Haptic-based bilateral teleoperation of underactuated Unmanned Aerial Vehicles

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Abstract: The bilateral teleoperation of underactuated aerial vehicles is addressed. A force feedback joystick is used to remotely pilot the aircraft and reflect an image of the environment to the operator. This feedback is a virtual force, function of the distance to an obstacle or the rate of approaching it. It can be computed using the measurements of on-board sensors (i.e. telemetric or optic flow). Input to State Stability of the teleoperation loop with respect to bounded or dissipative virtual environment forces is established based on Lyapunov analysis.

1. INTRODUCTION

Bilateral teleoperation can be used to ensure a precise and safe remote piloting of Unmanned Aerial Vehicles (UAVs), by a human operator, to approach and inspect a target infrastructure such as hydraulic dam walls, bridges, etc. Quite often, visual evaluation of the distance between the vehicle and the target/obstacle is not very accurate. Thus, even experienced pilots may fail to avoid collisions. Moreover, complex aerodynamic effects induced by strong and unpredictable wind gusts and/or interactions between the vehicle and the target complicate the matter even more. Therefore, a haptic force feedback joystick presents an appealing solution to bring to the operator sensation of nearby obstacles. The authors believe that the key to develop effective remote operations of UAVs is to treat the problem as the one of robotic teleoperation rather than more traditional approaches of augmented manual control or Fly-By-Wire systems.

A bilateral teleoperation loop includes an operator driving a slave robot by means of a master system (i.e. a manipulator in general or a joystick in the context of the present paper). Reference trajectories specified by the master are sent to the slave, operating in a remote environment, via a communication channel. In turn, an image of the the slave robot’s interaction with the environment is sent back, via the communication channel, to the master and consequently to the operator (Fig. 1). Different schemes can define the bilateral teleoperation, depending on the physical quantities (force, velocity, position, etc.) transmitted forth and back between the master and slave systems (see, e.g., Hokayem and Spong (2006) for a state-of-the-art of existing teleoperation schemes). Teleoperation has been widely studied for applications where the master and slave systems have the same dynamics. The stability of the closed-loop system is often proved by exploiting passivity properties, using a particular representation of the communication channel called “wave variables” (Hokayem and Spong (2006), Niemeyer and Slotine (1998)). However, only few works have dealt with underactuated systems (e.g. Vertical Taking-Off and Landing (VTOL) vehicles). In Lam et al. (2007), the environment feedback is taken into account through a modification of the joystick (master) stiffness. In Lam et al. (2009), an artificial force field for haptic feedback, that maps the environment constraints to repulsive force, is developed for UAV teleoperation. In Troy et al. (2009), a force feedback based on a motion capture tracking of the environment is presented. In Mahony et al. (2009), optical flow provided by onboard camera(s) is exploited to provide force feedback to an operator’s joystick to facilitate collision avoidance of a teleoperated UAV. In this reference, the coupling between optical flow and force is modeled as an optical impedance and the dual force/velocity is shown to be dissipative. The teleoperation of UAVs is investigated in Mettler et al. (2008), Andersh et al. (2009), Hurzeler et al. (2008), Pena et al. (2007) based on visual feedback, but haptic force feedback is not considered. In Stramigioli et al. (2010), the concept of network theory and port-Hamiltonian systems is applied to UAV teleoperation. Note, however, that only few of the aforementioned works have proved the stability of the teleoperation loop (including the operator, the master joystick, the slave UAV, and the environment) (Stramigioli et al. (2010)). Previous works have considered either simplistic model of an aerial vehicle as a point-mass servo-controlled by attitude reference and thrust input or the classical human-in-the-loop control scheme (Hurzeler et al. (2008)). Moreover, there are technical difficulties related to the difference of dynamic models, that is the slave UAV is underactuated whereas the master joystick is fully-actuated. In summary, even though bilateral teleoperation for VTOL UAVs has been addressed in the past four years, the specific case of teleoperation for underactuated UAVs...
with assistance of haptic force feedback received little
attention.

