Human dynamics and stability of teleoperator systems with generalized projection-based force reflection algorithms

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Abstract: A new general stability result for networked bilateral teleoperator systems with projection-based force reflection is presented. Contrary to the previous results, the human operator dynamics are taken into account in the stability analysis. The human dynamics are assumed to satisfy a general condition formulated in terms of finite distance between output of the human dynamics and a certain “passivity-based” stabilizing set. In particular, this assumption allows for both passive and non-passive behaviour of the human operator. The overall stability of the bilateral teleoperator system is proven in the presence of network-induced communication constraints for an arbitrary projection-based force reflection algorithm from a broad class.

1. INTRODUCTION

Design of high-performance networked bilateral teleoperators presents a number of substantial challenges. Some of the most important problems in this area include stability in the presence of network-induced communication constraints and the trade-off between stability and transparency. One particular formulation of the latter problem can be given in terms of trade-off between the overall stability and a high force reflection gain. Specifically, higher force reflection gain improves haptic perception of the remote object, however, it also increases the closed-loop gain which leads to instability. This problem was addressed extensively in the literature on teleoperator systems, including Hannaford (1989); Hannaford and Anderson (1988); Speich et al. (2000); Kuchenbecker and Niemeyer (2006), among other works. This problem motivated the introduction of the projection-based force reflection algorithms for bilateral teleoperation with communication constraints Polushin et al. (2007), which were further developed in Polushin et al. (2008a,b), with some experimental results presented in Polushin et al. (2009a). The idea behind these algorithms is to decompose the reflected force into “interaction” and “momentum-generating” components. The “interaction” component is directly compensated by the interaction with the human hand; this is the component that the human operator actually feels. The residual component of the reflected force is not compensated during the interaction with the human hand and is not explicitly felt by the human operator; this is the component that may generate momentum leading to instability. The job of the projection-based force reflection algorithms is to identify the “interaction” and the “momentum-generating” components and attenuate the latter while applying the former in full. Using this approach, the constraints on force reflection gain can be effectively removed without losing the overall stability of the teleoperator system.

However, a common shortcoming of the previously published results (Polushin et al. (2007, 2008a,b, 2009a)) is that the human operator forces were always considered as an external input; in other words, the dynamics of the human operator hand were not taken into account when analyzing stability of the bilateral teleoperator system with projection-based force reflection algorithms. The motivation for such a simplified approach is that in impedance controlled haptic systems (i.e., where movement is measured and force is reflected) which are addressed in the above cited works, the worst-case scenario for stability is the situation where the operator releases the haptic device (Niemeyer et al., 2008, p. 752). In other words, an impedance controlled haptic system which is stable without contact with the human hand would remain stable when interacting with a human hand. Thus, the dynamics of the human operator can be ignored in stability analysis. However, a more interesting and complete approach would be to incorporate a dynamical model of the human hand into the mathematical description of the teleoperator system. Specifically, it would be beneficial to define a significantly wide and well-motivated class of the human hand dynamics and show that the stability of the teleoperator system is achieved for all possible human dynamics from the above class. In particular, this is the approach taken in more conventional passivity-based design methods (Anderson and Spong (1989); Niemeyer and Slotine (2004); Stramigioli et al. (2002); Lee and Spong (2006)), where the human dynamics are assumed to belong to a general class of passive systems. Although such a passivity assumption is somehow restrictive and even counterintuitive, experimental evidences suggest that the human operator can be...
have passively (if she wish) when interacting with a passive manipulandum (see Hogan (1989)). It is obvious, however, that the human behaviour is not necessarily passive; more specifically, a human can exhibit both passive and nonpassive behaviour depending on the task at hand. A simple example is a human operator swinging up a pendulum which is initially at rest in the lower equilibrium point. To generate oscillations with significantly large amplitude, the operator must supply energy to the pendulum thus behaving as an active system. Therefore, although passivity is currently considered as a conventional assumption on the human operator dynamics, it may make sense to look for a possibly more appropriate assumption that, in particular, admits both passive and active behaviour of the human operator.

