Second Order Sliding Mode Block Control of Single-Phase Induction Motors

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Abstract: The authors propose a robust non-linear controller based on a block control linearization technique combined with a second order sliding modes super-twisting algorithm for controlling a rotor speed of single phase induction motors (SPIM). The controls objectives consist on the maximizing of the average torque operation and minimizing of the pulsating torque operation of the motor to achieve desirable performance. A nonlinear observer is designed to estimate the unmeasured variables (rotor flux linkages and torque load).

Keywords: Sliding mode control, Single-phase induction motor, Robust control, Nonlinear systems.

1. INTRODUCTION

Many domestic applications require low power motors operating essentially at a desired speed to control compressors, pumps, and other equipment that starts under load. Considering the potential for energy savings and broad application of the drivers in single-phase induction motors, the variable speed control is a topic of interest both economical and environmental.

The SPIM offers no starting torque, therefore a small auxiliary winding with a permanent split capacitor or a capacitor-start-capacitor-run is provided. Typically, the SPIM is provided with a centrifugal switch (SW) that operates when the motor speed reaches the threshold. As an alternative control scheme, a capacitor controlled by a control law to improve the functioning of the SPIM can be used instead of the centrifugal switch and starting capacitor. Using this scheme, the equivalent capacitance can be adjusted to achieve the maximum electromagnetic torque during start-up to get better performance in steady state.

Different controllers, based on power electronics, have been proposed to improve the performance of the SPIM. The most prominent are Law et al. (1986), Lettenmaier et al. (1991), Liu T. H. (1995), and Muljadi et al. (1993). Basically, these controllers are designed using a linear control technique ensuring local stability in a vicinity of the operation point.

A relatively simple technique, especially dealing with non-linear plants, is the use of sliding mode (SM) control, Utkin V. I. (1992) and Utkin et al. (1999). This method allows the decomposition of the design problem into two separate stages: first, proper selection of sliding manifold with the desired motion, and then, the design of discontinuous control such the sliding mode motion is forced along the desired manifold. During sliding mode, the effect of nonlinearities, uncertainties on parameters and external perturbations in the subspace of control can be rejected, providing the invariance of the system motion with respect to the system uncertainties.

In this paper we propose the control scheme which consists of two control loops and one nonlinear observer. The first control loop is designed in the main winding to control the speed and it is based on the block feedback linearization technique, Loukianov A. G. (1998), combined with a continuous SM super-twisting algorithm, Fridman L. et al. (2002), which can be applied directly to a PWM. The second loop is designed in the auxiliary winding, and a discontinuous SM algorithm is proposed to control the magnitude of flux.

The paper is organized as follows. In Section 2, the SPIM model is described and represented in a state variable space domain. In Section 3, first, a block feedback control technique is applied to design a desired sliding nonlinear manifold and a super-twisting control is used to ensure the designed manifold be attractive. Then, the auxiliary controller design based on first order SM algorithm is presented. In Section 4, a nonlinear observer is designed to estimate unmeasured variables: the rotor flux and load torque. Finally, in Section 5, simulation results are shown to support the designed controllers.

2. THE SPIM MODEL

The dynamic model of the unsymmetrical 2-phase induction machine \((a,b)\), Krause et al. (c2002), is described as follows

\[
\begin{align*}
\dot{v}_a &= v_{ia} + d \lambda_{ua} / dt \\
\dot{v}_b &= v_{ib} + d \lambda_{ub} / dt \\
\dot{v}_s &= v_{is} + d \lambda_{us} / dt \\
\dot{\alpha} &= \frac{n_v}{j}(T_e - T_i)
\end{align*}
\]

