying the factorization approach permits to give a polynomial description of the proposed functional observer.

The main reason of formulating the results of the time domain in the frequency one is the advantages that it presents for the observer-based control (see Hippe and al.
In fact, in this case, the compensator is driven by the input and the output of the system. So only the input-output behavior of the compensator (characterized by its transfer function) influences the properties of the closed-loop system. The additional degrees of freedom given by the frequency approach can then be used for robustness purpose for example (see Hippe and al. [2009]). Note that another interest of our approach, is the use of a functional observer (not a full order one) which estimates the desired functional of the state, without estimating all the state of the system contrary to full order. These benefits include time reduces computations and its use for large scale systems.

2. PROBLEM STATEMENT

Let’s consider the following continuous-time linear time-delay descriptor system described by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + A_dx(t - d) + Bu(t) + E_1w(t) \quad (1a) \\
y(t) &= Cx(t) \quad (1b) \\
z(t) &= Lx(t) \quad (1c)
\end{align*}
\]

with the initial state \(x(\theta) = \phi(\theta)\forall \theta \in [-d, 0]\) where \(x(t) \in \mathbb{R}^n\) is the state vector, \(y(t) \in \mathbb{R}^m\) is the measured output vector, \(u(t) \in \mathbb{R}^r\) is the input vector, \(w(t) \in \mathbb{R}^q\) represents the unknown input vector, \(z(t) \in \mathbb{R}^{m_z}\) is the vector to be estimated, with \(m_z \leq n\). \(d\) is a known constant time-delay of the system. Matrices \(E \in \mathbb{R}^{m \times n}\), \(A, A_d, B, E_1, C\) and \(L\) are known constant matrices of appropriate dimensions.

In the sequel of the paper we make the following assumption.

**Hypothesis 1.** Darouach and Boutayeb. [1995]

1) \(\text{rank } E = r_1 \leq n\)

2) \(\text{rank } \begin{bmatrix} E \\ C \end{bmatrix} = n\)

3. UIFO TIME DOMAIN DESIGN

Under assumption 1, there exists a nonsingular matrix,

\[
\begin{bmatrix}
a_0 & b_0 \\
c_0 & d_0
\end{bmatrix}
\]

such that,

\[
\begin{align*}
a_0 E + b_0 C &= I_n \\
c_0 E + d_0 C &= 0_{m \times n}
\end{align*}
\]

We aim to design a functional observer of order \((m_z)\) for system (1), of the form:

\[
\begin{align*}
e(t) &= Ne(t) + N_d e(t - d) + Hu(t) + D_1 y(t) + D_2 y(t - d) \\
\dot{z}(t) &= e(t) + L_0 y(t) + L_2 d_0 y(t)
\end{align*}
\]

where \(e(t) \in \mathbb{R}^r\) is the estimation error, \(z(t) \in \mathbb{R}^{m_z}\) is the estimate of \(z(t)\).

The problem of Unknown Input Functional Observer (UIFO) design to be solved in the paper can be stated as follows.

**Problem UIFO:** Given the system (1) and the functional observer (5), design the observer matrices \(N, N_d, H, D_1, D_2\) and \(E_2\) such that \(\dot{z}\) asymptotically converges to \(z\).

3.1 UIFO Conditions of Constant Time-Delay Descriptor Systems

Define \(e(t)\) as the time estimation error, using (3) and (4) it is given by

\[
\begin{align*}
e(t) &= z(t) - \hat{z}(t) \quad (6a) \\
&= L(I - b_0 C - E_2 d_0 C)x(t) - e(t) \quad (6b) \\
&= \psi_1 Ez(t) - e(t) \quad (6c)
\end{align*}
\]

with

\[
\psi_1 = L(a_0 + E_2 c_0)
\]

then, we have the following theorem.

