A Class of Strongly Stabilizing Bandpass Controllers for Flexible Structures

Alberto Cavallo, Giuseppe De Maria, Ciro Natale and Salvatore Pirozzi

Abstract: Along the line of past publications of the authors on the topic of active vibration control of flexible structures, the present paper proposes an improvement of the robust control strategy leading to strongly stabilizing bandpass controllers. The improvement consists in the extension of the approach to the case of active control systems with more sensors than actuators, which is very frequent in practical applications. In fact, quite often the application requirements do not allow the control engineer to install actuator/sensor couples in any desired location but only a sensor. In such cases, sensors can be usefully placed in the locations where the primary force fields act on the structure, so as to provide the controller with a direct information on the disturbance effects in terms of structural vibrations. This leads to a control problem with a rectangular plant, which complicates the design, especially if the same characteristics of strongly stabilizing bandpass controllers are desired. The design problem is here solved by resorting to an LMI (Linear Matrix Inequality) technique, which allows also to select the performance weights based on different design requirements. Experimental results are presented for a vibration reduction problem of a stiffened aeronautical panel controlled by piezoelectric actuators.

Keywords: Optimal control theory; Active vibration control; Flexible structure control; \( H_2 \) control; Strong stabilization; Industrial applications of optimal control; LMIs

1. INTRODUCTION

Reduction of vibrations in flexible systems has being considered for years a challenging problem, due to some specific aspects. First, as the model of the structure is described by partial differential equations, that can be treated as an infinite set of ordinary differential equations, any finite-order controller must rely on a limited (truncated) description of the plant. Thus, neglected dynamics actions must be taken into account by robustness properties of the controller. A second problem is that, in order to base the design technique for the controller on a meaningful model of the structure, many structural modes must be included in the model, especially for structures with high modal density, so that control problems for flexible structures are intrinsically large-scale problems. Finally, stability of the controller (in order to reduce sensitivity to disturbances) and limited bandwidth (to avoid spillover, due to interaction of unmodelled dynamics with control action) must be enforced. From a practical point of view, vibrations are often measured by using accelerometers, which are prone to offsets which can even saturate actuators if not suitably filtered out in the feedback loop. Thus the need for a filtering action even at low frequency arises. Hence a limited-bandwidth, strongly stabilizing controller is needed. Further discussions on this topic can be found in (Cavallo et al., 2008), (Cavallo et al., 2006), (Cavallo et al., 2010) and references therein.

In this paper the selection of a stable stabilizing controller with bandpass frequency shape is discussed, able to deal with rectangular plants (i.e. with more outputs than inputs), is proposed. The motivation is provided by the requirements of many applications, where the rejection of a disturbance acting on the structure is desired in locations where the installation of actuator/sensor couples is prohibited. Therefore, the possibility to exploit the measurement of the vibrating status in those locations appears the only viable option. The control strategy proposed in this paper is specifically designed to usefully exploit the knowledge of the disturbance entry point on the structure to place additional sensors able to provide the controller with the necessary information on the vibration status caused by the disturbance. The resulting optimal controller thus has more inputs than outputs, but it is still guaranteed to be strongly stabilizing and with bandpass frequency response. Moreover, selection of free control parameters in order to improve effectiveness of the control, by emphasizing the action of the most controllable and observable modes is discussed. To achieve such an objective, the control design procedure, still based on an \( H_2 \) approach, adopts an LMI (Linear Matrix Inequality) technique to compute the control parameters to allow the user to select the design weights based on various design requirements or by taking into account modal controllability/observability indices. The paper reports experimental results of broadband vibration reduction for a stiffened aeronautical panel controlled with piezoelectric actuators.
2. THE MODEL

In this paper a flexible structure with \( m \) control inputs and \( m + n_d \) measured (control) outputs is considered. The structure is assumed to be linear and time-invariant. Since in general \( n_d \geq 0 \), the system is not necessarily square, i.e. it has \( n_d \) more outputs than inputs. This assumption is quite reasonable in flexible systems control applications, where actuators are heavy and expensive, while sensors (accelerometers) are light and relatively cheap.