In this work, we present a general stability result for networked bilateral teleoperator systems with projection-based force reflection algorithms where, in particular, the human dynamics are included in the consideration. To this end, we introduce a new assumption on the human dynamics which, in particular, allows for both passive and nonpassive behaviour of the human operator. As in our previous work (Polushin et al. (2008b)), we assume that the communication between the master and the slave is subject to time-varying discontinuous possibly unbounded communication delays and possible packet losses. We also impose a mild assumption the human force measurement/observation process. Finally, we define a class of force-reflecting algorithms which includes the previously considered algorithms as special cases, and show that, under the above assumptions, the overall stability of the networked bilateral teleoperator system can be guaranteed for any force reflection algorithm from the above class, regardless of the subsystems gains.

The structure of this paper is as follows. In Section 2, we describe the teleoperator system under consideration and introduce all the assumptions. The main stability result is presented in Section 3. Conclusions are given in Section 4. All proofs are omitted due to the space limitations.

## 2. TELEOPERATOR SYSTEM

### 2.1 Notation

\[ \mathbb{R}_+ := [0, +\infty) \]

A continuous function \( \alpha : \mathbb{R}_+ \to \mathbb{R}_+ \) belongs to class \( K (\alpha \in \mathcal{K}) \) if and only if \( \alpha(0) = 0 \) and \( \alpha \) is strictly increasing; \( \alpha \in \mathcal{K}_\infty \) (\( \alpha \in \mathcal{K}_\infty \)) if and only if it is unbounded (\( \lim_{s \to +\infty} \alpha(s) = +\infty \)). Also, \( \beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+ \) belongs to class \( KL \) if and only if \( \beta(a, b) \) is a \( \mathcal{K}_\infty \)-class function in \( a \) for any fixed \( b \geq 0 \), strictly decreasing in \( b \) for any fixed \( a > 0 \), and \( \lim_{b \to +\infty} \beta(a, b) = 0 \) for each \( a \in \mathbb{R}_+ \). Finally, given \( x \in \mathbb{R}^m \) and a set \( \Omega \subset \mathbb{R}^m \), by \( |x|_\Omega \) denote the distance between \( x \) and \( \Omega \), \( |x|_\Omega := \inf \{|x-y|, y \in \Omega\} \). In particular, \( \Omega_0 = \{0\} \), \( |x|_{\partial \Omega} \) becomes usual Euclidean norm of \( x \), \( |x|_{\partial \Omega} = |x| \).

### 2.2 Master and Slave Robots

In this work, similarly to Polushin et al. (2008b), we do not restrict our consideration to any specific types of master and slave manipulators and their local control algorithms, but rather consider a general case where both the master and the slave subsystems are described as nonlinear systems that satisfy certain properties. In particular, we assume that the closed-loop “master manipulator plus local master controller” subsystem is described as an affine nonlinear system of the form

\[
\dot{x}_m = F_m(x_m) + G_m(x_m) u_m, \\
y_m^{(h)} = Y_m^{(h)}(x_m), \\
y_m^{(s)} = Y_m^{(s)}(x_m),
\]

(1)

where \( x_m \) is the state of the master subsystem, \( u_m \) is the master input,

\[
u_m = f_h - f_r,
\]

(2)

where \( f_h \) is the force/torque applied by the human operator, and \( f_r \) is the force/torque reflection signal. Also, \( y_m^{(h)} \) is an output signal that is perceived by the human operator, and \( y_m^{(s)} \) is the signal that is transmitted to a remotely located slave. On the other hand, the “slave plus environment plus local slave controller” interconnection is described as a general nonlinear system of the following form

\[
\dot{x}_s = F_s(x_s, u_s^{(m)}), w_{env}), \\
y_s = G_s(x_s, u_s^{(m)}), w_{env}),
\]

(3)

where \( x_s \) is a state of the slave-environment interconnection, \( u_s^{(m)} \) is the “master” input, \( w_{env} \) represents all external forces acting on the environment, and the output \( y_s \) represents the contact force due to environment. All \( F_s(\cdot), G_s(\cdot), Y_s^{(h)}(\cdot), Y_s^{(s)}(\cdot), F_r(\cdot) \) and \( G_r(\cdot) \) are locally Lipschitz continuous functions of their arguments.

