A sliding mode-based impedance control for bilateral teleoperation under time delay

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Abstract: Several control strategies have been proposed to deal with time delay, parametric uncertainties and nonlinearities for bilateral teleoperation. Furthermore, published works had shown that sliding mode control is a viable option to deal with parametric uncertainty and hard nonlinearities. In this paper, an impedance control based on sliding mode techniques is presented, to guarantee robust tracking under unknown constant time delay. Furthermore, in order to implement the proposed impedance controller, it is necessary to know the velocity and acceleration of the slave system, then to overcome this problem, i.e the estimation in teleoperation in presence the unknown time-delays in the communication channel, a nonlinear observer designed via the super twisting algorithm is presented in order to estimate velocity and acceleration. Simulation results are presented and discussed, which reveal the effectiveness of the proposed observer with the sliding mode control.

Keywords: Teleoperation, time-delay, sliding-mode control, impedance control, observer, super twisting algorithm.

1. INTRODUCTION

It is well-known that in Teleoperation systems stability is mainly affected by communication time-delays. Furthermore, in a system which is teleoperated, a human operator interacts with an interface, called master teleoperator, human drives master system in order to control the slave system. On the opposite side, another interface is in charge of directly implementing commands received from the operator on the remote environment (García Vaklovinos and Arteaga [2007]). In bilateral teleoperation, transparency refers to the matching degree between the impedance perceived by the operator and the environment impedance. Along with stability, transparency is a major objective of bilateral teleoperation system design, under any operating conditions and for any possible environment. Taking into account this objective, several bilateral control architectures have been developed (Hannahford [1989]; Hashtrud-Zaad and Salcudean [1999]). To deal with nonlinearities of the teleoperation system, nonlinear control strategies have been designed using different techniques, such as feedback linearization, adaptive control, sliding mode techniques to design the bilateral controller (Hashtrud-Zaad and Salcudean [1996]; Lee and Chung [1998]; Ryu and Kwon [2001]; Zhu and Salcudean [2000]).

A complete analysis of bilateral teleoperation should include time-delayed systems, several schemes have been proposed most of them for linear systems and some for the nonlinear case. Particularly, a significant interest on sliding mode control techniques has become popular in the control research community worldwide. One of the most interesting aspects of sliding mode is the discontinuous nature of the control action whose primary function is to switch between two different structures such that a new type of system motion, called sliding mode, exists in a manifold.

Recently, nonlinear observers based in super twisting sliding mode techniques have become an interesting option to be used in several kinds of systems. Thanks to the robustness properties and the finite-time convergence this technique is very attractive. Several works have been published using this technique (Floquet and Barbot [2007]), since the first result introduced by Levant (Levant [2003]; Levant [2005]). In sliding mode methodology applied to observers the systems trajectories are constrained to reach and stay, after a finite time, on a given sliding manifold for which the output error is zero. The sliding motion provides an estimation of the system state, asymptotically (Slotine and Misawa [1987]; Walcott and Žak [1987]) or in finite time. The finite time convergence property of sliding mode observers is often desirable in the framework of observation and particularly for the purpose of observer-based controller design for nonlinear systems, the observer can be designed separately from the controller and the separation principle does not need to be proved (Floquet and Barbot [2007]), which is an interesting property.

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Contribution
In this paper, we consider a second order sliding mode impedance control for the slave system designed to track the time-delay master trajectories. Then, for the nonlinear bilateral teleoperation system a sliding mode observer-controller scheme is proposed, which guarantees the tracking trajectory generated by the master system under an unknown constant time-delay and without using velocity and acceleration measurements of the slave system, avoiding expensive and bulky sensors.

Paper structure
The paper is organized as follows. In Section 2, the dynamical models of the bilateral teleoperated system is introduced. Next, in Section 3, the proposed second order sliding mode impedance control for the slave system is designed to track the master trajectories. Then, to implement this controller an observer is given in Section 4 in order to estimate the velocity and the acceleration of the slave system. Simulation results for the full control-observer bilateral teleoperation scheme are presented in Section 5. Finally, concluding remarks are given in Section 6.