The present paper deals with the bilateral teleoperation
of an underactuated VTOL UAV using a force feedback
 joystick. The main contribution of the paper consists in
proving the stability of the teleoperation loop with respect
to bounded external forces (i.e. operator and environment
forces). Two expressions of environment forces that reflect
the UAV’s approach to an obstacle are proposed using
telemetric and optic flow sensors. The paper is structured
as follows. The teleoperation loop is proposed in Section
2. Section 3 presents the control laws of the joystick and
the UAV. The stability of the teleoperation loop is proved
in Section 4. Applications are addressed in Section 5 along
with simulation results in Section 6. Finally, conclusions
and perspectives are given in Section 7.

2. MODELING OF THE TELEOPERATION LOOP

The considered haptic teleoperation scheme (illustrated in
Fig. 1 and 2) consists of a fully-actuated haptic joystick
(master), an underactuated VTOL UAV (slave), and a
communication link between them. Note that the com-
communication time-delay is not considered in the present
study. The position of the joystick is sent to the VTOL
UAV via the communication channel. This position value
defines the reference velocity \( \dot{x}_s \in \mathbb{R}^3 \) for the UAV. The
joystick device is manipulated by the human operator and
is capable of force feedback. This fully-actuated joystick is
subject to the operator force \( F_h \in \mathbb{R}^3 \), the control force
\( F_m \in \mathbb{R}^3 \), and the environment feedback force \( F_e \in \mathbb{R}^3 \).
The force \( F_e \) can be defined as a function of the UAV’s
translational velocities and/or the distance between the
UAV and the obstacle detected in its direction of flight.
Details on the definition of \( F_e \) are provided in Section 5.
This force is applied to the UAV as a repelling force, by
means of its control, allowing the UAV to avoid collisions
with the detected obstacle. It is also sent back via the
communication channel to the haptic joystick to be felt by
the operator (see Fig. 2).

Denote \( \mathcal{F}^l \) the inertial frame chosen as the NED frame
(North-East-Down), \( \mathcal{F}^m \) the frame attached to the master
joystick, \( \mathcal{F}^s \) the frame fixed to the slave UAV (see Fig.
3). In the following, the model of each component of the
teleoperation loop will be described.

2.1 Master joystick model

Let \( q_m \in \mathbb{R}^3 \), \( \dot{q}_m \in \mathbb{R}^3 \) be respectively the vectors of
coordinates of the position and velocity of the joystick’s
extremity, expressed in \( \mathcal{F}^l \). Using the Euler-Lagrange
formalism (Spong et al. (2006)), the dynamics of the
joystick are given by

\[
M(q_m)\ddot{q}_m + C(q_m, \dot{q}_m)\dot{q}_m + g(q_m) = F_m + F_h - K_c F_e
\]

with \( M(q_m) \) the joystick inertia matrix, \( C(q_m, \dot{q}_m) \) the
Coriolis and centrifugal effects and \( g(q_m) \) the vector
of gravitational forces, \( F_h \) the force generated by the human
operator, \( F_m \) the control force and \( F_e \) the environment
feedback force, all expressed in the inertial frame \( \mathcal{F}^l \). \( K_c \)
is a scaling positive diagonal matrix. Note that the damping
and spring parameters of most commercial joysticks can
be regulated by the user. For the sake of simplicity, the
enclosed parameter(s) of \( M(q_m) \), \( C(q_m, \dot{q}_m) \), and \( g(q_m) \) is
(are) omitted in the sequel.

Remark 1. (Spong et al. (2006)) The matrix defined by
\( M - 2C \) is skew symmetric, i.e. \( \forall x \in \mathbb{R}^3, x^\top(M - 2C)x = 0 \).

2.2 Slave VTOL UAV model

The slave system belongs to the family of VTOL vehicles.
The motors generate a vertical “thrust” force allowing the
vertical movement. A special mechanism is responsible of
the torques generation and consequently the rotational
movement of the UAV. The longitudinal and lateral move-
ments are ensured by a coupling between the thrust and
the UAV’s orientation through the control torque mech-
anism. The UAV is considered as a rigid body evolving
in 3D-space subject to three control torques and a thrust
force along a body-fixed direction.