The communication process between the master and the slave sites is described as follows. The master output \( y_m^{(s)} \) is transmitted through the forward communication channel with communication delay \( \tau_f(\cdot) \) to the slave site, where it is applied to the input of the slave according to the formula

\[
u_s^{(m)}(t) := y_m^{(s)}(t - \tau_f(t)).
\]

(4)

On the other hand, the slave output \( y_s \) is transmitted to the master site through backward communication channel with communication delay \( \tau_b(\cdot) \), according to the formula

\[
\dot{w}_{env}(t) := y_s(t - \tau_b(t))
\]

(5)

Based on \( f_{env} \), the force reflection signal \( f_r \) (2) is generated using projection-based force reflection algorithms described below in Section 2.5.

We assume that both master and slave robots are equipped with local controllers that make them input-to-state stable with respect to external signals. Specifically, we make use of the following definition.

**Definition 1.** (Lin et al. (1995); Sontag (2006)) A system of the form

\[
\dot{x} = F(x, u_1, \ldots, u_p)
\]

is said to be input-to-state stable (ISS) with respect to a (nonempty) compact set \( \Omega \) with ISS gains \( \gamma_i \in \mathcal{K}, i \in \{1, \ldots, p\} \) and offset \( \delta \geq 0 \) if there exists \( \beta \in \mathcal{K}_\infty \) such that the following two properties hold:

i) uniform boundedness:

\[
\sup_{t \geq 0} |x(t)| \Omega \leq \max \left\{ \beta\left( |x(0)| \Omega \right), \gamma_1\left( \sup_{t \geq 0} |u_1(t)| \right), \ldots, \gamma_p\left( \sup_{t \geq 0} |u_p(t)|, \delta \right) \right\},
\]

ii) input-to-state stability:

\[
\beta\left( |x(0)| \Omega \right), \gamma_1\left( \sup_{t \geq 0} |u_1(t)| \right), \ldots, \gamma_p\left( \sup_{t \geq 0} |u_p(t)|, \delta \right)
\]

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ii) convergence:
\[
\limsup_{t \to +\infty} |x(t)|_{\Omega} \leq \max \left\{ \gamma_1 \left( \sup_{t \geq 0} |u_1(t)| \right), \ldots, \\
\gamma_p \left( \sup_{t \geq 0} |u_p(t)| \right), \delta \right\}.
\]

Our first assumption is that the local controllers are designed to guarantee the input-to-state stability of both the master and the slave subsystems, as follows.

**Assumption 1.** The master subsystem (1) with input \( u_m \) is ISS with respect to some nonempty compact set \( \Omega_m \). The slave subsystem (3) with inputs \( u_{hm}^m \), \( w_{env} \), is ISS with respect to some nonempty compact set \( \Omega_s \).

In the above Assumption 1, offsets are assumed to be zero. Assumption 1 make use of the notion of input-to-state stability with respect to a compact set which is a generalization of the conventional input-to-state stability notion (Sontag (2006)). This generalization seems to be particularly suitable in the case of teleoperator systems, as it does not require the subsystem to have a single equilibrium point in the absence of external forces, which allows to consider statically balanced subsystems. Examples of the control algorithms that make a robot input-to-state stable are well-known and can be found, for example in (Angeli, 1999; Polushin et al., 2006).

### 2.3 Dynamics of the human operator

The choice of an appropriate dynamic model that describes the human operator is a complicated problem, as the dynamic behaviour of the human hand is highly variable and is generally difficult to characterize and/or quantify.

In this work we adopt a general model of the human dynamics, where the human force applied to the master manipulator is a function of mechanical state of the human arm \( x_h \), perceivable output from the master manipulator \( y_m^{(h)} \), as well as an additional term \( f^a \) which reflects the human “intentions”. We assume that the human force is a function of the current values of \( x_h, y_m^{(h)}, f^a \) as well as their history, and that this dependence has a general form. Specifically, we use the following notation (Teel (1998)).

