Robust Adaptive Regulation by Output Error Feedback for Uncertain Nonlinear Systems

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Abstract: Minimum phase uncertain nonlinear systems in output feedback form are considered: they are subject to disturbances and/or uncertainties and are required to track reference signals. Both disturbances and references are generated by an exosystem whose parameters are uncertain. Under the assumption that the regulator problem has a solution, but with no a priori assumption on the required control input (e.g. no ‘immersion’ assumption), a global robust error feedback regulator is designed. If its adaptive internal model can generate the required control input global asymptotic regulation is achieved, otherwise the regulation error tends to a residual set which decreases as the reference input modeling error decreases.

1. INTRODUCTION

Given a nonlinear system which is affected by disturbances and is required to track output reference signals, if both disturbances and reference signals are generated by an exosystem, the global regulator problem is to design a dynamic tracking error feedback control such that: for any initial condition of the system and of the exosystem and for any parameter (of both the system and the exosystem) in a given set, the closed loop system has bounded solutions and the output tracking error tends asymptotically to zero.

The existence of a regulator has been investigated for linear systems in Francis and Wonham (1976) where the internal model principle was formulated and in Isidori and Byrnes (1990) for nonlinear systems. The robustness of plant parameter variations has been addressed in Huang and Lin (1994) while the presence of uncertain parameters in the exosystem was addressed in Serrani et al. (2001). A global solution was first obtained in Serrani and Isidori (2000) for nonlinear systems in output feedback form and then extended by Xu and Huang (2009) for more general systems. All these results rely on the key ‘immersion’ assumption which restricts the required control input to be generated by finite dimensional systems of known order. The removal of this restrictive assumption has been investigated in Marconi et al. (2007) and in Marconi and Praly (2008) leading to uniform practical nonlinear output regulation.

In this paper we consider minimum phase nonlinear systems in the so called output feedback form introduced in Marino and Tomei (1993) for which many results are now available. They can be globally stabilized by output feedback [Marino and Tomei (1995)]. Global adaptive output feedback controls can be designed when uncertain parameters appear linearly [Marino and Tomei (1993)]. Global robust output regulation can be achieved for systems with uncertain parameters by error feedback, when the reference and/or the disturbances are generated by a linear exosystem with known parameters, under the so called ‘immersion’ assumption, that is the desired reference control is generated by a finite dimensional linear internal model [Serrani and Isidori (2000)]. When the exosystem parameters are unknown, global output feedback regulators were obtained by Ding (2001, 2003) while semiglobal results were presented in Serrani et al. (2001) for a more general class of systems, introducing an adaptive internal model. The basic assumptions are: the knowledge of an upper bound on the linear exosystem order (and, consequently, on the number of its unknown frequencies); the knowledge of the order of the internal model which generates the reference control.

However, in the presence of unknown disturbances the maximum number of sinusoidal components may be very large and possibly infinite in the case of periodic disturbances. Moreover, even a single sinusoidal disturbance in the presence of uncertain nonlinear terms may generate many or even infinite higher order harmonics. Hence, the two key assumptions of known orders for the exosystem and the internal model may be unrealistic. This paper addresses the basic question: how to design an adaptive internal model when the ‘true’ internal model dynamics are unknown? This question was addressed in Marino and Tomei (2008a) for uncertain linear systems. In Marino and Tomei (2008a) an adaptive regulator is designed which forces exponentially the tracking error to a residual bound which decreases as the order of the unmodeled exosystem dynamics decreases; the tracking error tends to zero asymptotically when the exosystem is overmodeled and exponentially when the exosystem is exactly modeled. The aim of this paper is to extend such a result to the class of uncertain nonlinear systems in output feedback form, on the basis of preliminary results obtained in Marino and Tomei (2008b). Since the nonlinearities are uncertain the assumption of knowing the maximum number of sinusoids to be generated by the internal model is certainly more
critical; in fact, the uncertain nonlinearities may generate many higher order harmonics to be compensated by the controller even from a single sinusoidal disturbance and/or reference. In this paper we do not make such a critical assumption. A robust adaptive regulator is designed which includes a robust nonlinear stabilizing control and an adaptive internal model. It is shown that if the adaptive internal model can generate the desired reference input then the regulation error tends to zero while, even if the adaptive internal model cannot generate the reference input, the regulation error tends to a residual set which decreases as the reference input modeling error decreases.

