LQG Control with Extended Kalman Filter for Power Systems with Unknown Time-Delays

Xuejiao Yang* Ognjen Marjanovic*

*Control System Centre, University of Manchester UK (e-mail: Xuejiao.Yang@postgrad.manchester.ac.uk).

Abstract: This paper considers the application of the supervisory discrete-time LQG control as a wide-area multi-machine power system controller subjected to unknown communication time-delays. These unknown time-delays are estimated using the Extended Kalman Filter algorithm and their estimates are then used to re-design LQG controller. The resulting LQG controller is then compared with the conventional LQG controller which ignores the presence of time-delays. The comparison is carried out on a non-linear simulation of a 2-area, 4-machine, 8-bus power system with a control objective of damping the electromechanical oscillations in the presence of unknown time-delays.

Keywords: Discrete-Time LQG, Power Systems, Rotor Angle Stability, Extended Kalman Filter, Time-Delay

1. INTRODUCTION

With increasing demand for power supply and the growing need for the restructuring of the power industry, electric power systems have become highly complex dynamic systems. However, the method of tuning standard Power System Stabilizers (PSSs) has in most cases remained unaltered despite the changes in network topology. More specifically, the PSSs are commonly tuned by considering only the local electromechanical modes of oscillation. As a result, inter-area electromechanical modes may be insufficiently damped, resulting in the deteriorated power system performance and even instability. Therefore, control of power systems using conventional PSS controllers has been brought into question, especially with the increased growth of inter-area incidents (CIGRE, 1996).

There has recently been an increased interest in applying supervisory wide-area controllers to damp inter-area electromechanical oscillations, thereby improving the overall power system stability. These control schemes rely on the Phasor Measurement Unit (PMU) technology and have been shown to significantly improve the damping of inter-area oscillatory modes (Aboul-Ela, 1996).

The stability control of a power system under Linear Quadratic Gaussian (LQG) control design has been discussed and applied using FACTS devices (Hu, 2007a, b; Zolotas, 2007), Static Var Compensators (Ferreira, 2007) and PSSs (Son, 2000). The LQG control design method is considered to be a cornerstone of the modern optimal control theory and is based on the minimization of a cost function that penalizes states’ deviations and actuators’ actions during transient periods. The main advantage of LQG control is its flexibility and usability when specifying the underlying trade-off between state regulation and control action.

However, one of the main obstacles in achieving adequate supervisory control of multi-machine power system is the inevitable presence of communication time-delays existing between a controller and individual generators dispersed over large geographical distances. In particular, it has been shown that these time-delays can result in deteriorating performance of the supervisory control scheme (Stahlhut, 2008). Therefore, time-delays should be considered when designing supervisory control schemes for multi-machine power systems. In literatures (Wu, 2002; Dotta, 2009), researchers have proposed supervisory controllers that do consider the presence of time-delays. These approaches tend to assume the time-delay is a known variable in the system. However, the time-delays existing between the local generator and the central supervisory controller are typically unknown. As a result, control systems which are built with an assumption that a time-delay is known at all times may be impractical.

This paper considers application of a supervisory discrete-time LQG with Extended Kalman Filter (EKF) to a nonlinear simulation of a 2-area, 4-machine, 8-bus power system while considering the time-delays which are unknown. The unknown time-delay is incorporated into the linearized model as an auxiliary state, which results in a nonlinear model. For this nonlinear problem, the EKF procedure is used to estimate the unknown time-delay and to feed that information into the supervisory LQG controller, which is assumed to be re-designed with an estimate of time-delay incorporated into the system model. Both the small signal stability and transient stability are evaluated using different values of unknown time-delays. The comparison between the supervisory
discrete-time LQG with EKF control and the conventional LQG control that ignores the presence of time-delays is also performed in order to illustrate the effectiveness of the proposed control design.

2. SYSTEM MODEL AND MODEL REDUCTION

2.1 Power System Model

This paper considers an 8-bus, 4-machine, 2-area power system simulation model. The single line diagram of the power system model is depicted in Fig. 1. The system consists of two areas, and each area includes two generating units: one containing generators 1 and 4, while the other area containing generators 2 and 3. Each generator unit is assumed to be equipped with AVR and PSS controllers. There is also a shunt impedance at bus 7 to help maintain the voltages in the system within statutory limits of ±5%. Machine parameters are listed in tables 2 and 3 in the Appendix A.