Given a nonnegative function \( \ell : \mathbb{R} \to \mathbb{R}^+ \), denote \( x_{sd}(t) = \{x_h(t - s) : 0 \leq s \leq t_d(t)\} \). Then, \( x_{sd}(t) \) is in general a piece of trajectory of \( x_h \) of length \( t_d(t) \geq 0 \).

\( y_{md} \) and \( f_{hd} \) are defined analogously. Using this notation, we arrive at the model of the human operator described by a functional equation of the form

\[
f_h(t) = F_h \left( x_{sd}(t), y_{md}(t), f_{hd}(t) \right),
\]

where \( F_h(\cdot) \) is a functional that describes the dynamics of the human operator. For regularity purposes, it is assumed that the inequality \( t_d(t_2) - t_d(t_1) \leq t_2 - t_1 \) holds for all \( t_1, t_2 \in \mathbb{R}, t_2 \geq t_1 \), and \( t - t_d(t) \to +\infty \) as \( t \to +\infty \).

In the literature on teleoperator systems, the dynamic properties of the human operator are frequently described in terms of passivity assumption. In fact, an apparent strict passivity of the human operator (more precisely, equivalence to zero of the antisymmetric component of the apparent stiffness matrix as well as positive definiteness of the symmetric one) was established experimentally in Hogan (1989) for the case where a human interacts with a passive manipulandum. It is worth to notice that, in this case, a strictly passive behaviour of the human operator is indeed the one that guarantees the overall stability. However, the choice of passivity as an “universal” assumption regarding the human operator dynamics does not seem to be convincing. On the contrary, a human is capable of both passive and active behaviour depending on the task to be executed. As a simple example, consider a human controlling a pendulum. If the task is to stabilize the lower equilibrium of initially oscillating pendulum, the human will normally apply the force against the pendulum’s velocity, thus behaving as a (strictly) passive system. On the other hand, if the task is to generate oscillations with significantly large amplitude starting from initial condition with lower energy (for example, if the pendulum is initially at rest in the lower equilibrium), then the human operator would naturally try to swing the pendulum up by applying forces along the direction of the pendulum’s velocity, effectively supplying energy to the pendulum and thus behaving as an active system (i.e., the one with an internal source of energy). In both cases, however, the human would try to stabilize the pendulum in the sense that is determined by the task. Overall, it seems meaningful to say that, depending on a particular task, the behavior of a human operator may or may not be passive, however, the natural reaction of the human operator is to stabilize (or at least, not to destabilize) the task at hand in the sense which is specific for each task. It is worth mentioning that this point of view agrees with the original interpretation of the experimental results presented in Hogan (1989), where it is said that: “... this experimental result strongly suggests that neural feedback in the human arm is carefully tuned to preserve stability under the widest possible set of conditions” (Hogan, 1989, p.1628). More recent studies (Kawato (1999); Burdet et al. (2001)) also indicate that a human operator tends to stabilize the manipulated object in the presence of external forces by learning an internal model of the dynamics.

In this work, we impose a general assumption on the behaviour of the human operator which, in particular, allows human to apply stabilizing as well as destabilizing actions to the master device. To formulate this assumption explicitly, note that the input-to-state stability of the master device (guaranteed by Assumption 1) is equivalent to the following property (Sontag (2006)): there exists a continuously differentiable function \( V_m : \mathbb{R}^n \to \mathbb{R}^+ \) such that

\[
\alpha_1 (|x_m|_{\Omega}) \leq V_m (x_m) \leq \alpha_2 (|x_m|_{\Omega}),
\]

and

\[
\frac{\partial V_m}{\partial x_m} [F_m (x_m) + G_m (x_m) u_m] \leq -\alpha_3 (|x_m|_{\Omega} + \sigma_m (|u_m|))
\]

hold for some \( \alpha_i \in \mathcal{K}_\infty, i = 1, 2, 3 \), and some \( \sigma_m \in \mathcal{K}_\infty \). Now, consider a set

\[
\Xi(x_m) := \left\{ \eta \cdot \frac{\partial V_m}{\partial x_m} G_m (x_m), \eta \in (-\infty, 0] \right\}.
\]