**Theorem 1.** The functional observer (5) is a UIFO for singular system (1) if and only if the following equations are satisfied:

\[
\begin{align*}
i) \quad &e(t) = Ne(t) + N_d e(t - d) \quad \text{asymptotically stable.} \\
ii) \quad &\psi_1 A - N\psi_1 E - D_1 C = 0 \\
iii) \quad &\psi_1 A_d - N_d \psi_1 E - D_2 C = 0 \\
iv) \quad &\psi_1 E_1 = 0 \\
v) \quad &H = \psi_1 B
\end{align*}
\]

**Proof.** It is easy to prove the previous theorem. In fact, from (1) and (5), the dynamics of the estimation error given by (6c), can be written as

\[
\begin{align*}
\dot{e}(t) &= \psi_1 E \dot{z}(t) - e(t) \quad (8a) \\
&= N e(t) + N_d e(t - d) + \psi_1 E_1 w(t) + (\psi_1 B - H) u(t) \\
&\quad + (\psi_1 A - N\psi_1 E - D_1 C)x(t) + (\psi_1 A_d - N_d \psi_1 E - D_2 C)x(t - d) \quad (8b)
\end{align*}
\]

with the initial condition \(e(t) = z(t) - \hat{z}(t), \forall t \in [-d, 0]\).

So, if the fifth conditions i) – vi) are satisfied, then the unknown inputs functional observer (5) will estimate asymptotically the functional \(z(t)\), for any initial conditions and any \(u(t)\).

3.2 UIFO Design of Constant Time-Delay Descriptor Systems

A new method is presented to design unknown inputs functional observers (5) for singular systems (1). Then, using LMI approach, the independence of delay conditions for the stability of the functional observer are given.

Now using the definition of \(\psi_1\) with equations (3) and (4), conditions ii) – iv) can be written as:

\[
\begin{align*}
NL a_0 E + K_1 C - LE_2 c_0 A &= La_0 A \quad (9) \\
N_d L a_0 E + K_2 C - LE_2 c_0 A_d &= La_0 A_d \quad (10) \\
-LE_2 c_0 E_1 &= La_0 E_1
\end{align*}
\]

where
\( K_1 = D_1 - NLE_2d_0 \) (12)

and

\( K_2 = D_2 - N_dLE_2d_0 \) (13)

Notice that once matrix \( E_2 \) is determined, matrix \( H \) is immediately deduced from condition \( r \) of theorem 1.

Equations (9)-(11) can be transformed to the following matrix form:

\[ X \Sigma = \Theta \] (14)

where,

\[ \begin{bmatrix} N & N_d & K_1 & K_2 & -LE_2 \end{bmatrix} = X \] (15)

\[ \begin{bmatrix} La_0 & 0 & 0 \\ 0 & La_0E & 0 \\ C & 0 & 0 \\ 0 & C & 0 \\ c_0A & c_0A_d & c_0E_1 \end{bmatrix} = \Sigma \] (16)

\[ \begin{bmatrix} La_0 & La_0A & La_0E_1 \end{bmatrix} = \Theta \] (17)

The matrix equation (14) has a solution \( X \) if and only if

\[ \text{rank} \left( \begin{bmatrix} \Sigma \\ \Theta \end{bmatrix} \right) = \text{rank} \Sigma \] (18)

and a general solution for (14), if it exists, is given by

\[ X = \Theta \Sigma^+ - Z(I - \Sigma \Sigma^+) \] (19)

where \( \Sigma^+ \) is a generalized inverse of matrix \( \Sigma \) given by (16) and \( Z \) is an arbitrary matrix of appropriate dimensions, that will be determined in the sequel using LMI approach.

Once matrix \( X \) is determined, it is easy to give the expressions of unknown matrices \( N, N_d, K_1, K_2 \) and \(-LE_2\).

In fact,

\[ N = X \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (20)

\[ = A_{11} - ZB_{11} \] (21)

where

\[ A_{11} = \Theta \Sigma^+ \] (22)

\[ B_{11} = (I - \Sigma \Sigma^+) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (23)

\[ N_d = X \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (24)

\[ = A_{22} - ZB_{22} \] (25)

where

\[ A_{22} = \Theta \Sigma^+ \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (26)

\[ B_{22} = (I - \Sigma \Sigma^+) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (27)

\[ K_1 = X \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (28)

\[ = A_{33} - ZB_{33} \] (29)

where

\[ A_{33} = \Theta \Sigma^+ \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (30)

\[ B_{33} = (I - \Sigma \Sigma^+) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (31)

\[ K_2 = X \begin{pmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{pmatrix} \] (32)

\[ = A_{44} - ZB_{44} \] (33)

where

\[ A_{44} = \Theta \Sigma^+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (34)

\[ B_{44} = (I - \Sigma \Sigma^+) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \] (35)
\[-LE_2 = X \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t \end{pmatrix} = A_{55} - ZB_{55} \] (36)

where

\[ A_{55} = \Theta \Sigma^+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{pmatrix} \] (37)

substituting (21) and (25) in (46), it can be easily obtained that (46) is equivalent to the LMI (42).