2. MAIN RESULT

Consider the observable minimum phase system of constant relative degree \( \rho \)

\[
\dot{x} = A_c \dot{x} + f(\dot{x}, w) + \frac{1}{\beta} bu, \; \dot{x} \in \mathbb{R}^n, \; u \in \mathbb{R}
\]

\[
\dot{w} = R(w), \; w \in \mathbb{R}^r
\]

\[
e = C_c \dot{x} - q(w), \quad e \in \mathbb{R}
\]

in which the pair \((A_c, C_c)\) is in observer canonical form

\[
A_c = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad C_c = [1 \; 0 \; \cdots \; 0]
\]

\(b = [0, \ldots, 0, 1, \bar{b}_{\rho+1}, \ldots, \bar{b}_n]^T\) is an unknown vector, \(\beta > 0\)
is an uncertain positive real such that \(\beta_m \leq \beta \leq \beta_M\), with \(\beta_m\) and \(\beta_M\) known positive reals; the polynomial \(s^{n-\rho} + \bar{b}_{\rho+1}s^{n-\rho-1} + \cdots + \bar{b}_n\) has all roots with negative real part. The functions \(f, R\) and \(q\) are uncertain but are assumed to be smooth. Since \(f\) is smooth we can write (see Nijmeijer and van der Schaft (1990))

\[
f(y_1, w) - f(y_2, w) = \tilde{f}(y_1, y_2, w)(y_1 - y_2).
\]

We assume that there exist known functions \(q_i\) such that

\[
|\tilde{f}(y_1, y_2, w)| \leq q_i(y_1, y_2, w), \quad 1 \leq i \leq n.
\]

Moreover, we assume that \(b \in \Omega_1 \subset \mathbb{R}^n\) and \(w \in \Omega_w \subset \mathbb{R}^r\), with \(\Omega_1\) and \(\Omega_w\) known bounded regions.

If \(\rho = 1\), we set \(b = \bar{b}\), \(u = \phi_1\) and \(x = \bar{x}\), otherwise when \(\rho \geq 2\), we consider the filtered transformation \((\lambda_i > 0, 1 \leq i \leq \rho - 1)\)

\[
\dot{\phi} = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\lambda_{\rho-1} \end{bmatrix} \phi + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u, \quad \phi(0) = 0
\]

(4)

\[
x_1 = \bar{x}_1
\]

\[
x_i = \bar{x}_i - \frac{1}{\beta} \sum_{j=2}^{\rho} b_j[j] \phi_{j-1}, \quad 2 \leq i \leq n.
\]

(5)

The vectors \(b[j] = [b_1[j], \ldots, b_n[j]]^T\) are recursively obtained by

\[
b[j] = \bar{b}
\]

\[
b[j-1] = A_c b[j] + \lambda_{j-1} b[j], \quad \rho \geq j \geq 2
\]

\[
b[1] = b = [1, b_2, \ldots, b_n]^T
\]

(6)

with \(b_i, 2 \leq i \leq n\) solutions of

\[
s^{n-\rho} + b_{\rho+1}s^{n-\rho-1} + \cdots + \bar{b}_n \prod_{i=1}^{\rho-1} (s + \lambda_i).
\]

(7)

From (1) and (5), we have

\[
\dot{x} = A_c x + f(x, w) + \frac{1}{\beta} b \phi_1
\]

\[
\dot{w} = R(w)
\]

\[
e = C_c x - q(w)
\]

(8)
in which \(b\) is a known vector. Assume that there exists an (unknown) bounded solution \(x_r = \Gamma(w), \phi_{1rg} = \gamma(w)\) for the regulator equations

\[
\dot{x}_r = A_c \dot{x}_r + f(y_r, w) + \frac{1}{\beta} b \phi_{1rg}
\]

\[
y_r = C_c x_r
\]

(9)
in which

\[
\phi_{1rg} = \{ u_r, \quad \text{if } \rho = 1 \\
\phi_{1r}, \quad \text{if } \rho > 1 
\}
\]

(10)

Let \(\bar{\phi}_{1rg}\) be an estimate of \(\phi_{1rg}\) given by a biased linear combination of \(m\) distinct sinusoids, i.e.

\[
\dot{\bar{w}}_c = \bar{R}_e \bar{w}_c, \quad \bar{w}_c \in \mathbb{R}^{2m+1}, \quad \bar{w}_c(0) = \bar{w}_{c0}
\]

\[
\bar{\phi}_{1rg} = \bar{w}_{c1}
\]

(11)

where

\[
\bar{R}_e = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ -\bar{\theta}_1 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\bar{\theta}_m & 0 & 0 & \cdots & 1 \end{bmatrix}
\]

(12)

and \(\bar{\theta}_i, 1 \leq i \leq m\), are positive reals satisfying

\[
\prod_{i=1}^{m} (s^2 + \omega_i^2) = s^{2m} + \sum_{i=0}^{m-1} s^{2i} \bar{\theta}_{m-i}
\]