2. TELEOPERATION SYSTEM

In a teleoperation general setting, the human imposes a force on the master manipulator which in turn results in a displacement that is transmitted to the slave that mimics that movement. If the slave possess force sensors, then it can reflect to the master reaction forces from the task being performed, which enters into the input torque of the master, and the teleoperator is said to be controlled bilaterally. Although reflecting the encountered forces back to the human to rely on his/her tactile senses along with visual senses, it may cause instability in the system if delays are present in the communication media (Hoyakem and Spong [2006]).

Position and force scale factors are used in the design and the analysis to generalize their results to cover many different types of teleoperation systems and applications. A time delay imposed on the communication channels is assumed to be constant.

The dynamics of the general nonlinear master/slave systems are the following mass-damper system equations.

\[ M_m(q_m) \ddot{q}_m + C_m(q_m, \dot{q}_m) \dot{q}_m = f_h + u_m \]

\[ M_s(q_s) \ddot{q}_s + C_s(q_s, \dot{q}_s) \dot{q}_s = u_s - f_e \]

where \( q_m, \dot{q}_m, \ddot{q}_m \in \mathbb{R}^n \) are the joint positions, velocities and accelerations; \( M_i(q_i) \in \mathbb{R}^{n \times n} \) are the inertia matrices; \( C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n} \) are the Coriolis and centrifugal effects, which are defined using the Christoffel symbols of the first kind; and \( u_i, \dot{u}_i \in \mathbb{R} \) are the control signals where subscript \( m \) and \( s \) denote the master and the slave; \( f_h \) and \( f_e \) are the force applied at the master by the human operator, and the force exerted on the slave by the environment, respectively. It is assumed that the manipulators are composed by actuated revolute joints and that gravitational forces and friction can be neglected (Núñio and Basaunità [2010]). This bilateral teleoperation system scheme can be represented by the block diagram shown in Figure 1 where the position and force of the master are transmitted to the slave and

Fig. 1. A block diagram of bilateral teleoperation

the contact force of the slave is sent to the master through the communication channel. Where there is a time delay, the signals from and to the channel are related as

\[ q_m^d(t) := q_m(t - T_1), \quad \dot{q}_m^d(t) := \dot{q}_m(t - T_1) \]

\[ f_h^d(t) := f_h(t - T_1), \quad f_e^d(t) := f_e(t - T_2) \]

where \( q_m^d, \dot{q}_m^d, \) and \( f_h^d \) are the position and velocity of the master, and the force exerted by a human operator, respectively, which are reflected to the slave through the communication channel; \( f_e^d \) is the external force at the slave reflected to the master through the communication channel; \( T_1 \) is a time delay of the signal flowing from master to the slave, and \( T_2 \) is in the opposite direction.

This delayed signals out of the communication block are then scaled up or down by some factors depending on teleoperation tasks. Using the scale factors, the position/velocity command to the slave and the force signal to the master are modified such that

\[ q_m^d = K_p q_m^d, \quad f_h = K_f f_e^d \]

where \( K_p \) and \( K_f \) are diagonal matrices representing position and force scale factors, respectively; and \( q_m^d \) and \( f_e^d \) are the position of the slave and the force reflected by the environment, respectively, which are reflected to the master through the communication channel.

3. IMPEDANCE CONTROLLERS

In this section, an impedance controller and a sliding-mode-based impedance controller are designed for the master and the slave, respectively. The force-controller master device reflects to a human operator the contact forces between the slave and its environment while the position-controlled slave follows the trajectory commanded from the master.

3.1 Impedance controller for the master

To expand the feasible applications of robots, it is necessary to control not only the motion but also the forces interacting between the manipulator and the environment. A distinction between impedance control and the more conventional approaches to manipulator control is that the controller attempts to implement a dynamic relation between manipulator variables such as end-point position and force rather than just control this variables alone (Hogan [1985]).