As extensively explained in (Cavallo et al., 2010), the mathematical model of a flexible structure can be written as

\[
\dot{x} = A x + B_u \bar{u}, \quad y = \bar{C}_y x
\]

where \( x = (x_1^T \ x_2^T)^T \in \mathbb{R}^{2n} \) is the state vector, \( \bar{u} \in \mathbb{R}^m \) the control input, \( \bar{y} \in \mathbb{R}^{m+n_d} \) the measured (control) output. By resorting to the “modal coordinates” the matrices of the state space model assume the form

\[
A = \begin{pmatrix} 0 & I \\ -\Omega & -\Lambda \end{pmatrix}, \quad B_u = \begin{pmatrix} 0 \\ B_{u2} \end{pmatrix}, \quad \bar{C}_y = \begin{pmatrix} 0 & B_{d2}^T & B_{u2}^T \end{pmatrix},
\]

where

\[
\Omega = \text{diag}(\omega_1^2, \ldots, \omega_n^2), \quad \Lambda = \text{diag}(2\zeta_1\omega_1, \ldots, 2\zeta_n\omega_n)
\]

being \( \omega_i \) and \( \zeta_i > 0 \) the natural frequency and damping coefficient of the \( i \)-th mode, \( (i = 1, \ldots, n) \). In this case \( x_1 \) are the modal displacements and \( x_2 \) the modal velocities. Moreover, assuming velocity measurement available, the output matrix \( \bar{C}_y \) has the form (5). By following the gray-box approach proposed by Cavallo et al. (2010), the model above can be accurately identified through a measurement campaign carried out on the structure.

3. THE CONTROL STRATEGY

In order to design a stabilizing controller for the flexible system it is convenient to resort to the standard control problem framework depicted in Fig. 1. In this figure, \( P \) is the system to control, with measured output \( y \), control input \( u \), generalized disturbance \( w \) and performance output \( z \). Augmenting the flexible system with suitable fictitious “disturbance” and “performance” matrices, the following generalized plant \( P \) is obtained

\[
\dot{x} = Ax + B_1 w + B_u u \\
z = C_1 x + D_{12} u \\
y = C_y x + D_{21} w
\]

where \( w \in \mathbb{R}^{m+n_d+n} \) is a fictitious disturbance, \( z \in \mathbb{R}^{m+n} \) a fictitious performance output, and

\[
u = R_1^{1/2} u \in \mathbb{R}^m, \quad y = R_2^{1/2} y \in \mathbb{R}^{m+n_d}
\]

\[
B_u = B_u R_1^{1/2} = \begin{pmatrix} 0 \\ B_{u2} \end{pmatrix}
\]

being \( R_1 \) and \( R_2 \) weighting matrices to be selected. The matrices \( B_1 \) and \( C_1 \) are \( 2n \times (m + n_d + n) \) and \( (m + n) \times 2n \) real matrices, respectively, with the following structure

\[
B_1 = \begin{pmatrix} 0_{n \times n} & 0_{n \times (n_c + m)} \\
W^{1/2} & 0_{n \times (n_c + m)} \end{pmatrix}
\]

\[
C_1 = \begin{pmatrix} 0_{n \times n} & Q^{1/2} \\
0_{m \times n} & 0_{m \times n} \end{pmatrix}
\]

where the matrices \( W \) and \( Q \) are to be selected. Finally, for the sake of simplicity, the matrices \( D_{12} \) and \( D_{21} \) are chosen to so as to satisfy the following conditions

\[
D_{12}^T C_1 = 0, \quad B_1 D_{21}^T = 0
\]

Although not essentials, conditions (15) are standard in \( H_2 \) or \( H_{\infty} \) control (Zhou and Doyle, 1995). It is possible to remove them by slightly increasing the complexity of the control strategy. A possible selection is

\[
D_{12} = \begin{pmatrix} 0_{n \times m} \\
D_{122} \end{pmatrix}, \quad D_{21} = \begin{pmatrix} 0_{n \times n} & D_{211} & 0_{n \times m} \\
0_{m \times n} & 0_{m \times n_d} & D_{212} \end{pmatrix}
\]

where

\[
R_1 = D_{12}^T D_{12} = D_{122}^T D_{122} > 0
\]

\[
R_2 = \begin{pmatrix} D_{211}^T & D_{212}^T \\
D_{212} & D_{212} \end{pmatrix} = \begin{pmatrix} R_2 & 0 \\
0 & R_2 \end{pmatrix} > 0
\]

Before computing the controller, a discussion on the required properties of the controller is in order. As stated in the introduction, stable bandpass controllers are preferred options in the control of flexible systems, due to their ability to filter out accelerometer biases. The following Lemma is useful for characterizing the state space representation of a general class of bandpass systems.