In modelling of this highly non-linear power system, the dynamic of each generator is represented by a 5th order state-space form. The AVR is designed as a lag compensator, of which the transfer function is:

\[ k_i(s) = \frac{198s}{1 + 0.0563s}, \quad i = 1, \ldots, 4 \]

The PSS is represented by two lead-lag blocks with washout and low pass filter. Its transfer function is given by:

\[ H_i(s) = K_{PSS} \left( \frac{1 + T_s}{1 + \alpha T_s} \right)^2 \left( \frac{5s}{1 + 5s} \right) \left( \frac{1 + 0.0563s}{1 + 0.1126s} \right), \quad i = 1, \ldots, 4 \]

The linearized complete model of the power system is represented by a 4-input, 12-output, 51st order state-space model. There are three oscillation modes, two of which are local modes and the third one is an inter-area mode. The three modes are shown in the first column of table 1.

Fig. 1. Single line diagram of the test system

2.2 Model Reduction

The complete 51st order model of the power system may contain variables which are not necessary for stability study. Also, this high order dimensional model is impractical and costly for computer simulation. Also, model-based controller design methods produce controllers of order at least equal to that of the power system, and usually higher with the incorporation of the extra weights. Model order reduction is then required to simplify the system dynamic model to accelerate the computation while keeping a good approximation of the complete power system model, thus, the complexity of the proposed LQG controller.

In this paper the optimal Hankel norm approximation method is used to get a reduced order model for controller design. The idea is to find the optimal reduced order model such that the Hankel norm of the approximation error between the nominal and the reduced order model is minimized (Glover, 1984). Fig. 2 shows the error bound between the 51st order nominal power system model and the reduced order model. It can be concluded from fig. 2 that 16th order reduced order model is a good approximation of the nominal power system. Table 1 shows the dominant oscillation modes are well retained in the reduced order model without much error.

Table 1 Model correspondence between the Nominal model and Reduced order model

<table>
<thead>
<tr>
<th>Nominal Model</th>
<th>Reduced Order Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>1</td>
<td>-1.6176 ± 8.5822i</td>
</tr>
<tr>
<td>2</td>
<td>-1.2607 ± 7.3485i</td>
</tr>
<tr>
<td>3</td>
<td>-0.1693 ± 3.5833i</td>
</tr>
</tbody>
</table>

Fig. 2. Error bound of the reduced order model

3. ESTIMATION OF UNKNOWN TIME-DELAY

3.1 Augmented Continuous-Time System

The standard description of the reduced continuous-time power system model is given by the following state-space equations:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

where \( x \in \mathbb{R}^{n_x} \), \( u \in \mathbb{R}^{n_u} \) and \( y \in \mathbb{R}^{n_y} \) are the system state, input and output vectors, respectively. Let \( \mathbb{R} \) denote the set of real numbers. A, B, C, D are appropriately dimensioned real constant matrices of the continuous-time system state-space model.

Consider an unknown input time-delay of \( \tau \) seconds. Such time-delay is expressed as \( e^{-\tau s} \) in Laplace domain. The
corresponding $k$-th order Pade approximation of the time-
delay can be represented by the following state-space model:

$$
\begin{align*}
\dot{x}_j(t) &= A_j x_j(y) + B_j u(t) \\
y_j(t) &= C_j x_j(t) + D_j u(t)
\end{align*}
$$

(4)

where $x_j \in \mathbb{R}^{n_{x_j}}$, $u \in \mathbb{R}^{n_{u}}$ and $y_j \in \mathbb{R}^{n_{y_j}}$ are the state, input and output vectors of the state-space representation of the input time-delay respectively. $A_j = \text{diag}\{a_1, a_2, \ldots, a_m\}$, $B_j = \text{diag}\{b_1, b_2, \ldots, b_m\}$, $C_j = \text{diag}\{c_1, c_2, \ldots, c_m\}$, $D_j = \text{diag}\{d_1, d_2, \ldots, d_m\}$. $a_i, b_i, c_i, d_i (i = 1, 2, \ldots, m)$ are the state-space models that represent $k$-th order Pade approximation for each input channel. $m$ is the input dimension as defined in the above continuous-time system. It is assumed that the delay for each signal is the same.