with

\[
\begin{align*}
\lambda_{ua} &= (L_u + L_{us})i_a + L_s \cos \theta i_s - L_c \sin \theta i_s \\
\lambda_{ub} &= (L_u + L_{us})i_b + L_s \sin \theta i_s + L_c \cos \theta i_s \\
\lambda_{us} &= (L_s + L_{us})i_s - L_s \sin \theta i_s + L_c \cos \theta i_s \\
\lambda_{us} &= (L_s + L_{us})i_s - L_s \sin \theta i_s + L_c \cos \theta i_s
\end{align*}
\]
where $\lambda_{sr}$ and $\lambda_{ar}$ are the stator and rotor magnetic-flux-linkage components, $i_{sr}$ and $i_{ar}$ are the stator and rotor current components, $\theta$ is the angular speed, $\omega_r$ is the rotor speed, $v_{sr}$ is the main winding voltage and $v_{ar}$ is the voltage of auxiliary winding, $J$ is the rotor moment of inertia, $r_e$ is the stator main winding resistance, $r_s$ is the stator auxiliary winding resistance, $r_p$ is the rotor resistance, $L_s$ and $L_a$ ( $L_{sr}$ and $L_{ar}$) are the leakage and magnetizing inductances of the main and auxiliary windings, $L_o$ and $L_{sr}$ are the leakage and magnetizing inductances of the rotor windings, $L_{ar}$ ( $L_{o}$) is the amplitude of the mutual inductance between the main and auxiliary windings, $n_p$ is the number of pole pairs, $T_r$ and $T_e$ is the load torque and electromagnetic torque respectively, finally $v_{ar}$ and $v_{sr}$ are the rotor voltages, which are zero.

Using the transformations to a stationary frame of reference known as $(\alpha-\beta)$ transformation, with $\theta = 0$ the matrix of transformation is $K_{2x} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ Krause et al. (c2002), the model (1) is represented of the form

$$d\omega_r/dt = dL/dT_r$$

$$d\lambda_{ar}/dt = -a_i\lambda_{ar} + n_c\omega_r\lambda_{br} + a_i\lambda_{ar}$$

$$d\lambda_{br}/dt = -n_i\omega_r\lambda_{ar} - a_i\lambda_{br} + a_i\lambda_{dr}$$

$$dL_{br}/dt = -c_ia_{dr} + c_4\lambda_{dr} - c_4\omega_r\lambda_{br} + c_4\lambda_{br}$$

$$dL_{ar}/dt = -c_ia_{ar} + c_4\lambda_{ar} + c_4\omega_r\lambda_{ar} + c_4\lambda_{ar}$$

where $\lambda_{ar}$ and $\lambda_{br}$ are the rotor magnetic-flux-linkage components, $i_{sr}$ and $i_{ar}$ are the stator current components, then $v_{sr}$ and $v_{br}$ are the voltage of the main and auxiliary stator winding respectively. The model constants depend on the motor parameters and they are given in the Appendix A.

The SPIM, in $\alpha\beta$-axis, with the stator current and rotor flux as the state variables, is presented in Fig.1.

![Single phase induction motor](image)

Fig.1. Single phase induction motor

The dynamics of the capacitor $v_c$ is governed by

$$dv_c/dt = \omega X_c \lambda_{br}$$

where $X_c$ is the capacitor reactance. The relation between the voltages of stator $v_{sr}$, $v_{br}$, in (5) and (6) and $v_c$ is given by

$$v_{sr} = v_c$$

$$v_{br} = n^{-1}v_c - v_c\rho$$

where the switching parameter $\rho$ is defined by

$$\rho = \begin{cases} 1 & v_{br} > n^{-1}v_c - v_c \\ 0 & v_{br} \leq n^{-1}v_c - v_c \end{cases}$$

and $n$ is the relation between the main and auxiliary windings. The load torque $T_L$ is assumed to be a slowly varying function of time. Thus,

$$T_L = 0.$$  \hfill (10)

Also the control input is bounded by

$$|v_c| \leq v_0$$  \hfill (11)

where $v_0$ is a positive scalar.

3. SLIDING MODE CONTROL FOR SPEED AND FLUX

Provided that the full state vector and load torque are known, the objective here is to design a SM controller which can effectively track the desired speed $\omega_{ref}$ and module to the square of rotor flux $\phi_{ref}$ reference signals by means of the basic continuous control $v_c$ and the auxiliary control $\rho$ as discontinuous function.