**Lemma 1.** Let \( B \) be a LTI 2n-(McMillan) degree system with minimal state space representation

\[
B = \begin{pmatrix} A & B \end{pmatrix}
\]

where

\[
A = \begin{pmatrix} 0 & I \\
A_1 & A_2 \end{pmatrix}, B = \begin{pmatrix} 0 \\
B_2 \end{pmatrix}, C = \begin{pmatrix} 0 & C_2 \end{pmatrix}
\]

where all the matrices are partitioned assuming the state split into two \( n \)-dimensional substates, and the square \( n \times n \)}
matrix $A_1$ is invertible. Then the system $B$ is bandpass, in the sense that
\[ \lim_{s \to 0} B(s) = \lim_{s \to \infty} B(s) = 0 \]  
(21)

**Proof.** Although the Lemma is an extension of the Theorem 1 in (Cavallaro et al., 2008), it can be easily proved observing that
\[ \lim_{s \to 0} (sI - A)^{-1} = \begin{pmatrix} A_1^{-1} A_2 & A_1^{-1} \\ -I & 0 \end{pmatrix}. \]  
(22)
Thus, direct computation shows that the static gain is zero. The high-frequency behavior is trivial.

The choice of a stabilizing controller with the structure in Lemma 1 can be carried out by exploiting the following Theorem.

**Theorem 1.** Any system of the form (7)–(9) with matrices given by (3), (11) and (12) is closed-loop stabilized by a controller with state space representation
\[ \mathbf{K} = \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}, \]  
(23)
where
\[ B_c = \begin{pmatrix} 0_{m \times n} & 0_{m \times n} \\ Y_2 B_d R_{21}^{-1} & Y_2 B_u R_{22}^{-1} \end{pmatrix}, \]  
(24)
\[ C_c = -R_{11}^{-1} \begin{pmatrix} 0_{m \times n} B_d^T X_2 \end{pmatrix}, \]  
(25)
\[ A_c = A + B_u C_c = B_c C_y \]  
(26)
where $X_2$ and $Y_2$ are $n \times n$ diagonal matrices with non-negative entries.

**Proof.** Let
\[ Q = 2X_2 A + X_2 B_u R_1^{-1} B_u^T X_2 \]  
(27)
and
\[ W = 2Y_2 A + Y_2 (B_d R_{21}^{-1} B_d^T + B_u R_{22}^{-1} B_u^T) Y_2. \]  
(28)
It is clear that both $Q$ and $W$ are positive semidefinite for any non-negative matrices $X_2$ and $Y_2$. Moreover, by defining the matrices
\[ X = \begin{pmatrix} \Omega X_2 & 0 \\ 0 & X_2 \end{pmatrix}, \]  
(29)
\[ Y = \begin{pmatrix} \Omega^{-1} Y_2 & 0 \\ 0 & Y_2 \end{pmatrix}, \]  
(30)
it is easy to show that $X = \text{Ric}(H_2)$ and $Y = \text{Ric}(J_2)$, where for a given Hamiltonian matrix $Z$, $\text{Ric}(Z)$ denotes the **stabilizing solution** of the Riccati equation associated to the Hamiltonian matrix (Zhou and Doyle, 1995), and the Hamiltonian matrices $H_2$ and $J_2$ are
\[ H_2 = \begin{pmatrix} A & -B_u R_1^{-1} B_u^T \\ -\hat{Q} & -A^T \end{pmatrix}, \]  
(31)
and
\[ J_2 = \begin{pmatrix} A^T & -C_y R_1^{-1} C_y \\ -\hat{W} & -A \end{pmatrix}. \]  
(32)
Finally, the matrices $\hat{Q}$ and $\hat{W}$ are
\[ \hat{Q} = \begin{pmatrix} 0 & 0 \\ 0 & \hat{Q} \end{pmatrix}, \]  
(33)

Thus, for any matrices $X_2$ and $Y_2$ the controller (24)–(26) solves an $H_2$ problem and is therefore stabilizing.

The last issue to address in order to design a useful controller for flexible structures is stability of the controller itself. It is well-known that inserting unstable elements into the control loop increases sensitivity to disturbances and call for high-bandwidth controller, thus also increasing the effect of measurement noise (Maciejowski, 1989).

Preliminarily, define
\[ \tilde{B}_X = B_u R_1^{-1} B_u^T, \]  
(35)
\[ \tilde{B}_Y = C_y R_1^{-1} C_y^T. \]  
(36)
From the above formulation it is easy to deduce the following Theorem.