The 1st order Pade approximation is applied for the estimation of the unknown time delay, where $a_i = -2/r$, $b_i = 2/r$, $c_i = 2$, $d_i = -1$, $i = 1, 2, \ldots, m$. Denoted as $x_1 = 1/r$, an auxiliary state is provided, which represents time delay and can be incorporated into the overall augmented system model, as given in (5):

$$
\begin{align*}
\dot{x}(t) &= \hat{A}x(t) + \hat{B}u(t) \\
y(t) &= \hat{C}x(t) + \hat{D}u(t)
\end{align*}
$$

(5)

Where

$$
\hat{A} = \begin{bmatrix} A & B C & 0 \\ 0 & A_j & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B_j \\ 0 \\ 0 \end{bmatrix},
$$

(6)

This augmented system model incorporates the power system model, the state-space model of the time delay, and the time delay. Note that the auxiliary state $x_1$ is presented in the state vector as well as the state matrix $\hat{A}$. As a result, the augmented system model in (6) is a nonlinear model.

3.2 Discretization

The discretization of the augmented system (5) was performed using the zero-order-hold (ZOH) (Astrom, 1984). Equation (5) is converted into the following discrete-time form with the sampling period $T$:

$$
\begin{align*}
\tilde{x}(k+1) &= \Phi \tilde{x}(k) + \Gamma u(k) \\
y(k) &= \tilde{C} \tilde{x}(k) + \tilde{D} u(k)
\end{align*}
$$

(7)

with $\tilde{x}(k) = \begin{bmatrix} x^T(k) & x_1^T(k) & x_2^T(k) \end{bmatrix}^T$.

$$
\begin{align*}
\Phi &= e^{AT} = I + \hat{A}T \\
\Gamma &= \int_0^T e^{\hat{A}s} ds \tilde{B} = \Psi \tilde{B}
\end{align*}
$$

(8)

where $\Psi = \int_0^T e^{\hat{A}s} ds = IT + \frac{\hat{A}T^2}{2!} + \frac{\hat{A}^2 T^3}{3!} + \cdots + \frac{\hat{A}^{k-1} T^k}{(k-1)!} + \cdots$ (9)

Substituting (6), (8) and (9) into (7) results in the discretized augmented system. This discretized augmented system model is also nonlinear and represented by the following general form:

$$
\begin{align*}
\tilde{x}(k+1) &= f(\tilde{x}(k), u(k)) + w \\
y(k) &= h(\tilde{x}(k), u(k)) + v
\end{align*}
$$

(10)

where $f(\cdot)$ and $h(\cdot)$ are the nonlinear functions of states and inputs. $w$ and $v$ represents the process noise and measurement noise with the known covariance matrix $W = E(ww^T)$ and $V = E(vv^T)$ respectively. Both $w$ and $v$ are assumed to be white Gaussian processes and uncorrelated.

3.2 Unknown Time-Delay Estimation

The EKF algorithm is developed for the estimation of the unknown time-delay in the augmented model given in (10). The Kalman Filter method is the well-known recursive algorithm to achieve the optimal estimation of the states of the linear model by using the measured output. For nonlinear problems, a linearization procedure is usually performed to derive the filter equations in order to get an approximated linear model. The idea of EKF is to apply the standard Kalman Filter to the approximated linear model to get the state estimation.

Let $F(k)$ and $H(k)$ be the Jacobian matrices of $f(\cdot)$ and $h(\cdot)$, respectively, denoted by:

$$
F_k = \frac{\partial f(\tilde{x}, u)}{\partial \tilde{x}} \bigg|_{(\tilde{x}_k, u)}, \quad F_s = \frac{\partial f(\tilde{x}, u)}{\partial u} \bigg|_{(\tilde{x}_k, u)}, \quad H_k = \frac{\partial H(\tilde{x})}{\partial \tilde{x}}
$$

(11)

The discrete EKF recursive algorithm is performed by the following steps.