Consider the following general master control structure

\[ u_m = -f_h + C_m(q_m, \dot{q}_m) \ddot{q}_m + M_m(q_m)M_m^{-1}(q_m)A \]
\[ \Lambda = f_h - K_f f_e' - C_m(q_m, \dot{q}_m)q_m - K_m q_m \]
and \( \bar{M}_m, \bar{C}_m, \bar{K}_m > 0 \) are desired inertia, damping coefficient, and stiffness, respectively; of a desired impedance.

Substituting (3) into (1), it follows that the closed-loop master system is given by
\[ M(q_m) \ddot{q}_m = f_h - K_f 
\bar{d}_e - C(q_m, \dot{q}_m) \dot{q}_m - \bar{K}_m q_m \]
(4)

Notice that the master control imposes a desired impedance dynamics in the master teleoperator, between the speed of the master and the linear combination of the human force and the delayed contact force.

### 3.2 Sliding-mode based impedance controller for the slave

Under a similar rationale as the master controller, consider the slave control design based on second order sliding mode approach to produce a desired impedance behavior modulated by the environmental contact forces, robust to unknown time-delay. Thus we design the slave controller based on sliding mode control techniques. To this end, consider the desired slave impedance
\[ M_s(q_s) \ddot{q}_s + C_s(q_s, \dot{q}_s) \dot{q}_s + K_s \ddot{q}_s = -f_e \]
(5)
where \( M_s, C_s, K_s > 0 \) are the desired inertia, damping, and stiffness, respectively; and \( \ddot{q}_s := \ddot{q}_s - K_p \ddot{d}_m, \dot{q}_s := \dot{q}_s - K_p \dot{d}_m, q_s := q_s - K_p q_m \)
are the slave tracking errors for acceleration, velocity, and position, respectively. Since we want to obtain (5) in closed-loop with a suitable slave controller, then we introduce the following sliding surface
\[ \Omega = \int_0^\tau L_e(d)dt + A \int_0^\tau \int_0^\sigma \text{sign}(L_e(\tau))d\tau d\sigma \]
(6)
where \( L_e = M_s(q_s) \ddot{q}_s + C_s(q_s, \dot{q}_s) \dot{q}_s + K_s \ddot{q}_s + f_e = 0 \)
(7)
\( K_i > 0 \) is a diagonal matrix.

Then, a stabilizing slave controller \( u_s \) which attains the desired slave impedance is given by
\[ u_s = M_s(q_s)K_s M_m^{-1} \Psi - M_s(q_s)M_s^{-1}(q_s) \Psi + f_e + C_s(q_s, \dot{q}_s) \dot{q}_s - K_s \ddot{q}_s \]
(8)
where
\[ \Psi = C_s(q_s, \dot{q}_s)[\dot{q}_s - K_p \dot{d}_m] + K_s \dot{q}_s + f_e \]
\[ \Upsilon = -C_m(q_m, \dot{q}_m) \dot{q}_m - K_m \dot{q}_m + u_m - K_p f_e' \]
(9)
and \( f_e' = f_e(t - 2T) \), the superscript \( dd \) stands for the round trip delay \( 2T \), \( K_g > 0 \), and \( \text{sign}(\cdot) \) is the discontinuous signum function. The term \( K_s \Omega \) has been added to achieve stability as will be seen afterwards.

Notice that (6) requires acceleration measurement because \( L_e \) depends on acceleration and terms depending on the time-delay signals of the master system. To deal with this inconvenience, as will be seen in section 4, acceleration and velocity will be estimated by means of super twisting observer.

### 3.3 Stability analysis

Now, we analyze the stability of the slave system in closed-loop with the slave controller (8).