**Theorem 2.** Consider the system $C$ with minimal state space representation
\[ C = \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}, \]  
(37)
where the matrices $A_c$, $B_c$ and $C_c$ are given in (24)–(26). If the diagonal matrices $X_2$ and $Y_2$ are chosen so as to satisfy the LMI
\[ X_2 \tilde{B}_X + \tilde{B}_X X_2 + Y_2 \tilde{B}_Y + \tilde{B}_Y Y_2 > 0 \]  
(38)
\[ X_2 > 0 \]  
(39)
\[ Y_2 > 0 \]  
(40)
Then the system $C$ strongly stabilizes the system (7)–(9) and is bandpass.

**Proof.** Can be easily proved by considering the Lyapunov function
\[ V(x) = x^T \begin{pmatrix} \Omega^{-1} & 0 \\ 0 & I \end{pmatrix} x. \]  
(41)

The rest of the theorem follows from Theorem 1 and Lemma 1.

The Theorem above shows that a parametrization of stable, stabilizing and bandpass controller is obtained, with $2n$ free parameters, namely the diagonal entries of the matrices $X_2$ and $Y_2$. Since these matrices define the controller gain, a further constraint in the form of LMI
\[ \text{tr}(X_2) + \text{tr}(Y_2) < \theta, \]  
(42)
where $\text{tr}(X)$ is the trace of the matrix $X$ and $\theta$ is an upper bound on the controller gain depending on the problem, can be imposed. Moreover, also the matrices $R_1$ and $R_2$ have still to be selected. Expressions (27) and (28) can suggest how to choose the degrees of freedom. According to them, $X_2$ and $Y_2$ can weights modes on which to operate, while the matrices $R_1$ and $R_2$ define the authority of the control.

The above considerations can be summarised in a procedure for controller design, namely:

\[
\hat{W} = \begin{pmatrix} 0 & 0 \\ 0 & W \end{pmatrix}.
\]  
(34)
(1) Preliminarily, collect the diagonal elements of $X_2$ and $Y_2$ in the vector $\xi = (x_{211}, \ldots, x_{2nn}, y_{211}, \ldots, y_{2nn})^T$, define a vector of performance weights $c \in \mathbb{R}^n$, whose $j$-th element is the weight of the $j$-th mode.

(2) Solve the optimisation problem

$$\max (c^T \, c^T) \, \xi, \quad (43)$$

subject to

$$X_2(\xi) \bar{B}_X + \bar{B}_X X_2(\xi) + Y_2(\xi) \bar{B}_Y + \bar{B}_Y Y_2(\xi) + 2A > 0, \quad (44)$$

$$X_2(\xi) > 0, \quad (45)$$

$$Y_2(\xi) > 0, \quad (46)$$

$$\sum_{h=1}^{M} |\Phi_k(\omega_h)|^2, \quad k = 1, \ldots, 5, \quad (47)$$

where $\delta > 0$ and $1_n$ is the $n$-dimensional column vector with all elements equal to 1.

(3) Compute the controller according to (23)–(26).

In the above procedure, the scalar $\delta > 0$ is chosen to limit the controller gain, since (47) can be written as

$$\text{tr}(X_2) + \text{tr}(Y_2) < \delta. \quad (48)$$

Obviously, in the above algorithm $c_j$, the $j$-th element of $c$, can assume any real value. For instance, choosing $c_1 = 10$, $c_2 = 1$ and $c_3 = 0.1$ would pose different penalties on different modes, thus focussing the control action more on mode one, than on mode two and on mode three. Cavallo et al. (2007b) showed that the optimal choice of the input scaling matrices is performed based on the controllability Gramian of the open loop and closed loop systems. However, it is clear that the efficiency of the control action depend on both the controllability and the observability Gramians. As an example, a well known strategy for model order reduction (Moore, 1981) consists of the computation of a balanced realisation and a reduction based on neglecting the modes associated to the Hankel singular values, i.e., square roots of the eigenvalues of the product of the controllability Gramian and the observability Gramian, with lowest magnitude. This kind of consideration leads to the idea of selecting the modal performance weights in (43) based on the Hankel singular values. Another aspect of the presented control strategy to highlight is the selection of the matrices $\bar{R}_1$ and, even more importantly, $\bar{R}_2$ in (17) and (18), respectively. The former is responsible for the authority of the control. The latter is responsible for the relative weights of colocated measurements with respect to non-colocated ones. By varying the magnitude of $\bar{R}_2$, relative to $\bar{R}_1$, the case of non-colocated feedback can be considered with stability guaranteed.