Step 1 Initialization

Initialize $\hat{x}(0) = \begin{bmatrix} \tilde{x}^T(0) & \tilde{x}_1^T(0) & \tilde{x}_2^T(0) \end{bmatrix}^T$ and $P(0|0)$. Define $\hat{x}(0|0) = \tilde{x}(0)$, and $P$ is defined as the error covariance of states $\tilde{x}$.

Step 2 Prediction

$$
\hat{x}(k+1|k) = f(\tilde{x}(k|k), u(k))
$$

$$
\hat{y}(k) = h(\tilde{x}(k|k))
$$

$$
P(k+1|k) = F_s(k) P(k|k) F_s(k)^T + F_s(k) W_s F_s(k)^T + W_k
$$

Step 3 Measurement Update

$$
L(k+1) = P(k+1|k) H_k^T (H_k P(k+1|k) H_k^T + V)^{-1}
$$

$$
P(k+1|k) = (I - L(k+1) H_k) P(k+1|k)
$$

$$
\hat{x}(k+1|k+1) = f(\hat{x}(k+1|k), u(k) + L(k+1)(y(k) - \hat{y}(k))
$$

Step 4 Increment the time constant, and go back to step 2.

Here $W_s, W_k$ are nonnegative definite symmetric which are defined as $W = W_s W_k^T I$, and $V$ is positive definite symmetric. $L(k)$ is the Kalman Filter gain at iteration $k$. $\hat{x}(k|k)$ is the estimation of $\tilde{x}(k|k)$.
The algorithm contains two main steps: prediction and measurement update. In the prediction step, the next predicted state \( \hat{x}(k+1 | k) \) and the predicted state error covariance matrix \( P(k+1 | k) \) are computed. In the measurement update step, next estimated state \( \hat{x}(k+1 | k+1) \) is obtained as the sum of the next predicted state and the correction term; and next estimated state error covariance matrix \( P(k+1 | k+1) \) is based on Kalman Filter gain \( L(k+1) \). After the iterations, the estimated time-delay \( \hat{\tau} \) is obtained from the estimated state \( \hat{x} \).

4. DISCRETE-TIME LQG DESIGN

The proposed discrete-time LQG controller is designed with consideration of the unknown time-delay. The control algorithm and the EKF function executed in parallel also can achieve to save the calculation time. The discrete-time LQG controller assumes no time-delay until EKF provides the estimate of the time-delay. The estimated time delay is then used to update the state-space model (7) and the LQG controller is then re-designed using this modified model. Fig. 3 shows the configuration of the proposed discrete-time LQG control scheme. By incorporating the input time-delay, the proposed discrete-time LQG controller is developed based on the sub-augmented system, which is derived from the overall augmented system (7) and (8). The sub-augmented system is represented in the following state-space form:

\[
\tau(k+1) = \bar{\Phi} \tau(k) + \bar{\Gamma} u(k) + w
\]

\[
y(k) = \bar{C} \tau(k) + \bar{D} u(k) + v
\]

with \( \tau(k) = [x^T(k) \ x_j^T(k)]^T \), \( \bar{\Phi}, \bar{\Gamma}, \bar{C}, \bar{D} \) are the sub-matrix blocks of \( \Phi, \Gamma, \bar{C}, \bar{D} \) corresponding to the first two states respectively.

The LQG control problem is to find the optimal control law which minimizes the following cost function:

\[
J = \sum_{k=0}^{\infty} [x^T(k)Qx(k) + u^T(k)Ru(k)]
\]

Here, \( Q \) is the nonnegative definite state weighting matrix, and \( R \) is the positive definite control weighting matrix.

The optimal state-feedback control law is given by

\[
u(k) = -K_x \tau(k), \quad K_x = [\bar{\Gamma}^T \bar{\Phi} + R]^{-1} \bar{\Gamma}^T P \bar{\Phi}
\]

Typically not all state variables are measured. Therefore the Kalman Filter is used to estimate these unmeasured system states. The structure of the Kalman Filter is given as follows:

\[
\hat{\tau}(k+1) = \bar{\Phi} \hat{\tau}(k) + \bar{\Gamma} u(k) + L(y(k) - \bar{C} \hat{\tau}(k))
\]