(12)

with \(\omega_i, 1 \leq i \leq m\), being distinct positive reals such that \(\omega_1 \leq \omega_M, 1 \leq i \leq m\), and \(\omega_M\) a known positive real. From (12) and the definition of \(\omega_M\), it follows that \(|\bar{\theta}| \leq \theta_M\), with \(\theta_M\) a known positive real. Define

\[
\epsilon_M = \min_{\bar{w}_{c0} \in \mathbb{R}^{2m+1}} \left\{ \sup_{t \geq 0} |\phi_{1rg}(t) - \bar{\phi}_{1rg}(t)| \right\}
\]

(13)

with \(\bar{\phi}_{1rg}(t)\) generated by
\[ \dot{w}_c = R_c w_c, \quad w_c(0) = w_{c0} \]
\[ \hat{\phi}_{1rg} = w_{c1} \] (14)

in which
\[ R_c = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
-\theta_1 & 0 & 1 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-\theta_m & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix} \]

\[ \sum_{i=0}^{m-1} s^2 \theta_{m-i} = \prod_{i=1}^{m} (s^2 + \omega_i^2). \] (15)

Defining the regulation error \( \hat{x} = x - x_r \), from (8) and (9) we have
\[ \dot{\hat{x}} = A_c \hat{x} + f(x_1, w) - f(y_r, w) + \frac{1}{\beta} b(\phi_1 - \phi_{1rg}) \]
\[ \hat{w} = R(w) \]
\[ e = C_c \hat{x}. \] (16)

Define \( \epsilon(t) = \phi_{1rg}(t) - \hat{\phi}_{1rg}(t) = \phi_{1rg}(t) - w_{c1}(t) \).

**Theorem 2.1.** Consider system (16). There exists an adaptive learning output error feedback control such that all closed loop signals are bounded and, moreover, for any \( t \geq 0 \), for suitable positive real \( \alpha_i, 0 \leq i \leq 5 \):

(i) \( |\epsilon(t)| \leq a_0 e^{-\alpha_t} + a_2 \sup_{\tau \in [0,t]} \| \epsilon(\tau) \| \).

(ii) if \( \phi_{1rg}(t) \) is a sufficiently rich signal of order \( 2m \), then
\[ |\epsilon(t)| \leq a_3 e^{-\alpha_t} + a_5 \sup_{\tau \in [0,t]} \| \epsilon(\tau) \| \] which implies that when \( \epsilon(t) = 0 \), \( \forall t \geq t \), \( \epsilon(t) \) converges exponentially to zero.

**Proof.** By means of the change of coordinates
\[ \eta_i = \hat{x}_{i+1} - b_{i+1} \hat{x}_1, \quad 1 \leq i \leq n - 1 \] (17)

system (16) is transformed into the normal form
\[ \dot{\epsilon} = \eta + b_2 \epsilon + f_1(x_1, w) - f_1(y_r, w) + \frac{1}{\beta} (\phi_1 - w_{c1} - \epsilon) \]
\[ \dot{\eta} = \begin{bmatrix}
-b_2 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-b_{n-1} & 0 & \cdots & 0 \\
-b_n & 0 & \cdots & 0
\end{bmatrix} \eta + \begin{bmatrix}
b_3 - b_2^2 \\
\vdots \\
b_n - b_2 b_{n-1} \\
\vdots
\end{bmatrix} \epsilon \]
\[ + \begin{bmatrix}
f_2(x_1, y_r, w) - b_2 f_1(x_1, y_r, w) \\
\vdots \\
f_n(x_1, y_r, w) - b_2 f_1(x_1, y_r, w)
\end{bmatrix} \epsilon \]
\[ \hat{\epsilon} = \dot{\eta} = \Gamma \eta + \gamma \epsilon \]
\[ \hat{w}_c = R_c w_c \] (18)

with
\[ \Gamma = \begin{bmatrix}
-b_2 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-b_{n-1} & 0 & \cdots & 1 \\
-b_n & 0 & \cdots & 0
\end{bmatrix} \]
\[ \gamma = \begin{bmatrix}
b_3 - b_2^2 + f_2 - b_2 f_1 \\
\vdots \\
g \end{bmatrix}, \quad g = \begin{cases}
0 & \text{if } \phi_1 - w_{c1} - \epsilon \leq 0 \\
1 & \text{if } \phi_1 - w_{c1} - \epsilon > 0
\end{cases} \]