By definition, \( \Xi(x_m) \) can be interpreted as a set of “speed-gradient” (Frädkov et al. (1999)) or “passivity-based” (van der Schaft (1999)) stabilizing control actions. We assume that the human action always lies within bounded distance from \( \Xi(x_m) \). Specifically, this assumption can be formulated as follows.
Assumption 2 implies that there exists a finite number \( \rho^* > 0 \) such that, for all time instants \( t \geq 0 \), the distance between output of the human dynamics \( f_m(t) \) and the “passivity-based” stabilizing set \( \Xi (x_m(t)) \) is uniformly bounded by \( \rho^* \). Note that distance \( \rho^* > 0 \) is by no means assumed to be “small” in any sense. Thus, Assumption 2 allows the human operator to apply both stabilizing and destabilizing actions to the master device; speaking in terms of passivity, it allows passive as well as nonpassive behaviour of the human operator. See Figure 1 for illustration. Overall, this appears to be a very general assumption which in fact excludes only those destabilizing human actions that grow without bound (more precisely, the human actions that are unbounded in the sense of their distance to the “passivity-based” stabilizing set \( \Xi \)).

2.4 Communication Channel

The next assumption, borrowed from Polushin et al. (2009b), is imposed on the communication process between the master and the slave.

Assumption 3. The communication delays \( \tau_f, \tau_h : \mathbb{R} \to \mathbb{R}_+ \) are Lebesgue measured functions with the following properties:

i) there exist \( \tau_* > 0 \) and a piecewise continuous function \( \tau : \mathbb{R} \to \mathbb{R}_+ \) satisfying \( \tau^*(t_2) - \tau^*(t_1) \leq t_2 - t_1 \), such that the inequalities \( \tau_* \leq \min \{ \tau_f(t), \tau_h(t) \} \leq \max \{ \tau_f(t), \tau_h(t) \} \leq \tau^*(t) \) hold for all \( t \geq 0 \); 

ii) \( t - \max \{ \tau_f(t), \tau_h(t) \} \to +\infty \) as \( t \to +\infty \).

As discussed in Polushin et al. (2009b), Assumption 3 does not impose any significant restriction on communication process, and can always be satisfied for any communication channel by implementing standard features such as packet numbering and/or time stamping, unless the communication is lost on a semi-infinite time interval.

2.5 Force reflection algorithms

The main idea behind the projection-based force reflection principle is to decompose the force reflection signal into the “interaction” and “momentum-generating” components and attenuate the latter while applying the former in full. For details, motivation, and additional explanations, see Polushin et al. (2007, 2008a,b). A force-reflection scheme is described by the following formula

\[
f_r = \alpha \left( \frac{|f_{env}|}{f_{env}} f_{env} + \frac{1 - \alpha}{|\phi_{env}|} \phi_{env} \right),
\]

where \( f_r \) is the force reflection signal applied to the motors of the master, \( f_{env} \) is the force signal that is arrived directly from the remote slave-environment subsystem, \( \phi_{env} \) is the signal generated by the projection-based force reflection algorithm, and \( \alpha \in \mathbb{K} \) is the corresponding weighting function; \( \alpha \) should be chosen to satisfy \( [1 - \alpha] \in \mathbb{K} \), where \( I : \mathbb{R}_+ \to \mathbb{R}_+ \) is the identity function, \( I(r) = r \) for all \( r \geq 0 \). Essentially, \( \phi_{env} \) is an estimate of the interaction component of the force reflection signal. Two specific algorithms have been considered earlier Polushin et al. (2008b). The first one is described by the formula

\[
\phi_{env} := \text{Sat}_{[0,1]} \left\{ \frac{f_{env}^T f_h}{\max \{ |f_h|^2, \epsilon_1 \}} \right\} f_h,
\]