4. EXPERIMENTAL RESULTS

The control procedure described in previous section has been experimentally tested on a very complex flexible structure. In particular, the selected test article is a fuselage skin panel of a BOEING 717 (depicted in Figure 2). The panel is stiffened by two bulkheads and three orthogonal stiffeners rivetted to the rear of the panel itself. In order to reproduce the conditions usually adopted in the structural testing phase, the panel has been suspended by a couple of soft springs to simulate the free boundary conditions. In the experiments presented in this section piezoelectric ceramic actuators have been used both for producing the primary disturbance field and for generating the control forces on the structure aimed at counteracting the effects of the primary field.

In detail, the control inputs are produced using three of the four piezoelectric patches bonded on the panel surface; the output acceleration is measured by means of the three colocated accelerometers placed on the rear part of the panel behind the piezoelectric actuators. The structure has a high modal density in the considered frequency range $\Omega = [100, 600]$ Hz and a model with 148 modes has been identified as proposed by Cavallo et al. (2007a). The experiments were performed by using a dSPACE rapid prototyping real-time control system with 16-bit A/D channels and 14-bit D/A channels at a sampling frequency of 20 kHz.

The fourth piezo is used only as a disturbance input so as to introduce a disturbance contribution not belonging to the range of the input matrix $B_u$ in (7). A fourth accelerometer placed behind the fourth piezo has been used as additional sensor for the implementation of the proposed non square controller. One of the known drawbacks in the application of optimal feedback control to vibration reduction problems is that the control action may focus only on the performance objectives, reducing vibrations only at the controlled points, while increasing vibration levels at other points of the structure. In order to check that the controller actually has a beneficial effect on the whole structure, a fifth accelerometer is used to measure the acceleration in a performance point far from both control and disturbance inputs. The effectiveness of the control strategy has been evaluated comparing the Frequency Response Functions (FRFs) from disturbance input to acceleration outputs and in terms of an energy-based index defined as follows. First, the energy of the measured acceleration in the given frequency range $\Omega$ has been estimated for the $k$-th of the five used sensors as

$$J_k = \sqrt{\sum_{h=1}^{M} |\Phi_k(\omega_h)|^2}, \quad k = 1, \ldots, 5, \quad (49)$$
Table 1. Values of the reduction energy index $R_{k\%}$ in (50) obtained in the frequency range [100, 600] Hz for disturbance and performance points

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$R_{k%}$-controller A</td>
<td>30.74</td>
</tr>
<tr>
<td>$R_{k%}$-controller B</td>
<td>39.16</td>
</tr>
</tbody>
</table>

where $\Phi_k(\omega_k)$ is the acceleration spectrum measured at the $k$-th point at the frequency $\omega_k \in \{\omega_1, \ldots, \omega_M\}$, a set of $M = 2500$ linearly spaced frequencies within $\Omega$. Then, the relative energy reduction at the $k$-th point has been quantified as

$$R_{k\%} = \frac{J_{k,ol} - J_{k,cl}}{J_{k,ol}} \times 100, \quad (50)$$

where the subscripts “ol” and “cl” stand for “open loop” and “closed loop”, respectively.

A band-limited white noise in the range [100, 600] Hz has been chosen as primary field, in order to excite all the system natural modes in the frequency range $\Omega$. The noise has been applied to the structure only through the fourth piezo actuator not used as control input, thus the disturbance is totally out of the range of the input matrix $B_u$ in (7). This choice allows better highlighting the feature of the controller when used with additional sensors located near the entry point of the primary field. Since the controller computation depends on the choice of modal performance weights and on the number of feedback sensors, the guidelines described at the end of Section 3 have been followed. As a consequence, three controllers have been computed and compared:

A Colocated controller with modal performance weights based on the Henkel singular values.

B Rectangular controller (with an additional sensor) with modal performance weights based on the Henkel singular values.

C Non-colocated controller with modal performance weights based on the Henkel singular values.