Define the tracking error as

$$x_1 = \begin{bmatrix} \omega_{ref} - \omega_c \\ \varphi - \varphi_{ref} \end{bmatrix}$$

and $x_2 = \begin{bmatrix} i_{ar} \\ i_{br} \end{bmatrix}$

where $\varphi = \sqrt{\omega_{ref}^2 + \lambda_{br}^2}$ is the module to the square of rotor flux. Then the system (2)-(7) can be represented in the Nonlinear Block Control form with perturbation, Loukianov A. G. (1998) that consists of two blocks:

$$\dot{x}_1 = f_1(x_1) + B_1(x_1)x_2 + DT_1 + h(t)$$

$$\dot{x}_2 = f_2(x_1, x_2) + Bu$$

where rank $B_1(x_1) = n_1$, rank $B_1 = n_1$ and $n_1 = n_2 = 2$ with $f_1 = \begin{bmatrix} f_{11} \\ f_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ -2a_2\omega_c \end{bmatrix} B_1(x_1) = \begin{bmatrix} d_1d_2 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -d_2 \\ 0 \end{bmatrix}, h(t) = \begin{bmatrix} \omega_{ref} \\ \varphi_{ref} \end{bmatrix}, f_2 = \begin{bmatrix} f_{21} \\ f_{22} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$u = \begin{bmatrix} v_{sr} \\ v_{br} \end{bmatrix} = \begin{bmatrix} i_{ar} \\ i_{br} \end{bmatrix} = c_1\lambda_{br} - c_1\omega_r\lambda_{br} - c_1\omega_r\lambda_{br} - c_1\omega_r\lambda_{br}$$

Setting $z_1 = x_1$, with the vector $z_1 = \begin{bmatrix} z_{x1} \\ z_{x2} \end{bmatrix}$ and using the block control technique, the desired value $x_{des}$ for the virtual control using a currents formulation for $x_1$, in the first block (12) is formed by
\[ x_{2,\text{dev}} = -B_1^{-1}(x)(f_i + DT_x + h(t)) + B_1^{-1}(x)(Kz_i) \] (14)

with the controller gains matrix

\[ K = \begin{bmatrix} -k_1 & 0 \\ 0 & -k_2 \end{bmatrix}, \quad k_1 > 0, \quad k_2 > 0. \]

A sliding variable \( z_2 \) is defined as follows

\[ z_2 = x_2 - x_{2,\text{dev}} \] (15)

where \( z_2 = (z_{21}, z_{22})^T \), \( z_{21} = (i_{\text{us}} - i_{\text{ds}}) \), \( z_{22} = (i_{\beta\text{us}} - i_{\beta\text{ds}}) \).

Then transformed first block (12) is represented as

\[ \begin{align*}
\dot{z}_1 &= -Kz_1 + B_1(x_1)z_2 \\
\dot{z}_2 &= f_2(x_1, x_2) + B_2u - x_{2,\text{dev}}
\end{align*} \] (16)

Substituting control law (20) and (21) in (19), the closed-loop system becomes

\[ \begin{align*}
\dot{z}_1 &= f_1 - c_1\alpha_1 z_1 \text{sign}(z_1) + v_{\text{rel}}(t) \\
\dot{v}_{\text{rel}} &= -\alpha_2 \text{sign}(z_1)
\end{align*} \] (20)

with

\[ v_{\text{rel}} = -\alpha_1 \text{sign}(z_1) \] (21)

and

\[ z_1 = \begin{cases} 
\begin{align*}
\text{for} & \quad z_1 \leq z_{1,\text{lim}} \\
\text{for} & \quad z_1 > z_{1,\text{lim}}
\end{align*}
\end{cases} \]

where \( z_{1,\text{lim}} = I_{\text{max}} \approx 3 \times I_{\text{nom}} \) to satisfy (11).