where \( f_h \) is a measurement/estimate of the human force applied to the master manipulator, \( \epsilon_1 > 0 \) is a sufficiently small constant, and \( \text{Sat}\{x\} := \max\{a, \min\{x, b\}\} \). Algorithm (11) estimates the interaction component of the environmental force as the component that is directed against the human force with magnitude bounded by the magnitude of the human force; more precisely, it calculates \( \phi_{env} \) as the projection of \( f_{env} \) onto the subspace spanned by the human force estimate \( f_h \); as a result, \( \phi_{env} \) is always collinear to \( f_h \). It is possible to construct a similar force reflection algorithm, where the resulting vector \( \phi_{env} \) would preserve the direction of the environmental force \( f_{env} \), however, its magnitude would depend on the magnitude of the projection of \( f_h \) onto the subspace spanned by \( f_{env} \). Such an algorithm is described by the formula

\[
\phi_{env} := \text{Sat}_{[0,1]} \left\{ \frac{f_{env}^T f_h}{\max \{ |f_{env}|^2, \epsilon_1 \}} \right\} f_{env}.
\]

Note that algorithms (11) and (12) give the same result if \( f_{env} \) is collinear to \( f_h \). It is also possible to use any convex combination of the algorithms (11) and (12).

One can generalize the above considered force-reflecting algorithms as follows. It can be directly checked that \( \phi_{env} \) generated by any of the algorithms (11), (12), (as well as by any convex combination of (11), (12)) satisfies the inequality

\[
|f_h - \phi_{env}|^2 \leq f_h^T (f_h - \phi_{env}).
\]
One, therefore, can consider a class of force reflection algorithms whose outcomes satisfy (13). In our work, we assume that the force reflection signal is generated by any of the algorithms from the above described class. More precisely, our assumption is the following.

Assumption 4. The force reflection signal \( f_r \) is described by formula (10), where \( \alpha \in \mathcal{G} \) is an arbitrary weighting function such that \( 1 - \alpha \in \mathcal{G} \), and \( \varphi_{env} \) is the projection-based component satisfying the inequality
\[
|f_h - \varphi_{env}|^2 \leq f_h^2 k_h (f_h - \varphi_{env}),
\]
(14)

Remark 1. Assuming \( f_h - \varphi_{env} \neq 0 \) and dividing both sides of (13) by \( |f_h - \varphi_{env}| \), we see that (13) is equivalent to
\[
|f_h - \varphi_{env}| \leq |f_h| \cos \angle(f_h, f_h - \varphi_{env}),
\]
(15)
where \( \cos \angle(f_h, f_h - \varphi_{env}) \) is the cosine of the angle formed by vectors \( f_h \) and \( f_h - \varphi_{env} \). In particular, (15) implies \( \cos \angle(f_h, f_h - \varphi_{env}) \geq 0 \) or, equivalently, \( \angle(f_h, f_h - \varphi_{env}) \in [-\pi/2, \pi/2] \). On the other hand, taking into account the law of cosines
\[
|f_h - \varphi_{env}|^2 = |f_h|^2 + |\varphi_{env}|^2 - 2f_h \varphi_{env},
\]
we see that inequality (13) is equivalent to
\[
|\varphi_{env}|^2 \leq f_h^2 \varphi_{env}.
\]
(16)
These properties of the projection-based force reflection algorithms will be utilized below.

2.6 Human Force Measurement/Estimation

Our last assumption is related to the force measurement/estimation process on the master side. In the results presented below, we do not restrict our consideration to the situation where the human force/torque is perfectly measured. On the contrary, we consider the human force measurement/estimation as a process whose accuracy may depend on a values of the human force and its derivatives as well as on disturbance level. More precisely, a sort of input-to-state stability assumption is imposed on the human force measurement/estimation process, as follows.

Assumption 5. Human force (torque) measurement (estimation) process satisfies the following estimate
\[
|f_h(t) - f_h(t)| \leq \max \left\{ \beta \left( |f_h(0) - f_h(0)|, t \right), \gamma_u \left( \sup_{s \leq t} |w_m(s)| \right) \right\}
\]
(17)
for all \( t \geq 0 \), where \( f_h \) is the human force applied to the master, \( w_m \) are the disturbances that affect the measurement/estimation process, \( \beta \in \mathcal{K} \), and \( \gamma_u \in \mathcal{K} \).