Here, \( L = \bar{\Phi} P \bar{\Phi}^T [\bar{C} P \bar{\Phi}^T + V]^{-1} \) is the Kalman Filter Gain that minimises estimation error variance \( E[\tau - \hat{\tau}^T (\tau - \hat{\tau})] \). \( P \) and \( P_\tau \) is the unique symmetric nonnegative definite solutions of the two Discrete Algebraic Riccati Equations (DARE):

\[
P = \bar{\Phi}^T P \bar{\Phi} - (\bar{\Phi}^T P \bar{\Phi} + R)^{-1} (\bar{\Phi}^T P \bar{\Phi}) + Q
\]

\[
P_\tau = \bar{\Phi} P \bar{\Phi}^T - \bar{\Phi} P C^T [\bar{C} P \bar{\Phi}^T + V]^{-1} \bar{C} P \bar{\Phi}^T + W
\]

The optimal LQG control law is then given by:

\[
u(k) = -K_x \hat{\tau}(k)
\]

Fig. 3. Configuration of the discrete-time LQG scheme

5. SIMULATION RESULTS

The 2-area, 4-machine power system model, described in section 2, is used to demonstrate the benefits of the LQG controller that utilises EKF. The proposed supervisory discrete-time LQG controller and EKF are designed based on the linearized reduced 16th order model with input time-delay. The simulations were used to assess the ability of EKF to estimate the unknown time-delays and to assess the ability of the resulting LQG controller, which utilises time-delay estimate, to control nonlinear full-order power system model.

5.1 Estimation of Unknown Time-Delays

Simulated time delay of 0.8 seconds was used in this case study. The delay was implemented using the “transport delay” block in SIMULINK. The sampling period \( T_s \) is set as 0.01 for the simulation. The initial conditions of the states and the error covariance of the states, the covariance matrices of the process noise and the measurement noise are set to be as following:

\[
\hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad P(0 | 0) = 0.1 I_{(r+s+m+1)(r+s+m+1)},
\]

\[
W = \begin{bmatrix} 10^{-4} & 0 \ 0 & 10^{-6} \end{bmatrix}, \quad V = I_{m+n}.
\]

Here, \( r, m \) and \( n \) are the state, input and output dimensions respectively and defined as in model (3). \( C \) is a constant which is set to be different values in order to evaluate the EKF estimation performance. As the time-delay is defined as \( \chi = 1/\tau = C \), the estimated time-delay \( \hat{\tau} \) is the inverse of \( C \). Fig. 4 depicts the estimation of the unknown time-delays with different initial conditions. It can be concluded that EKF provides fast and accurate estimation of the time delay. Also, it is apparent from Figure 4 that large initial values give better and faster convergence of the estimates.
5.2 Performance of the Discrete-Time LQG Controller

The estimated time-delay is fed into the supervisory discrete-time LQG controller, which is denoted as $LQG_{\text{EKF}}$, in order to adequately control the power system in the presence of the unknown time-delays. The state weighting $Q$ and control weighting $R$ are chosen as:

$$Q = 10^{-3} \times I_{r'}, \quad R = 10^{-4} \times I_{mvn}.$$ 

Here, $r'$ is the state dimension of the model used in designing the controller. The conventional discrete-time LQG controller without consideration of the time-delay, namely $LQG_0$, is also designed with the same quantity as the weightings of controller $LQG_{\text{EKF}}$. The simulation tests are obtained by applying different disturbances and time-delays into the power system. The comparisons are performed between the proposed supervisory discrete-time LQG controller $LQG_{\text{EKF}}$ and the conventional discrete-LQG controller $LQG_0$ which ignores the presence of the time delays.

**Case 1 Small Signal Stability**

A 2.5% reference voltage increase of generator 2 is applied into the power system at 1 second. The nominal steady-state electric power $P_e$ and the terminal voltage $E_t$ of generator 2 are 0.9 p.u. and 1 p.u. respectively. Therefore, the steady-state terminal voltage $E_t$ of generator 2 after the disturbance is applied is equal to 1.025 p.u.. Fig. 5 shows the electric power and terminal voltage of generator 2 for the cases where the overall power system is controlled by the supervisory controllers $LQG_{\text{EKF}}$ and $LQG_0$. Two different values of input time-delays are implemented, namely 0.8s and 1.5s, and the corresponding results are shown in Fig. 5(a) and Fig. 5(b) respectively. It can be clearly seen that the conventional discrete-time LQG controller designed that ignores time delay $LQG_0$, cannot stabilize the power system following a small signal disturbance. In particular, note that the final terminal voltage cannot reach the steady-state value. On the other hand, based on the estimation of the unknown time-delay, the final electric power and the terminal voltage of the power system with unknown input time-delays reach the steady-state values controlled by the proposed LQG controller $LQG_{\text{EKF}}$ for both 0.8s and 1.5s input time-delays.