Substituting control law (20) and (21) in (19), the closed-loop system becomes

\[ \begin{align*}
\dot{z}_1 &= f_1 - c_1\alpha_1 z_1 \text{sign}(z_1) + v_{\text{rel}}(t) \\
\dot{v}_{\text{rel}} &= -\alpha_2 \text{sign}(z_1)
\end{align*} \] (22)

with \( f_1 = f_{21} - a_c i_{\text{us}} - di_{\text{ds}}/dt \), \( \alpha_1 > 0 \) and \( \alpha_2 > 0. \)

**Theorem 1:** Consider an admissible region \( \Omega \) where the term \( f_i \), considered as a perturbation in (22), is bounded by

\[ |f_i| \leq \delta_1 |z_1| \] (23)

for a constant \( \delta_1 > 0 \) and choose the control gains \( \alpha_1 \) and \( \alpha_2 \) such that the following conditions hold:

\[ \alpha_1 > 2\delta_1 \quad \text{and} \quad \alpha_2 > \alpha_1^2 \frac{\delta_1^2}{16} \] (24)

Then the state vector of the closed-loop system (22) reaches the manifold \( z_{21} = 0 \) in finite time, having semi-global asymptotic stability, Moreno, et al. (2008).

**Remark:** The proposed control algorithm ensures the robustness of the closed-loop system, alleviating the chattering effect and providing a continuous control signal suitable for deployment in a conventional PWM system.

### 3.2 Auxiliary control design

In SM motion on the manifold \( z_{21} = 0 \), the equivalent value \( v_{\text{eq}} \) of the control \( v_i \) is calculated as a solution of \( z_{21} = 0 \) (22) Utkin V. I. et al. (1999), as

\[ v_{\text{eq}} = c_1^i f_i \] (25)

Substituting (25) in the second equation of (19) yields

\[ \dot{z}_2 = -a_2 z_{22} + f_2(z_{22}) - c_2 v_i \rho \] (26)

where \( \rho = (z_{21}, z_{22})^T \), \( a_2 = a_1 \), and

\[ f_2(z_{22}) = f_{22} + c_1 (z_{22}) \quad f_{22} = -a_2 z_{22} - di_{\text{ds}}/dt \]

Thus, a SM motion on \( z_{21} = 0 \) is governed by the following reduced order system:

\[ \begin{align*}
\dot{z}_{11} &= -k_1 z_{11} + f_{11}(\xi) \\
\dot{z}_{12} &= -k_2 z_{12} + f_{12}(\xi) \\
\dot{z}_{22} &= -a_2 z_{22} + f_{22}(\xi) - c_2 v_{\text{rel}} \rho
\end{align*} \] (27)

with

\[ f_{11}(\xi) = -d_x d_t \zeta_{\text{rel}} z_{22} \quad \text{and} \quad f_{12}(\xi) = 2a_4 z_{22} z_{22} \]

The system (27) can be represented as a linear perturbed system:

\[ \dot{\xi} = A\xi + \zeta\rho + g(\xi) \] (28)

where

\[ A = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -a_2 \end{bmatrix}, \quad \zeta = \begin{bmatrix} 0 \\ 0 \\ -c_2 v_{\text{rel}} \end{bmatrix}, \quad g(\xi) = \begin{bmatrix} f_{11}(\xi) \\ f_{12}(\xi) \end{bmatrix}. \]

We assume that the perturbation \( g(\xi) \) satisfies in \( \Omega \) the standard bound

\[ \|g(\xi)\| \leq \gamma_1 \|\xi\| + \gamma_2, \quad \gamma_1 > 0, \quad \gamma_2 > 0. \] (29)

To design the auxiliary control \( \rho \) we apply the following Lyapunov function candidate:
\[ \dot{V} = \frac{1}{2} (z_{11}^2 + z_{12}^2 + z_{22}^2). \]  
(30)

The time derivative of (30) along the trajectories of (27) or (28) is calculated as

\[ \dot{V} = -k_1z_{11}^2 - k_2z_{12}^2 - a_{22}z_{22}^2 + z_1^T g(z) - c_zz_{22}v_\rho. \]  
(31)