The resulting closed-loop system obtained form the slave system (2) in closed-loop with the controller (8), and after straightforward calculation, is given by
\[ \dot{\Omega} = -M_s M_s^{-1}K_s \Omega \]
Now, defining the following candidate Lyapunov function
\[ V(\Omega) = \frac{1}{2} \Omega^\top \Omega \]
(10)
Then, taking the time derivative, we have
\[ \dot{V}(\Omega) = \Omega^\top \dot{\Omega} = -\Omega^\top M_s K_s \Omega \]
(11)
where \( M_s, K_s \) are definite positive matrices. Let \( \mu = \lambda_{\min}(M_s K_s) \) be an eigenvalue of matrix \( M_s K_s \), then
\[ \dot{V}(\Omega) \leq -\mu \Omega^\top \Omega \leq -2\mu V(\Omega) \]
(12)
Thus
\[ V(\Omega(t)) \leq V(\Omega(t_0))e^{-2\mu(t-t_0)} \]
which proves that the trajectories of closed-loop slave system converge exponentially to the sliding surface \( \Omega = 0 \). Hence \( \dot{\Omega} = 0 \), which satisfies the desired slave impedance. This can be summarized as follows

**Proposition 4.1.** Consider the slave system in closed-loop with the control \( u_s \). Then, the trajectories of slave system converges exponentially to zero.

**Remark 1.** The robust impedance controller can be achieved by designing a sliding-mode controller such that the desired impedance model becomes exact on the sliding surface. This proposed controller uses a robust property of a sliding control against uncertainties such as parametric uncertainty and unmodeled dynamics. The controller \( u_s \) can be modified using
\[ u_s = M_s(q_s)K_s M_m^{-1} \Psi - M_s(q_s)M_s^{-1}(q_s) \Psi + f_e + C_s(q_s, \dot{q}_s) \dot{q}_s - K_s \text{sign}(\Omega) \]
(13)
Then the proof of the proposition is
\[ \dot{V}(\Omega) \leq \Omega^\top \Omega \leq -\mu \Omega \]
(14)
If the sliding condition is satisfied, then the slave system trajectories tends to the surface \( \Omega \), where \( \mu \) is a strictly positive constant.

### 4. OBSERVER DESIGN

To implement the impedance control law it is necessary to know the velocity and acceleration of the slave system. In order to avoid expensive and bulky sensors which may add noise to the system, we will design an observer which allows to avoid using sensors measuring velocity and acceleration, and then by replacing the no measurable variables for the estimated states given by the observer.

Among different methods used for estimating the state of a nonlinear system, it follows that sliding mode observers is one of the most interesting techniques thanks to its robustness and convergence in finite-time. In this paper we introduce an observer based on sliding mode technique(Floquet and Barbot [2007]; Li and Elbuluk [2001]).
Let us consider a nonlinear system in triangular form

\[ \Sigma : \begin{cases} \dot{x}_j = x_{j+1}, & \text{for } j=1, n-1. \\ \dot{x}_n = \alpha(x) + \beta(x)u \end{cases} \tag{15} \]

where \( x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n \) is the state vector, \( y = x_1 \in \mathbb{R} \) is the output vector and \( u \in \mathbb{R} \) is the unknown input. \( \alpha(x) \) and \( \beta(x) \) are bounded smooth scalar functions.

Now let assume that the state of the system is uniformly bounded, i.e. \( \forall t > 0, |x_i(t)| < d_i \), and \( \forall t > 0 \). This means the state and its derivatives are bounded. Then, the following system \( \hat{O} \) is an observer for system (15)

\[ \begin{aligned} \dot{x}_1 &= \hat{x}_2 + \lambda_1 |e_1|^{1/2} \text{sign}(e_1) \\ \dot{x}_2 &= \alpha_1 \text{sign}(e_1) \\ \vdots \\ \dot{x}_{n-1} &= E_{n-3} \alpha_{n-2} \text{sign}(e_{n-2}) \\ \dot{x}_{n} &= E_{n-2} \alpha_{n-1} \text{sign}(e_{n-1}) \\ \dot{\hat{x}}_n &= E_{n-1} \alpha_n \text{sign}(e_n) \end{aligned} \tag{16} \]