**Case 2 Large Disturbance Stability**

A three-phase fault at bus 7 takes place at 1 second, with a self-clearing time 80 ms. Fig. 6 shows the performance of generator 2 of the power system controlled by the two controllers $LQG_{\text{EKF}}$ and $LQG_0$ respectively. The same input time-delays as in case 1 are implemented. As the large disturbance occurs and a 0.8s input time-delay exists in the power system, Fig. 6(a) shows that the oscillations of the power system cannot be damped when the system is controlled by the conventional discrete-time LQG controller $LQG_0$. The damping is even worse when a longer input time-delay 1.5s is applied as shown in Fig. 6(b). On the other hand, based on the estimated time-delay, the post-fault rotor angle speed and the electric power of the power system controlled by controller $LQG_{\text{EKF}}$ can reach the nominal steady-state values. This means the transient stability of the power system is greatly improved with the application of the supervisory discrete-time LQG controller that does consider time delays $LQG_{\text{EKF}}$. 

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Fig. 4. Estimated time-delay with different initial conditions

Fig. 5. Electric Power & Terminal Voltage of Gen 2: (a) 0.8s (b) 1.5s time-delay

Fig. 6. Speed Deviation & Electric power of Gen 2: (a) 0.8s (b) 1.5s time-delay
6. CONCLUSIONS

This paper considered supervisory discrete-time LQG of a multi-machine power system in the presence of unknown time-delays. For the particular simulation model of 2-area, 4-machine, 8-bus nonlinear power system, the complete linearized 51st order system model has been appropriately reduced to a 16th order model by using optimal Hankel norm approximation and discretized for discrete-time control design. This reduced order model was deemed to be a good approximation of the nominal system. The unknown time-delay has been successfully estimated by implementing the Extended Kalman Filter procedure and using the Padé approximation method for modelling of the time delays. Based on the estimated time-delay and the derived delay-free model, a supervisory discrete-time LQG controller with consideration of the time-delay was developed. The simulation on the nonlinear power system was performed by applying different faults and time-delays. The results showed the conventional LQG controller was not able to achieve stable operation in the presence of disturbance and relatively large time-delays. On the other hand, the proposed supervisory discrete-time LQG that employs EKF observer was found to effectively improve both small signal stability and transient stability of the wide-area power system subjected to severe disturbances and large time-delays.

REFERENCES


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Appendix A

Table 2 Generator Data

<table>
<thead>
<tr>
<th>Gen</th>
<th>$R_d$</th>
<th>$X_d$</th>
<th>$X_d'$</th>
<th>$T_{av}$</th>
<th>$T_{av}'$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.86</td>
<td>0.121</td>
<td>0.089</td>
<td>5.9</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.445</td>
<td>0.316</td>
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</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.86</td>
<td>0.121</td>
<td>0.089</td>
<td>5.9</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.86</td>
<td>0.121</td>
<td>0.089</td>
<td>5.9</td>
</tr>
</tbody>
</table>

$X_q$ $X_q'$ $X_q''$ $T_{av}''$ $T_{av}'' 

| 1   | 0.828 | 0.198 | 0.089 | 0.535 | 0.078 | 13.3    |
| 2   | 0.959 | 0     | 0.162 | 0     | 0.159 | 4.27    |
| 3   | 0.828 | 0.198 | 0.089 | 0.535 | 0.078 | 6.34    |
| 4   | 0.828 | 0.198 | 0.089 | 0.535 | 0.078 | 10.34   |

Table 3 AVR Data & PSS Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$K_{avr}$</th>
<th>$T_{avr}$</th>
<th>$T_w$</th>
<th>$T_s$</th>
<th>$T_p$</th>
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<tbody>
<tr>
<td>Values</td>
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<td>0.055</td>
<td>5</td>
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