By summing up all external forces acting on the vehicle
(gravity, dissipative aerodynamic forces, etc.) in a vector
\( F_s \), expressed in the inertial frame \( \mathcal{F}^l \), and by denoting
the resulting torque induced by these external forces
(expressed in the UAV’s frame \( \mathcal{F}^s \)) as \( \Gamma_s \), the equations
of motion can be derived from the fundamental theorems
of Mechanics (i.e. Euler-Newton formalism) (see e.g. Hua
et al. (2009))

\[
\dot{x}_s = R_s v_s \tag{2}
\]

\[
m_s \dot{v}_s = -m_s \omega_s \times v_s - T_c e_3 + R_s^\top F_s \tag{3}
\]

\[
\dot{R}_s = R_s S(\omega_s) \tag{4}
\]

\[
J_s \omega_s = -\omega_s \times J_s \omega_s + \Gamma_c + \Gamma_s \tag{5}
\]

with \( m_s \in \mathbb{R} \) and \( J_s \in \mathbb{R}^{3 \times 3} \) the UAV’s mass and inertia
matrix in \( \mathcal{F}^s \), \( x_s \in \mathbb{R}^3 \) the UAV’s position vector expressed
in \( \mathcal{F}^l \), \( \dot{x}_s \in \mathbb{R}^3 \) and \( v_s \in \mathbb{R}^3 \) the UAV’s translational
velocity vectors expressed in \( \mathcal{F}^l \) and \( \mathcal{F}^s \) respectively, \( R_s \)
the rotation matrix from the frame \( \mathcal{F}^s \) to \( \mathcal{F}^l \), \( \omega_s \in \mathbb{R}^3 \)
the UAV’s angular velocity in \( \mathcal{F}^s \), \( T_c \in \mathbb{R} \) and \( \Gamma_c \in \mathbb{R}^3 \)
the thrust control force and the vector of control torques
of the UAV expressed in \( \mathcal{F}^s \), \( S(\cdot) \) the skew-symmetric
matrix associated with the cross product \( \times \), i.e. \( S(u)v =
\end{flushleft}

Fig. 3. Inertial frame \( \mathcal{F}^l \), joystick-fixed frame \( \mathcal{F}^m \), UAV-
fixed frame \( \mathcal{F}^s \).
The knowledge of the external force \( F_e \) is very important for the control design. Several approaches can be used, depending on the available embedded sensors: direct measurement, estimation, or combination of both. Some solutions to this problem can be found in (Hua et al. (2007); Hua (2009)). In the present paper, \( F_e \) is assumed to be known with good accuracy.

From (5), the vehicle’s orientation is fully actuated, and the exponential convergence of \( \omega \) to any desired reference value can easily be achieved. As a consequence, \( \omega_\alpha \) can be seen as an intermediary vector control input of subsystem (2)–(4). This control strategy corresponds to the classical decoupled control architecture for inner and outer loops. In the sequel, the control of the subsystem (2)–(4) is detailed, using \( T_c \) and \( \omega_\alpha \) as the control inputs.

### 3. CONTROL DESIGN

In the sequel, the control of the master and slave systems is addressed in order to achieve the stability of the teleoperation loop in presence of human and environment forces.

#### 3.1 Control design for the master joystick

A control force is applied to the joystick in order to ensure its stabilization at \( (q_m = 0, \dot{q}_m = 0) \) when the sum of the external forces is null. The synchronization of the master relative to the slave position is not considered in order to guarantee a full operator control depending on the environment feedback. The control force is based on a proportional derivative controller and has the following expression:

\[
F_m = -(C + K)q_m - (MA + K)\dot{q}_m + g
\]

\( \Lambda \in \mathbb{R}^{3 \times 3} \) and \( K \in \mathbb{R}^{3 \times 3} \) are diagonal matrices of positive control gains.

#### 3.2 Control design for the slave VTOL UAV

The control law applied to the UAV is designed such that the velocity \( \dot{x} \) is stabilized to the desired value defined by the joystick deflection \( \ddot{x}_{sd} = K_m q_m \), with \( K_m \) a scaling diagonal matrix of positive parameters. The control design is inspired by (Hua et al. (2009)). Let \( \ddot{x} := \dot{x}_s - \dot{x}_{sd} \) and \( \ddot{v} := R_\gamma ^T \ddot{x} \) denote the velocity error variables, expressed in the frames \( \mathcal{F}^I \) and \( \mathcal{F}^s \) respectively. Define

\[
\gamma := \frac{F_s}{m_s} - \frac{F_e}{m_s} + h(\dot{|\ddot{x}|}) \ddot{x}_s
\]

\[
\ddot{\gamma} := R_\gamma ^T \gamma
\]

with \( \ddot{x}_{sd} = K_m q_m \) the desired acceleration, \( F_e \) the external environment virtual force and \( h(\cdot) \) some smooth bounded positive function defined on \( [0, +\infty) \) such that for some positive functions \( \alpha, \beta \)

\[
|h(s^2)s| < \alpha, \quad 0 < \frac{\partial}{\partial s} \left( h(s^2)s \right) < \beta, \quad \forall s \in \mathbb{R}
\]