where \( e_i = \hat{x}_i - x_i \) for \( i = 1, \ldots, n \) with \( \hat{x}_1 = x_1 \) and \( [\hat{x}, \dot{\hat{x}}]^T = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n, \dot{\hat{x}}]^T \) is the output of the observer. For \( i = 1, \ldots, n-1 \), the scalar functions \( E_i \) are defined as

\[ E_i = 1 \text{ if } |e_j| = |\hat{x}_j - x_j| \leq \varepsilon, \forall j \leq 1 \text{ else } E_i = 0 \tag{17} \]

where \( \varepsilon \) is a small positive constant and \( \lambda_i \) and \( \alpha_j > 0 \) are the observer gains. The convergence of the state observation error is obtained in \((n-1)\) steps in finite time.

The recursive scheme based on differentiator given in equations (16) is used to reconstruct the non-measurable variables. The sliding mode differentiator provided an exact differentiation using a recursive method with a finite-time convergence. The proof of convergence in finite time of this observer follows the same steps given in (Floquet and Barbot [2007]).

Next, following the same ideas we extend the above observer design for the class of mechanical systems considered in this paper.

Let us consider the differential equation of slave system (2). Introducing the following change of coordinates \( \mathbf{X}_{s2} = q_i; \mathbf{X}_{s2} = \mathbf{q}_i \), then the slave system can be rewritten in the following state space representation

\[ \Sigma_s : \begin{cases} \mathbf{\dot{X}}_s = \mathbf{X}_{s2} \\ \mathbf{\dot{X}}_{s2} = -\mathbf{M}_s^{-1}(\mathbf{X}_{s1}) \{-\mathbf{C}_s(\mathbf{X}_{s1}, \mathbf{X}_{s2}) + \mathbf{u}_s - \mathbf{f}_s\} \end{cases} \tag{18} \]

In order to design a sliding mode observer for slave system (18) estimating the velocity and the acceleration, equations can be written in a canonical form such that achieve conditions presented in (15).

\[ \Sigma_s : \begin{cases} \mathbf{\dot{X}}_s = \mathbf{X}_{s2} \\ \mathbf{\dot{X}}_{s2} = \mathbf{F}(\mathbf{X}_{s1}, \mathbf{X}_{s2}, \mathbf{f}_s) + \Delta \end{cases} \tag{19} \]

with

\[ \mathbf{F}(\mathbf{X}_{s1}, \mathbf{X}_{s2}, \mathbf{f}_s) = \mathbf{M}_s^{-1}\{\mathbf{C}_s(\mathbf{X}_{s1}, \mathbf{X}_{s2})\mathbf{X}_{s2} + \mathbf{f}_s\} \tag{20} \]

\[ \Delta = -\mathbf{M}_s^{-1}(\mathbf{u}_s) \tag{21} \]

Suppose that the system states can be assumed to be bounded, then there exists a constant \( f \) such that the inequality \( |\mathbf{F}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{f}_i)| < f \) holds for any state. Furthermore, \( \Delta \) is considered as an uncertain term depending on the time-delayed signals which are assumed uniformly bounded in a compact, i.e. \( \|\Delta\| \leq \alpha \). It is clear that the slave system is observable.

The aim in this section is to design an observer for the slave system \( \Sigma_s \) which converges in finite-time to the actual states of the system, when only the position and the force exerted on the slave by the environment are available.