The switching logic for the capacitor is chosen as follows:

\[ \rho = \begin{cases} 
1 & \text{if } z_{22} > 0 \text{ and } v_\rho > 0 \\
0 & \text{if } z_{22} > 0 \text{ and } v_\rho < 0 \\
1 & \text{if } z_{22} < 0 \text{ and } v_\rho > 0 \\
0 & \text{if } z_{22} < 0 \text{ and } v_\rho < 0 
\end{cases} \]

that results in

\[ \rho = 0.5 \text{sign}(z_{22}v_\rho) + 0.5 \]  
(32)

Substituting (32) in (31) and using \( z_{22}v_\rho \) sign \( (z_{22}v_\rho) = \lvert z_{22}v_\rho \rvert \), the derivative (31) becomes

\[ \dot{V} \leq -k_1\lvert z_{11} \rvert^2 - k_2\lvert z_{12} \rvert^2 - a_{22}\lvert z_{22} \rvert^2 \\
+ \lvert \bar{F} \rvert \lvert g(z) \rvert - 0.5c_z \lvert z_{22}v_\rho \rvert + z_{22}^2 v_\rho \]  
(33)

where the properties \( z_{22}v_\rho \leq \lvert z_{22}v_\rho \rvert \) and \( \alpha = \min \{k_1,k_2,a_{22} \} \)

were used. Substituting now (29) in (33) and adding and subtracting \( (\alpha - \gamma_2) \beta \lvert \bar{F} \rvert \) we have

\[ \dot{V} \leq -((\alpha - \gamma_2))(1-\beta) \lvert \bar{F} \rvert - \lvert (\alpha - \gamma_2) \beta \lvert \bar{F} \rvert - \gamma_2 \]  
(34)

Therefore, a solution of the system (27) is ultimately bounded by

\[ \lvert \bar{F}(t) \rvert \leq \delta_0, \quad \delta_0 = \frac{\gamma_2}{(\alpha - \gamma_2) \beta} \]

Finally, to limit the stator currents we propose the following logic for the sliding variables \( z_{11} \) and \( z_{22} \):

\[ z_{11} = \begin{cases} 
i_{\text{ar}} - I_{\text{max}} & \text{for } \lvert i \rvert \leq I_{\text{max}} \\
i_{\text{ar}} & \text{for } \lvert i \rvert > I_{\text{max}} 
\end{cases} \]

\[ z_{22} = \begin{cases} 
i_{\text{pr}} - I_{\text{max}} & \text{for } \lvert i \rvert \leq I_{\text{max}} \\
i_{\text{pr}} & \text{for } \lvert i \rvert > I_{\text{max}} 
\end{cases} \]

where \( \lvert i \rvert = \sqrt{i_{\text{ar}}^2 + i_{\text{pr}}^2} \) is the module of the currents.

This currents limit provides maximum motor torque during the speed transient process.

### 4. NONLINEAR OBSERVER

Having the rotor speed \( \omega_r \) and stator current \( i_{s\alpha} \) and \( i_{s\beta} \) measurements only, in this section a nonlinear observer is designed to estimate the rotor flux and load torque. Consider the following transformation:

\[ \lambda_{\text{ar}} = \lambda_{\text{ar}} - l_i i_{s\alpha}, \quad \lambda_{\text{pr}} = \lambda_{\text{pr}} - l_i i_{s\beta}, \]  
(35)

where \( l_i \) and \( l_s \) are positive constants. Using (35) the speed and flux dynamics (2) - (4) are represented in new variables \( \lambda_{\text{ar}} \) and \( \lambda_{\text{pr}} \) as,

\[ \dot{\lambda}_{\text{ar}} = -l_i \lambda_{\text{ar}} + l_i \omega_r \lambda_{\text{pr}} + \gamma_1 \lambda_{\text{pr}} + \gamma_2 l_i \lambda_{\text{pr}} - \gamma_3 v_{\text{ar}} \]

\[ \dot{\lambda}_{\text{pr}} = -l_i \lambda_{\text{pr}} - l_i \omega_r \lambda_{\text{ar}} - \gamma_1 \lambda_{\text{ar}} + \gamma_2 l_i \lambda_{\text{ar}} - \gamma_3 v_{\text{pr}} \]