Let \( \hat{\theta} \in [-\pi, \pi] \) denote the angle between the two vectors \( e_3 \) and \( \ddot{\gamma} \), so that \( \cos \hat{\theta} = \ddot{\gamma}_3 \) with \( \ddot{\gamma}_3 \) the third component of \( \ddot{\gamma} \).

The control thrust and angular velocities are given by

\[
\begin{align*}
\omega_{s,1} &= -|\gamma| k_2 \ddot{v}_2 - \frac{k_3 |\gamma| \ddot{\gamma}_2}{(|\gamma| + \gamma_0)^2} - \frac{1}{|\gamma|^2} \gamma^T S(R_s e_1) \ddot{\gamma} \\
\omega_{s,2} &= |\gamma| k_2 \ddot{v}_1 + \frac{k_3 |\gamma| \ddot{\gamma}_1}{(|\gamma| + \gamma_0)^2} - \frac{1}{|\gamma|^2} \gamma^T S(R_s e_2) \ddot{\gamma}
\end{align*}
\]

with \( k_1, k_2, k_3 \) positive gains, \( \ddot{v} = [\ddot{v}_1, \ddot{v}_2, \ddot{v}_3]^T \), \( \ddot{\gamma} = [\ddot{\gamma}_1, \ddot{\gamma}_2, \ddot{\gamma}_3]^T \) defined by (8), \( e_1 = [1,0,0]^T \) and \( e_2 = [0,1,0]^T \).

**Remark 2.** \( \gamma \) as defined in (7) regroups all external forces acting on the VTOL UAV including the constant gravity force. A good choice of \( \alpha \), defined as the bound of \( h(\dot{|\ddot{x}|}) \ddot{x} \), can limit the possibilities for \( \gamma \) to cross zero. Therefore, the hypothesis \( \gamma \neq 0 \) is considered in the following analyses.

### 4. STABILITY OF THE TELEOPERATION LOOP

The stability of the teleoperated system is studied for two situations. First, an equilibrium point is shown to be asymptotically stable for the unforced and free moving system. Then, the teleoperation loop is proven to be input-to-state stable (Khalil (2002)) in presence of \( i \) bounded operator and environment forces (i.e. \( F_h \) and \( F_e \)), and \( s \) bounded operator force \( F_h \) and dissipative environment force \( F_e \).

#### 4.1 Free moving system

Let us study the teleoperation loop represented in Fig. 2 where the master and slave systems operate in free space: i.e. the operator and environment do not exert any forces (i.e. \( F_h = F_e = 0 \)).

**Proposition 4.1.** Consider the teleoperation system where the master and slave subsystems defined respectively by (1) and (2)–(5) operate in free space, i.e. \( F_h = F_e = 0 \). Assume that \( \ddot{x}_{sd}, \ddot{x}_{sd}^{(3)}, \ddot{x}_s^{(3)} \) are bounded and \( \gamma \) is always different from zero. Applying the control force (6) to the joystick, the control thrust and angular velocities (9) to the UAV ensures the asymptotic stability of the teleoperation loop at \( X = (q_m, \dot{q}_m, \ddot{x}, \ddot{\theta}) = (0,0,0,0) \) with domain of attraction equal to \( \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^3 \times (-\pi, \pi) \).