The proposed super twisting observer has the form

\[ \begin{aligned} \dot{\mathbf{X}}_s &= \mathbf{X}_{s2} + \Lambda_1 \sqrt{|\mathbf{X}_{s1} - \mathbf{X}_{s1}|} \text{sign}(\mathbf{X}_{s1} - \mathbf{X}_{s1}) \\ \dot{\Theta} &= \mathbf{E}_1 (\dot{\Theta}_1 + \Lambda_2 (\sqrt{|\mathbf{X}_{s2} - \mathbf{X}_{s2}|} \text{sign}(\mathbf{X}_{s2} - \mathbf{X}_{s2}))) \\ \Theta &= \mathbf{E}_2 \alpha_2 \text{sign}(\mathbf{X}_{s2} - \mathbf{X}_{s2}) \end{aligned} \tag{22} \]

where \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \) are the state estimated of the state vectors \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \), \( \Lambda_1 = \text{diag}(\lambda_1, \ldots, \lambda_n) \), \( \Lambda_2 = \text{diag}(\lambda_2, \ldots, \lambda_n) \), \( \alpha_1 = \text{diag}(\alpha_{11}, \ldots, \alpha_{nn}) \) and \( \alpha_2 = \text{diag}(\alpha_{11}, \ldots, \alpha_{nn}) \) are the gains of the observer, \( \sqrt{|\mathbf{X}_{s1} - \mathbf{X}_{s1}|} = \text{diag}(\sqrt{|\mathbf{x}_{s1, 1} - \mathbf{x}_{s1, 1}|}, \ldots, \sqrt{|\mathbf{x}_{s1, n} - \mathbf{x}_{s1, n}|}), \) \( \text{sign}(\mathbf{X}_{s1} - \mathbf{X}_{s1}) = \text{diag}(\text{sign}(\mathbf{x}_{s1, 1} - \mathbf{x}_{s1, 1}), \ldots, \text{sign}(\mathbf{x}_{s1, n} - \mathbf{x}_{s1, n})) \), for \( i = 1, \ldots, n \), with \( \mathbf{X}_{s1} = \mathbf{X}_{s1} \).

Observer \( \Theta \) estimates velocity \( \dot{x}_s \) and acceleration \( \ddot{x}_s \) of the slave system, which are very difficult to measure due to noises, using only measures of position \( x_s \) and environment force \( \mathbf{f}_s \), as it is shown in the Figure [??]. Notice that the time-delay signals appear in the system in such a way that it can be concentrated in a term which can be bounded by a constant.

**Proposition 4.2** Consider system \( \Sigma_s \) and assume that position \( \mathbf{X}_{s1} \) and environment force \( \mathbf{f}_s \) are measurable, the time-delayed signals are bounded and the time-delay is constant and unknown. Then, the system \( \hat{O} \) is an observer for system \( \Sigma_s \), where the states of the observer converge in finite-time to the states of the system \( \Sigma_s \).

**Proof.** The proof of convergence of this observer, taking into account the time-delay present signals sent by the master system to slave system, can be straightforward proved following the same procedure given in (Floquet and Barbot [2007]).

Now, we can establish the main result of the paper.

**Theorem 4.1.** Consider the slave system (2) in closed-loop with the impedance control (8) and using estimated states given by the sliding mode observer (16). Then, the closed loop slave system is exponentially stable, under time-delay signals of the master slave. Furthermore, the slave system converges exponentially to the time-delayed trajectories of the master system.

**Proof.** Since the finite-time convergence of the observer allows to design the observer and the control law separately,
i.e. the separation principle is satisfied. Furthermore, since the slave control is known to stabilize the slave system, then, the stability of the closed-loop system is proved.

5. SIMULATION

Now, in order to illustrate the proposed methodology, consider the following case of a 1-dof system in order to simplify simulation analysis, even though results can surely be prove for more degrees of freedom.

\[ m_m \ddot{q}_m + c_m \dot{q}_m = f_h + u_m \]  
\[ m_s \ddot{q}_s + c_s \dot{q}_s = u_s - f_c \]