\[ \dot{\omega}_r = d_i d_2 \left( \lambda_{\text{pr}} - \lambda_{\text{ar}} \right) - d_2 T_L \]

\[ \dot{T}_L = 0 \]  
(36)

where \( l_i = l_s \),

\[ l_1 = (a_1 + l_c c_1 c_1), \quad l_2 = (1 + l_c c_1 c_1), \quad \gamma_1 = l_c, \]

\[ l_1' = (a_1 + l_c c_1 c_1), \quad l_2' = (1 + l_c c_1 c_1), \quad \gamma_1' = l_c, \]

\[ \gamma_2 = (l_1 + l_c c_1 c_1), \quad \gamma_2' = (1 + l_c c_1 c_1), \quad \gamma_3 = a_1 l_c c_1 c_1 c_1. \]

Using (36), a nonlinear observer is proposed as follows

\[ \dot{\lambda}_{\text{ar}} = -l_i \dot{\lambda}_{\text{ar}} + l_i \omega_r \dot{\lambda}_{\text{pr}} + \gamma_1 \dot{\lambda}_{\text{pr}} + \gamma_2 l_i \dot{\lambda}_{\text{pr}} - \gamma_3 v_{\text{ar}} \]

\[ \dot{\lambda}_{\text{pr}} = -l_i \dot{\lambda}_{\text{pr}} - l_i \omega_r \dot{\lambda}_{\text{ar}} - \gamma_1 \dot{\lambda}_{\text{ar}} + \gamma_2 l_i \dot{\lambda}_{\text{ar}} - \gamma_3 v_{\text{pr}} \]

\[ \dot{\omega}_r = d_i d_2 \left( \lambda_{\text{pr}} - \lambda_{\text{ar}} \right) - d_2 T_L + l_1 \omega_r - \omega_r \]

\[ \dot{T}_L = l_1 (\omega_r - \omega_r) \]  
(37)

where \( \dot{\lambda}_{\text{ar}}, \dot{\lambda}_{\text{pr}}, \dot{\omega}_r \) and \( \dot{T}_L \) are estimates of \( \lambda_{\text{ar}}, \lambda_{\text{pr}}, \omega_r \) and \( T_L \), respectively. Define the estimation error as

\[ e_{\alpha r} = \lambda_{\text{ar}} - \dot{\lambda}_{\text{ar}}, \quad e_{\alpha p} = \lambda_{\text{pr}} - \dot{\lambda}_{\text{pr}}, \quad e_{\omega r} = \omega_r - \dot{\omega}_r \]

Then, from (36) and (37), the observer error dynamics are obtained of the form

\[ \dot{e}_{\alpha r} = -l_i e_{\alpha r} + l_i \omega_r e_{\alpha p}, \quad \dot{e}_{\omega r} = -l_i e_{\omega r} - l_i \omega_r e_{\omega r} \]  
(38)

\[ \dot{e}_{\alpha p} = -d_i e_{\omega r} - l_i e_{\omega r}, \quad \dot{e}_{\omega r} = -l_i e_{\omega r} \]  
(39)

The gains \( l_i \) and \( l_s \) in the autonomous system (39) are chosen such that \( \begin{bmatrix} -l_i & -d_2 \\ 0 & 0 \end{bmatrix} \) is a Hurwitz matrix, that results in

\[ \lim_{t \to \infty} e_{\omega r} (t) = 0 \quad \text{and} \quad \lim_{t \to \infty} e_{\omega r} (t) = 0 \]

The stability of the subsystem (38) is determined by using the following Lyapunov function candidate:
\[ V_r = \frac{1}{2}(e_{ar}^2 + e_{pr}^2), \quad (40) \]