**Proof** Consider the following candidate Lyapunov function

\[
V := \frac{1}{2} (\dot{q}_m + \Lambda q_m)^T M (\dot{q}_m + \Lambda q_m) + \dot{q}_m^T \Lambda K q_m
\]

\[
+ \frac{1}{2} \ddot{v}^T \ddot{v} + \frac{1}{k_2} \left( 1 - \frac{\gamma_3}{|\gamma|} \right) v_1
\]

\[
- \frac{v_2}{k_3} \left( 1 - \frac{\gamma_3}{|\gamma|} \right)
\]

The derivative of the Lyapunov function satisfies

\[
\dot{V} = (\dot{\dot{q}}_m + \Lambda \dot{q}_m)^T M (\dot{\dot{q}}_m + \Lambda \dot{q}_m) + \frac{1}{2} (q_m + \Lambda q_m)^T \dot{M} (q_m + \Lambda q_m)
\]

\[
+ 2 \dot{q}_m^T \Lambda K q_m + \ddot{v}^T \ddot{v} + \frac{1}{k_2} \frac{d}{dt} \left( 1 - \frac{\gamma_3}{|\gamma|} \right)
\]

From (3) and (7), one can verify that

\[
\ddot{v} = -\omega_s \times \ddot{v} - \frac{T_c}{m_s} e_3 + \frac{1}{m_s} R_\gamma ^T F_c - h(\dot{|\ddot{x}|}) \ddot{v}
\]
Note that (see Hua et al. (2009) for details of the proof)
\[
\frac{d}{dt} \left(1 - \frac{\gamma_1}{|\gamma|}\right) = \frac{1}{|\gamma|} \gamma_1 \left[ (-\omega_2,1) + \frac{1}{|\gamma|} \left( -\gamma^\top S(R_{\kappa e}) \gamma \right) \right]
\]
with $\gamma_{1,2} = (\gamma_1, \gamma_2)\top$. When no external forces are considered, (1) and (6) lead to
\[
M(\ddot{q}_m + \Lambda q_m) = -(C + K)q_m + \Lambda q_m \tag{14}
\]
Substituting (9), (12), (13), (14) in (11) and recalling Remark 1 under the assumption $(F_h = F_e = 0)$, the derivative of the function $V$ becomes
\[
\dot{V} = -\dot{q}_m^\top K \dot{q}_m - \lambda_{\min}(K)q_m^2 - \lambda_{\min}(\Lambda)q_m^2
\]
```
| + Λv(0,0,0) is an equilibrium point of the teleoperation loop in free moving system. Furthermore (q_m,q_m) = (0,0) is an exponentially stable equilibrium of (14). Thus, one deduces that \( x \) converges to zero. Consequently, \( x \) converges to a constant. Hence, in absence of external forces the UAV is stabilized in hovering mode. ■
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4.2 Forced system

Proposition 4.2. Consider the teleoperation system with the master and slave subsystems defined respectively by (1) and (2)–(5). Apply the control laws (6) and (9) to this system. Assume that \( \dot{x}_m, \dot{x}_s, \dot{x}_h \) are bounded and \( q_m \) is always different from zero. Assume also that \( \exists \alpha_h, \alpha_e \in \mathbb{R}^+ \) such that \( |F_h| \leq \alpha_h, |F_e| \leq \alpha_e \) (i.e. the operator and the virtual environment forces are bounded). Then, for the following cases:

1. the virtual environment force \( F_e \) is passive, i.e. \( \exists \kappa_e \in \mathbb{R}^+ \) such that \( V(t) \geq 0 \),
\[
\int_0^t \dot{x}(s)\top F_e(s)ds \geq -\kappa_e \tag{15}
\]
2. the bound value \( \alpha_e \) of \( F_e \) is sufficiently small compared to \( m_s \lim_{s \to \infty} h(s^2) \); the teleoperation loop is input-to-state stable (I.S.S.) w.r.t. \( F_h \) and \( F_e \), and the system solutions are ultimately bounded.

Proof First, let’s study the case where the virtual environment force \( F_e \) is passive. Consider the following candidate Lyapunov function
\[
L := V + \frac{1}{m_s} (\kappa_e + \int_0^t \dot{x}_s(\sigma)\top F_e(\sigma)d\sigma) \tag{16}
\]
with $V$ defined by (10). One can verify that the time-derivative of the function $L$ satisfies
\[
\dot{L} = -\dot{q}_m^\top K \dot{q}_m - q_m^\top \Lambda K q_m
\]
```
Applying Barbalat Lemma (Khalil (2002)), one can prove that