Thus equations presented before, for simulation are simplify to de 1-dof equations. The objective of this simulations is prove the performance of the controllers based in a sliding mode observer with a constant time delay. Previous nonlinear equations are now reduce in the follow master and slave controls

\[ u_m = -f_h + c_m \dot{q}_m \]  
\[ = \frac{m_m}{m_m} \{ f_h - k_f f_c - c_m \dot{q}_m - k_m q_m \} \]  
\[ u_s = -\frac{m_s}{m_m} \left\{ c_s \ddot{q}_s + k_s \dot{q}_s + f_e + k_p \int_0^t \text{sign}(E_c(\tau))d\tau \right\} \]  
\[ + \frac{m_s}{m_m} k_p \{ f_d - k_f f_c - c_m \dot{q}_m - k_m q_m \} \]  
\[ + f_c + c_s \dot{q}_s - k_s \Omega \]

Changing equation (2) to state variable the 1-dof equation for sliding mode observer has the following form,

\[ \begin{align*}
\dot{x}_1 &= \dot{x}_2 + \lambda_1 \sqrt{|x_1 - \dot{x}_1|} \text{sign}(x_1 - \dot{x}_1) \\
\dot{x}_2 &= \alpha_1 \text{sign}(\dot{x}_1 - \dot{x}_2) \\
\dot{\Theta} &= E_1 \alpha_2 \text{sign}(\dot{x}_2 - \dot{x}_2)
\end{align*} \]

The parameters applied in the simulation for master and slave system, are presented in table 1. The delay in the communication block between this two systems are considered as 1 s. Thus, in the case of double delay used in master system for control its value for simulation is 2 s. In the same way parameters applied in control laws at master and slave, as well as in observer are presented in table 2 where no units are needed since the scalars used are all gains. Furthermore, in practical situations human operator applied a non continuous force to master systems as reference, so the force applied by the human in simulation is supposed to be a pulse signal with amplitude 5 and a period of 20 seconds, and the environment torque \( f_e \), a sinusoidal signal with similar frequency that \( f_h \). Moreover, \( E_1 \) is selected according to condition (17) and choosing \( \varepsilon = 0.01 \). Simulations results can be seen in figures.

First the behavior of the impedance control applied at the master systems can bee seen in the velocity graphic Figure 2, once the reference \( f_h \) has been applied in the form illustrated in the same figure, as mentioned \( f_h \) is a pulse signal. At Figure 3 can be seen how the tracking control is working, also can be seen the tracking error at the slave side specially when fast move is generated by the pulse signal. Moreover the expected constant

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<td>( x_{s2} = 1 )</td>
<td>( \dot{\theta} = 1 )</td>
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Table 1. Master and Slave system parameters

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<th>Units</th>
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<td>( m_s )</td>
</tr>
<tr>
<td>( c_m )</td>
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<td>( c_s )</td>
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<td>( \bar{k}_s )</td>
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<td>( k_f )</td>
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</tbody>
</table>

Fig. 2. Behavior of master system under impedance control. Human operator \( f_h \) is a pulse signal. Master position is tracking reference human reference \( 'fh' \)

Fig. 3. Behavior of slave system under a tracking control, where there exist a time delay of \( T = 1 \) s
delay between both systems appears. Even scheme fails in producing a perfect transparency, reach to provide a good approximation of remote environment, even under a constant delay. Moreover the control shows a good performance using the state information provided by the sliding modes observer, which in a practical experiment will allow us avoid the use of some sensors. The Figure 5 shows specifically how observer rapidly reach slaves states when it stabilize. Since there is no signal at the beginning on the control during simulations because of delay, $u_s$ presents large overshoots during the transient. After the transient has finished, control leads slave system to converge to master trajectories.

6. CONCLUSIONS

A nonlinear control-observer scheme for a nonlinear bilateral teleoperated system based on sliding mode techniques in presence of time delays in the communication channel has been presented. An impedance controller using a second order sliding mode observer has been proposed and applied the slave system in order to track the time-delayed signals sent by the master system. The states estimated by the observer are well reconstructed so that the control applied to slave does not depend of velocity nor acceleration measures.

REFERENCES