Taking the derivative time of the Lyapunov function (40), along the trajectories of (39), and taking into account \[ |\omega| \leq \omega_{nom}, \]
yields
\[
\dot{V}_e = -\left[ l_1 e_{ar}^2 + 2 l_2 e_{ar} e_{pr} + (l_{12} - l_{22}) \omega e_{ar} e_{pr} \right] + (l_0 - \frac{1}{2} (l_{12} - l_{22}) \omega_{nom}) \| e \|^2 < 0
\]
where \( l_0 = \min(l_{11}, l_{21}) \) and the stability condition \( \left( l_0 - \frac{1}{2} (l_{12} - l_{22}) \omega_{nom} \right) > 0 \) is achieved by an appropriate selecting of the observer gains \( l_{12} \) and \( l_{22} \). As a result, the equilibrium point \( e_{ar} = 0 \) and \( e_{pr} = 0 \) of the system (38) is exponentially stable, that is
\[
\lim_{t \to \infty} \dot{\hat{\lambda}}_{ar} (t) = \lambda_{ar} (t) \quad \text{and} \quad \lim_{t \to \infty} \dot{\hat{\lambda}}_{pr} (t) = \lambda_{pr} (t).
\]

Finally, using (35) the rotor flux estimates \( \dot{\hat{\lambda}}_{ar} \) and \( \dot{\hat{\lambda}}_{pr} \) are obtained as
\[
\dot{\hat{\lambda}}_{ar} = \dot{\lambda}_{ar} + l_1 \dot{e}_{ar} \quad \text{and} \quad \dot{\hat{\lambda}}_{pr} = \dot{\lambda}_{pr} + l_2 \dot{e}_{pr}.
\]

These estimates, as well the load torque estimate \( \hat{T}_L \), are used then in the control algorithm (15) and (14) instead of the real variables.

5. SIMULATION RESULTS

In order to analyze the performance of the designed controller and the closed-loop system response, simulations in Matlab/Simulink have been designed. Parameters and data of the SPIM are as follows, Krause et al. (c2002):

- H.P. = 0.25, \( V_{rms} = 110 \) (volts), \( f = 60 \) (Hz), \( n_p = 2 \), \( n = 1.18 \)
- \( r_{ar} = 2.02 \) (\( \Omega \)), \( r_{pr} = 5.13 \) (\( \Omega \)), \( r = 4.12 \) (\( \Omega \)), \( L_{ar} = 0.1772 \) (H), \( L_{pr} = 0.1846 \) (H), \( L_{pr} = 0.1833 \) (H), \( L_i = 0.1828 \) (H), \( J = 0.0146 \) (Kg-m^2), \( I_{max} = 15 \) (amps), \( C_{run} = 35 \ \mu f \).

The controller gains are adjusted as \( k_i = k_z = 1000 \) and \( \alpha_t = \alpha_s = 33 \), the gains of the observer are \( l_1 = l_z = 0.1 \), \( l_2 = 50 \) and \( l_4 = -5 \).

The system presents disturbances, one of them are a change of load torque. The controlled variables are angular rotor velocity \( \omega_r \), and the module to the square of rotor flux \( \varphi \).

The disturbances are presented as

1) At the first second, a change of reference speed – in step form – from 100 rad/sec to 140 rad/sec is presented.

2) At 1.5 seconds, a change of load torque from 0.5 Nw-m to 1 Nw-m during one second is applied.
A nonlinear observer is designed to estimate the rotor flux and the load torque. The stability condition of the closed-loop system is derived. The simulation results show a robust performance of the proposed controller with respect to the perturbations caused by a change of load torque. Moreover, the proposed controller ensures the constriactions on the stator currents.

REFERENCES


Appendix A. CONSTANTS OF SPIM MODEL

\[ a_i = r_{si} + \frac{r_{Lm}}{L_s}, \quad a_2 = r_{si} + \frac{r_{Lm}}{L_s}, \quad a_3 = \frac{r_{Lm}}{L_s}, \quad a_4 = \frac{r_{Lm}}{L_s}, \]

\[ c_1 = \frac{L_r}{L_{sr}L_r - L_m}, \quad c_2 = \frac{L_r}{L_{sr}L_r - L_m}, \quad c_3 = \frac{L_r}{L_{sr}L_r - L_m}, \quad c_4 = \frac{r_{Lm}}{L_s}, \]

\[ d_1 = n_r \frac{L_m}{L_r}, \quad d_2 = \frac{n_r}{J}. \]