As a consequence, one obtains
\[
\dot{L} \leq -\frac{1}{2} \lambda_{\min}(K)|\dot{q}_m|^2 - \frac{1}{2} \lambda_{\min}(\Lambda)|q_m|^2
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\[
\dot{V} \leq -\frac{1}{2}\lambda_{\min}(\Lambda K)\|\dot{q}_m\|^2 - \frac{1}{2}\lambda_{\min}(K)\|\dot{q}_m\|^2
- |\gamma|k_1\bar{v}^2 - \frac{k_1^2}{k_2}\pi^2(\bar{\theta}/2) \\
- \frac{h(\|\dot{\vec{x}}\|^2)|\dot{\vec{x}}|^2 + |\ddot{\vec{x}}|^2|F_c|}{m_s} + \frac{F_{h,c}^2\lambda^2_{\max}(\Lambda)}{2\lambda_{\min}(\Lambda)} + \frac{F_{h,c}^2}{2\lambda_{\min}(K)}
\]

with \( \Delta_e = \lambda_{\min}(K)e\alpha_e \). If the bound \( \alpha_e \) of \( F_c \) is sufficiently small compared to \( m_s \), then there exists a positive constant \( \Delta_{\ddot{z}} \) such that: if \( |\ddot{z}| > \Delta_{\ddot{z}} \), then the term \( h(\|\ddot{\vec{x}}\|^2)|\ddot{\vec{x}}|^2 \) becomes dominant w.r.t. \( |\ddot{\vec{x}}|^2|F_c| \), making the term \( W \) negative. Note that the term \( W \) tends to \(-\infty\) when \( |\ddot{\vec{x}}| \) tends to \(+\infty\), and that the term \( \delta_{h,c} \) remains bounded. In view of these facts and relations (10), (18), the I.S.S. property can be directly deduced. \( \square \)

5. APPLICATION

The UAV’s environment contains some fixed and well-defined obstacles that can be detected by onboard sensors. An obstacle surface is supposed locally planar. Let \( \eta_i \in F^7 \) be the unit vector normal to the plane surface and pointing to the obstacle. Two kinds of sensors are considered in the present work: telemetric and optic flow sensors. Using the measurements of the onboard sensors, an environment force \( F_c \) can be computed. On one hand, \( F_c \) can be seen as a virtual force acting on the UAV (slave), by means of the control, in order to push it away and avoid collisions consequently. On the other hand, \( F_c \) is a haptic force acting on the joystick (master) in order to inform the operator of obstacle presence. To prevent the saturation of the joystick and UAV’s motors, the environment force \( F_c \) should be bounded.

5.1 Telemetric sensors

A telemetric sensor (e.g. radar, sonar, laser range finder, etc.) delivers the distance \( d \) to the obstacle in the direction of measurement. The telemetric sensor (e.g. laser range finder) executes \( n \) shots (i.e. \( n \) measurements) during a turn, with fixed step in the UAV frame \( F^7 \). The orientation of each shot is given by the unit vector \( \eta_i \in F^7 \) and the delivered distance is noted \( d_i \), \( i \in \{1,\ldots,n\} \). The virtual force \( F_{c,i} \) in the \( i^{th} \) direction, is modeled as a repelling force derived from an artificial potential field. It can be defined as

\[
F_{c,i} = \chi_{d_0}(d_i)\eta_i
\]

do a threshold distance between the UAV and an obstacle; \( \chi_{d_0}(\cdot) \) a non-increasing smooth function defined on \((0, +\infty)\) and satisfying the following three properties (see e.g. Hua and Rifai (2010)):

\[
\frac{\partial \chi_{d_0}(s)}{\partial d} < 0, \forall s \in (0, d_0), \quad \chi_{d_0}(s) = 0, \forall s \geq d_0,
\]

\[
\int_0^{d_0} \chi_{d_0}(\sigma) d\sigma = +\infty,
\]

For example, one can take

\[
\chi_{d_0}(d_i) = \left\{ \begin{array}{ll}
\kappa_0 d_0^2/(4d_i), & \text{if } 0 < d_i \leq d_0/2 \\
\kappa_0 (d_0 - d_i), & \text{if } d_0/2 < d_i \leq d_0 \\
0, & \text{if } d_i > d_0
\end{array} \right.
\]

\[ (19) \]

The environment force \( F_c \) is then defined as

\[
F_c := \text{sat}_{\alpha_e} \left( \frac{1}{n} \sum_{i=1}^{n} F_{c,i} \right)
\]

with \( \text{sat}_{\alpha_e}(\cdot) \) a classical saturation function between \( \pm \alpha_e \). The derivative of \( d_i \) necessary for the computation of \( \dot{\gamma} \) in (9) can be obtained using a special filter as explained in (Hua and Rifai (2010)).