4. FREQUENCY DOMAIN DESIGN

In this section we propose an easier technique of designing an unknown inputs functional observer described in the frequency domain by applying the factorization approach Vidyasagar. [1985] (see also Ding and al. [1990]) and Yao and al. [1997].

4.1 Review of factorization approach: case with delay

Consider a transfer function matrix \( G(s) = G_0(s)e^{-\tau s} \), where \( G_0 \in IR(s) \) is a strictly proper rational transfer matrix, with the state-space realization \( G(s) = C(sI - A)^{-1}Be^{-\tau s} \), and \( \tau \) is the constant time delay. The double coprime factorization of \( G(s) \) can be written as (Yao and al. [1997]), see also (Nobuyama and Kitamori. [1990]- Nobuyama. [1992]):

\[ G(s) = G_1(s)G_2^{-1}(s) = \hat{G}_1^{-1}(s)\hat{G}_2(s) \] (47)

The four matrices above can be calculated by the standard algorithms in the state-space construction and are given below

\[ G_2(s) = K(sI - A - BK)^{-1}B + I \] (48)

\[ G_1(s) = C(sI - A - BK)^{-1}Be^{-\tau s} \] (49)

\[ \hat{G}_1(s) = C(sI - A - L_1C)^{-1}L_1 + I \] (50)

\[ \hat{G}_2(s) = C(sI - A - L_1C)^{-1}Be^{-\tau s} \] (51)

where \( K \) and \( L_1 \) are chosen such that \( \text{det}(sI - A - BK) \) and \( \text{det}(sI - A - L_1C) \) are stable.

Remark 1. Note that, for the case of transfer function "without delay", one can refer to the corresponding factorization approach given by Vidyasagar. [1985] and Ding and al. [1990].

4.2 Frequency domain synthesis

The next theorem presents the second result of the paper by giving a polynomial description of the proposed functional observer for delay descriptor systems, which is based mainly on the factorization approach.

Theorem 3. Consider the following right coprime factorization based on Matrix Fraction Descriptions (MFDs)

\[ (sI - N_1(s))^{-1}D_2e^{-ds} = N_2(s)M_2^{-1}(s) \] (52)
\( N_1(s) = N + N_d e^{-ds} \)

b) The two polynomial matrix \( N_2(s) \) and \( M_2(s) \) have the specification to be right coprime. These transfer functions, as mentioned above, can be calculated from the factorization approach, as in (Nobuyama and Kitamori. [1990]- Nobuyama. [1992]).

\[ (sI - N_1(s))^{-1}H = N_3(s)M_3^{-1}(s) \]  

(53)

\[ (sI - N_1(s))^{-1}D_1 + D_0 = N_4(s)M_4^{-1}(s) \]  

with \( D_0 = Ld_0 + LE_2d_0 \).

Note that, polynomial matrices \( N_3(s), M_3(s), N_4(s) \) and \( M_4(s) \) can be calculated from Vidyasagar. [1985].

Then, a frequency domain representation of the functional observer (5) of order \( m_z \), \( m_z \leq n \), related to singular system (1) is given by,

\[ \dot{z}(s) = N_3(s)M_3^{-1}(s) \times u(s) + [N_2(s)M_2^{-1}(s) + N_4(s)M_4^{-1}(s)] \times y(s) \]  

(55)

Proof. The s-transform of (5a) reads

\[ \epsilon(s) = (sI - N_1(s))^{-1}Hu(s) + (sI - N_1(s))^{-1}D_1y(s) + (sI - N_1(s))^{-1}D_2e^{-ds}y(s) \]  

(56)

So, the s-transform of the estimated vector \( \dot{z}(s) \) (5b), can be written by taking into account (56) as

\[ \dot{z}(s) = (sI - N_1(s))^{-1}Hu(s) + (sI - N_1(s))^{-1}D_2e^{-ds}y(s) + [(sI - N_1(s))^{-1}D_1 + D_0]y(s) \]  

(57)

Therefore and in view of the right coprime MFDs (52)-(54), the frequency domain description (55) holds.