3. STABILITY ANALYSIS

In this section, we formulate and prove our main stability result for the above described bilateral teleoperator system. Consider the system (10), (1)–(6). Since this system involves communication delays (4), (5), it can appropriately be described by a system of functional differential equations (FDEs). Consider a system of FDEs of a general form
\[
\dot{x} = F(x_d, u_d, t),
\]
(18)
where, as before, \( x_d(t) = \{x(t - s): 0 \leq s \leq t_d(t)\} \), \( u_d(t) = \{u(t - s): 0 \leq s \leq t_d(t)\} \), and \( t_d(\cdot) \) is a nonnegative function defined on \( \mathbb{R}_+ \). For regularity purposes, it is assumed that \( F(\cdot) \) is a Lipschitz continuous operator; also, the inequality \( t_d(t) - t_d(t_1) \leq t_2 - t_1 \) holds for all \( t_1, t_2 \in \mathbb{R} \), \( t_2 \geq t_1 \), and \( t - t_d(t) \to +\infty \) as \( t \to +\infty \). Analogously to Definition 1, the input-to-state stability property for (18) can be defined as follows.

Definition 2. The system (18) is said to be input-to-state stable with respect to a nonempty compact set \( \Omega \) with offset \( \delta \geq 0 \) if there exists \( \beta \in \mathcal{K}_\infty \) and \( \gamma \in \mathcal{K} \) such that the following inequalities hold:
\[
\sup_{t \geq 0} |x_d(t)|_{\Omega} \leq \max \left\{ \beta \left( |x_d(0)|_{\Omega} \right), \gamma \left( \sup_{t \geq 0} |u_d(t)| \right), \delta \right\},
\]
\[
\lim sup_{t \to +\infty} |x_d(t)|_{\Omega} \leq \max \left\{ \gamma \left( \lim sup_{t \to +\infty} |u_d(t)| \right), \delta \right\}.
\]
The main result of this work can be formulated as follows.

Theorem 1. Consider the closed-loop force reflecting teleoperator system (1)–(6) with force reflection algorithm (10). Suppose Assumptions 1–5 are satisfied. Then there exists \( \alpha_\ast \in \mathcal{K}_\infty \) such that if \( \alpha(\cdot) \in \mathcal{K} \) in (10) satisfies \( \alpha(s) \leq \alpha_\ast(s) \) for all \( s \geq 0 \) then the closed-loop force reflecting teleoperator system with state \( x_d = (x_d^T, x_s^T)^T \) and input \( u = (\rho, w_m^T, w_{env}^T)^T \) is input-to-state stable in the sense of Definition 2 with respect to \( \Omega = \Omega_m \times \Omega_s \) with some offset \( \delta \geq 0 \).

The result analogous to Theorem 1 was established earlier in Polushin et al. (2008b) for the simplified case where the human force was considered an external input, i.e., the human dynamics were not taken into account. The most important and interesting feature of these results is that they do not impose any restrictions on the ISS (IOS) gains of the master and slave subsystem. Instead, given the master and the slave gains, the overall stability can always be achieved by an appropriate choice of the weighting function \( \alpha(\cdot) \in \mathcal{K} \) in (10). This has very important consequences for the design of teleoperator systems; in particular, stability can be achieved for arbitrarily high force reflection gain and arbitrarily low damping and stiffness of the master manipulator.

4. CONCLUSIONS

This paper presents a general stability result for networked bilateral teleoperator systems with projection-based force reflection where, unlike the previous work on this topic, the dynamic behaviour of the human operator is explicitly taken into account in the stability analysis. Specifically, we impose a general assumption on the behaviour of the human operator which, in particular, allows to apply stabilizing as well as destabilizing actions to the master device. The overall stability of the networked teleoperator system is then proven for an arbitrary force reflection algorithm from a broad class. The important property of the projection-based force reflection is that it effectively removes constraints on subsystem’s gains, which in particular allows for arbitrarily high force reflection gain and arbitrarily low damping/stiffness of the master manipulator. The application of this principle may lead to design of new teleoperator systems and haptic interfaces with improved stability and transparency characteristics.
REFERENCES