5.2 Optic flow sensors

An optic flow sensor detects the relative change between speed and depth. It measures the linear optic flow rate which is equivalent to the ratio of the UAV’s linear velocity and the distance to an approaching obstacle. In this work, we assume that \( n \) optic flow sensors are embarked on the UAV. Consider the unit vector \( \eta_i \in F^7 \) in the direction of observation \( i \), \( i \in \{1,\ldots,n\} \). The translational optic flow measured by the sensor \( i \) is given by

\[
w_i = M(\eta_0, \eta_i) \frac{\dot{x}_s}{d_i}
\]

with \( \dot{x}_s \) the UAV velocity, \( d_i \) the distance to the obstacle in the direction of observation \( \eta_i \). One has \( M(\eta_0, \eta_i) = I_3 \) if \( \eta_i = \eta_0 \) with \( I_3 \) the identity matrix. The direction of the translational optic flow, resultant of the \( n \) optic flow measurements delivered by the on-board sensors, is defined as (Herisse et al. (2010b), Herisse et al. (2010a))

\[
\beta = \frac{1}{n} \sum_{i=1}^{n} w_i \eta_i = \frac{\|\dot{x}_s\|}{n} \sum_{i=1}^{n} \frac{\eta_i}{d_i}
\]

Here, \( \beta \) corresponds to the sum of the observation directions weighted by the corresponding optic flow measurements. A measurement direction has a higher weight as the encountered obstacle is closer to the UAV. The optical flow has two components: the normal one, in the direction of \( \beta \) and the tangential one, in the plane orthogonal to \( \beta \). The normal component is of interest in this work since it defines the virtual interaction force with the obstacle. It is given by

\[
w_\perp = \frac{1}{n} \sum_{i=1}^{n} w_i \eta_i > \beta = -\frac{1}{n} \sum_{i=1}^{n} \frac{d_i}{d_i} \frac{\beta}{|\beta|}, \quad \text{if } |\beta| \neq 0
\]

The virtual environment force \( F_c \) is modeled as the sum of two components: a damping force and an elastic repulsive force. The first component is a function of the rate of the UAV’s velocity to the obstacle distance, it helps dissipate the energy in order to stabilize the virtual interaction. The second one is a function of the distance to the obstacles, ensuring the UAV’s stop before collision. The elastic repulsive component, function of the UAV’s distance to the obstacle, can be defined as a repulsive potential field. The distance can be measured by some on-board telemetric sensors. This solution has the disadvantage of increasing the slave’s load. Another solution consists in integrating the optic flow measurements giving then information about \( (\ln d_i) \):

\[
\int w_\perp = -\frac{1}{n} \sum_{i=1}^{n} \int \frac{d_i}{d_i} \frac{\beta}{|\beta|}, \quad \text{if } |\beta| \neq 0
\]

The virtual interaction force between the environment and the UAV is the sum of the damping and elastic repulsive components.
This section illustrates the proposed bilateral teleoperation approach for a model of a VTOL ducted-fan tail-sitter (see e.g. Hua (2009), Pflimlin (2006)). Simulations are performed using Matlab Simulink. The master system is the joystick Logitech Force 3D Pro, capable of force feedback in two axes of the joystick. The damping and spring parameters are regulated so that the joystick’s position automatically returns to the origin when no external force is applied. Two joystick outputs, corresponding to the two variables which are capable of force feedback and whose range value is \([-1; 1]\), are multiplied by the scaling matrix $K_m = \text{diag}(3, 3, 0)$ to obtain the first two components of the reference velocity vector $\dot{x}_{sd}$ of the UAV, i.e. $\dot{x}_{sd,1}, \dot{x}_{sd,2}$, and set the third component $\dot{x}_{sd,3}$ to zero.