Note, that polynomial matrices implemented in the proposed MFDs (see Yao and al. [1997]) and Vidyasagar. [1985]), are given as follows:

\[ N_2(s) = (sI - N_1(s) - D_2K)^{-1}D_2e^{-ds} \]  

(58)

\[ M_2(s) = K(sI - N_1(s) - D_2K)^{-1}D_2 + I \]  

(59)

\[ N_3(s) = (sI - N_1K_1(s))^{-1}H \]  

(60)

\[ M_3(s) = K_1(sI - N_1K_1(s))^{-1}H + I \]  

(61)

\[ N_4(s) = (I + D_0K_2)(sI - N_1K_2(s))^{-1}D_4 + D_0 \]  

(62)

\[ M_4(s) = K_2(sI - N_1K_2(s))^{-1}D_4 + I \]  

(63)

\( K_1, K_2 \) and \( K \) are such that \( N_1K_1(s) = N_1(s) + HK_1, N_1K_2(s) = N_1(s) + D_1K_2 \) and \( \text{det}(sI - N_1(s) - D_2K) \) are stable.

5. NUMERICAL EXAMPLE

Consider the time-delay singular system presented in section 2, where

\[ E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \ A_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ L = \begin{bmatrix} 1 & 1 \end{bmatrix}. \]

1 - Time domain functional observer design

By applying the proposed method, we obtain the following results:

\[ a_0 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \ b_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ c_0 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ d_0 = 0 \]

and the functional observer matrices values are given as follows, after solving the LMI (42):

\[ Z = 10^{15} \begin{bmatrix} -0.0336 & -1.8879 & -0.7062 & -0.3653 & -1.8925 \end{bmatrix} \]

\[ N = -2.5815, \ N_d = -2.4775, \ H = 1 \]

\[ D_1 = 1.6788, \ D_2 = 3.9433, \ LE_2 = 0.1706 \]

2 - Frequency domain functional observer design

For that, let \( d = 0.1 \). By using the factorization approach as mentioned in section (4), see ((58)-(63)), and with \( K = 1, K_1 = 12.5815, K_2 = 7.4943 \) and \( N_1(s) = -2.5815 - 2.4775e^{-0.1s} \), the polynomial matrices implemented in the observer design are given as follows:

\[ N_2(s) = \frac{3.9433e^{-0.1s}}{s + 2.4775e^{-0.1s} - 1.3618} \]  

(64)

\[ M_2(s) = \frac{s + 2.4775e^{-0.1s} + 2.5815}{s + 2.4775e^{-0.1s} - 1.3618} \]  

(65)

\[ N_3(s) = \frac{1}{s + 2.4775e^{-0.1s} - 10} \]  

(66)

\[ M_3(s) = \frac{s + 2.4775e^{-0.1s} + 2.5815}{s + 2.4775e^{-0.1s} - 10} \]  

(67)

\[ N_4(s) = \frac{1.6788}{s + 2.4775e^{-0.1s} - 10} \]  

(68)

\[ M_4(s) = \frac{s + 2.4775e^{-0.1s} + 11.1218}{s + 2.4775e^{-0.1s} - 10} \]  

(69)

so, the frequency domain description (see (55)) related to singular system (1) is determined.

6. CONCLUSION

In this paper two new methods for functional observers design for a class of descriptor time delay systems with unknown inputs has been developed. The time domain method is based on Lyapunov-Krasovskii stability theory
where we give the existence condition of such observers, and an algorithm that summarizes main steps of resolution is given. Then, based on time domain results and by applying the factorization approach, a new frequency domain description of the unknown inputs functional observer is derived. Demonstrative example proves the effectiveness of the proposed algorithm. Further works will concern the design of functional filters for systems where measurements are affected by disturbances and the observer-based compensator design as it is the main motivation of the frequency domain approach Hippe and al. [2009].

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