All these controllers use the same three piezoelectric actuators as control inputs and the four accelerometers as measured outputs and are designed by selecting the modal performance weights in (43) according to the Henkel singular values. However, the controller A practically uses, as output accelerations, only the measurements provided by the three colocated sensors, since weighting matrices are selected as $R_1 = I$, $R_2 = 10^8 I$ and $R_2 = I$. The large value of $R_2$ makes the information of the fourth sensor meaningless for the control action. Controller B takes into account the problem of vibration reduction when information on the location of the application point of the disturbance force is available. Of course, in such a case, the optimal solution would be to locate an actuator/sensor pair in this entry point. However, assume that mounting of the actuator in such a location is not possible for some reason, while placement of the sensor is still allowed. Under this assumption, controller B uses the information provided by the fourth sensor by selecting the weighting matrices $R_1 = I$, $R_2 = 1$ and $R_2 = I$, where a lower value for $R_2$ has been set. The controller C has been considered to assess the behaviour of the active vibration control system when the application at hand does not allow to place actuators and sensors in colocated pairs. In particular, it practically uses only the signal of the fourth sensor placed in the disturbance entry point, by selecting the weighting matrices $R_1 = 10^{-4} I$, $R_2 = 10^{-4}$ and $R_2 = 10^8 I$, where a large value for $R_2$ has been set. The rectangular non-colocated controller C has no information on what happens to the structure in the control points.

The performance obtained with the controllers above are evaluated both in terms of vibration reduction quantified according to the index in (50) and by comparing the FRFs from disturbance input point to a control point, the disturbance point and the performance point. In a first experiment the controller A, used as reference, has been compared to the controller B. Table 1 shows that the reduction energy index for controller B is significantly higher in the disturbance and performance points. Moreover, the effects of controller B, compared with controller A and open loop, on the entire frequency range $\Omega$ is shown in Figure 3, where over 20 dB of vibration reduction peaks are visible and no spillover appears.

The controllers have been also compared in terms of required control energy according to the index defined below. First, the energy $E_s$ of a generic discrete-time signal $s(k)$ has been evaluated as

$$E_s = \sum_{k=1}^{N} s^2(k), \quad (51)$$

where $N$ is a fixed number of samples equal for all experiments. Taking into account that the primary field

![Fig. 3. FRFs from disturbance input to output acceleration in a control point (a), in the disturbance point (b), in the performance point (c): open loop (solid black lines), controller A (solid grey lines), controller B (dashed black lines)
The control energy required by controllers B and C has been compared to the one required by the controller A, by considering the control energy reduction index

$$R_{\%} = \frac{E_{u,Y}}{E_{u,X}} \times 100 \quad r = 1, 2, 3, \quad X = B, C. \quad (53)$$

The results are summarised in Table 2, which shows that controller B requires less control energy than controller A despite its higher performance. It is evident that the information from the additional sensor improves the control action.

In a second experiment the controller C has been tested. Although the design phase the weighting matrices have been selected positive definite according to (17)–(18), in the real experiment, only the fourth sensor placed in the disturbance point has been physically used, in order to demonstrate the possibility to use the proposed controller as an actual non-colocated one. The performance in terms of reduction energy index is reported in Table 3. The reduction energy index values are lower than in the previous cases, but the same applies to the required control energy indicated in Table 2. Interestingly enough, even in this actually non-colocated control no spillover phenomenon occurs, as evident from the FRFs reported in Figure 4, still obtaining over 10 dB of vibration reduction peaks.

### Table 2. Control reduction index $R_{\%}$ for controllers B and C

<table>
<thead>
<tr>
<th>Channel</th>
<th>Controller B</th>
<th>Controller C</th>
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<tbody>
<tr>
<td>1</td>
<td>11.18</td>
<td>56.35</td>
</tr>
<tr>
<td>2</td>
<td>12.83</td>
<td>64.59</td>
</tr>
<tr>
<td>3</td>
<td>10.14</td>
<td>75.57</td>
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### Table 3. Values of the reduction energy index $R_{\%}$ in (50) obtained for non-colocated controller in the frequency range [100, 600] Hz for disturbance and performance points

<table>
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<tbody>
<tr>
<td>Controller C</td>
<td>27.44</td>
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<td></td>
<td>18.25</td>
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5. CONCLUSIONS

The paper reported an extension of the control approach originally presented by the authors to tackle the vibration reduction problem of flexible structure with the strong requirement of stable and bandpass controller. The original control law is here extended to the case of more sensors than actuators. The more complex design is tackled by solving an LMI problem, which allows also to select the performance weights based on controllability/observability indices. The experimental results obtained for a vibration reduction problem of a stiffened aeronautical panel controlled by piezoelectric actuators confirm the applicability of the strategy in actual systems and its advantages with respect to a controller assuming only colocated actuator/sensor pairs.

### REFERENCES