The controller (9) is applied to the UAV with the following gains and functions: $k_1 = 0.306$, $k_2 = 0.0798$, $k_3 = 20.8$, $h(s) = \beta/\sqrt{1 + \beta^2 s/\alpha^2}$, with $\alpha = 6$, $\beta = 0.4431$. The desired yaw angular velocity is set to zero (i.e. $\omega_{sd,1} = 0$). Then, a high gain controller is applied to subsystem (5) to stabilize the angular velocity at the desired value $\omega_{sd}$ whose first two components are generated by the first controller (i.e. controller (9)). The applied control torque is computed according to $\Gamma_c = [\omega_s]_s J_d \omega_{sd} - J_c K_c (\omega_s - \omega_{sd})$, with $K_c = \text{diag}(20, 20, 20)$. The external force $F_e$ and its time-derivative are not known exactly. They are estimated using a high gain observer, based on the measurement of the vehicle’s velocity $\dot{x}_s$ and attitude $R_s$, as proposed in (Hua (2009)).

The following scenario is considered. There are two vertical walls (crossing the origin of the inertial frame $\mathcal{F}^I$) whose normal vectors are respectively $e_1$ and $e_2$. The initial position of the UAV is $x_s(0) = (15, 14, 0)^T$. The UAV is equipped with a high frequency laser range finder so that it can approximately provide instantaneous measurement of $j$) the distances between the UAV and nearby obstacles, and $ii)$ the normal vectors of the observed (planar) obstacles. $d_1$ and $d_2$ denote the distances between the UAV and the two walls. The virtual environment force $F_e$ is defined as

$$F_e = \text{sat}_{\alpha_e} \left( w_\perp + \int w_\perp \right)$$

with $\text{sat}_{\alpha_e}(\cdot)$ a classical saturation function between $\pm \alpha_e$.

### 6. SIMULATION RESULTS

This experiment aims to avoid remote obstacles via a force feedback joystick. The obstacles are detected by UAV’s on-board sensors. In the first simulation, the joystick’s extremity is moved from the origin to the (2D) position $(-1, -1)^T$ which corresponds to $(\dot{x}_{sd,1}, \dot{x}_{sd,2})^T = (-3, -3)^T$ (m/s). Then, the joystick’s extremity is maintained at this position by the operator so that the reference velocity $\dot{x}_{sd}$ remains constant despite the environment feedback force acting on the joystick. Fig. 4 and 5 show that before entering the area where the environment force $F_e$ is activated (i.e. $d_1$ or $d_2$ is smaller than $d_0 = 8$) the UAV’s velocities follow very closely the corresponding reference values, specified by the joystick. In turn, once the UAV enters this area, the repelling virtual environment force $F_e$ makes the UAV decelerate until it stops in front of the obstacles and thus collision is avoided.

Fig. 4. Simulation 1 – UAV’s reference and actual horizontal velocities.

In the second simulation, the operator pilots the UAV to approach the two walls, like the first simulation, by means of the joystick. However, the operator is now more passive. When a strong feedback force is felt via the joystick, the operator reacts by reducing its force $F_h$. As a consequence, the joystick motion changes. Then, when the feedback force becomes small enough, the operator will increase its force $F_h$ to command the UAV to approach the obstacles again. Fig. 6 and 7 show such a situation with several changes of reference velocities. The effectiveness of haptic force feedback for bilateral teleoperation is justified, allowing the operator to safely pilot the UAV without collisions with obstacles.

### 7. CONCLUSION

The paper presented a rigorous stability analysis of a haptic-based teleoperation of underactuated aerial vehicles with respect to bounded external forces (i.e. operator and environment forces). The teleportation loop aims to avoid remote obstacles via a force feedback joystick. The obstacles are detected by UAV’s on-board sensors.
Two cases were proposed: telemetric and optic flow. The stability of the teleoperation loop in presence of obstacles was validated through some simulations in the presence of active and passive human operators. Among possible future works, some are proposed: i) the extension of the stability analysis to prove the impossibility of collision between the UAV and obstacles based on a previous work (Hua and Rifai (2010)), ii) the study of the stability in presence of delay in transmission along the communication channel, iii) the experimental validation is also envisaged.

REFERENCES