By virtue of (3) and, since \( b \in \Omega_1 \) and \( w \in \Omega_w \), it is possible to determine a function \( \gamma_M(e) \) such that
\[ \| \gamma \| \leq \gamma_M(e), \quad \forall e \in \mathbb{R}, \forall w \in \Omega_w, \forall b \in \Omega_1. \]

The further change of coordinates
\[ \beta = \begin{bmatrix}
1 \\
\theta_1 \\
\vdots \\
\theta_m
\end{bmatrix} \]
\[ \hat{\beta} = \begin{bmatrix}
\beta \epsilon \\
\dot{\beta} \epsilon
\end{bmatrix} \]
\[ \hat{\beta} = A_c \hat{\beta} + \beta \eta_2 + \beta \eta_3 \]
\[ = \begin{bmatrix}
0 \\
\theta_1 \\
\vdots \\
\theta_m
\end{bmatrix} \]

transforms (18) into
\[ \hat{\beta} \dot{\epsilon} = \begin{bmatrix}
0 \\
\theta_1 \\
\vdots \\
\theta_m
\end{bmatrix} \epsilon + \begin{bmatrix}
1 \\
\theta_1 \\
\vdots \\
\theta_m
\end{bmatrix} \beta \epsilon + \begin{bmatrix}
\beta \eta_2 + \beta \eta_3 \\
\dot{\beta} \epsilon
\end{bmatrix} \]
\[ = \begin{bmatrix}
0 \\
\theta_1 \\
\vdots \\
\theta_m
\end{bmatrix} \epsilon + \begin{bmatrix}
1 \\
\theta_1 \\
\vdots \\
\theta_m
\end{bmatrix} \beta \epsilon + \begin{bmatrix}
\beta \eta_2 + \beta \eta_3 \\
\dot{\beta} \epsilon
\end{bmatrix} \]

Define the filtered transformation \( \xi_i \in \mathbb{R}^{2m+1} \)
\[ \dot{\xi}_i = D \xi_i + [0 I_{2m+1}] E_{2i+1} \phi_1, \quad \xi_i(0) = 0 \]
\[ \mu_i = [1 0 \cdots 0] \xi_i, \quad 1 \leq i \leq m \]
\[ z = \begin{bmatrix}
\beta \epsilon \\
\dot{\beta} \epsilon
\end{bmatrix} - \sum_{i=1}^{m} 0 \xi_i, \quad z \in \mathbb{R}^{2m+2} \]
(21)

with
\[ D = \begin{bmatrix}
-d_2 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-d_{2m+1} & 0 & \cdots & 1 \\
-d_{2m+2} & 0 & \cdots & 0
\end{bmatrix} \]
a Hurwitz matrix, \( I_{2m+1} \) the \((2m+1) \times (2m+1)\) identity matrix and \( E_j \) the j-th column of an identity matrix of proper dimension. From (20) and (21), we have
\[ \dot{z} = A_c z + \sum_{i=1}^{m} \theta_i E_{2i+1} \left[ \beta \eta_1 + \beta b_2 e - e \right] + \sum_{i=1}^{m} \theta_i E_{2i+1} \beta \tilde{f}_1(x_1, y_r, w) e + d \mu^T \theta + E_1 \left[ \phi_1 + \beta \eta_1 + \beta b_2 e - e \right] - e \sum_{i=1}^{m} \theta_i E_{2i} \]
\[ \beta e = C_c z \]
with \( \mu = [\mu_1, \ldots, \mu_m]^T \) and \( d = [1, d_2, \ldots, d_{2m+2}]^T \). Now, make the linear change of coordinates

\[ \chi_i = z_{i+1} - d_{i+1} z_1, \quad 1 \leq i \leq 2m + 1 \quad (22) \]
so that from (20), (22) and (23), we obtain

\[ \beta \dot{e} = \chi_1 + \phi_1 + \beta \eta_1 + \beta b_2 e + \tilde{f}_1(x_1, y_r, w)e + d_2 e \]
\[ + \mu^T \theta - e \tilde{\chi}_1 + \phi_1 - e + \beta \eta_1 + \phi_1 + \chi_{1+1} e + \chi_{2+1}^T \theta \]
\[ \dot{\chi} = D \chi + p_1 e + p_2 \eta_1 - \bar{d} \phi_1 + p_3 e \]
\[ \dot{\eta} = \Gamma \eta + \gamma e \]
(24)
with
\[ d = [d_2, \ldots, d_{2m+2}]^T \]
\[ c_1(x_1, y_r, w) = \beta b_2 + \beta \tilde{f}_1 + d_2 \beta \]
\[ p_1(x_1, y_r, w) = \beta \]
\[ d_{2m+2} - d_2 d_{2m+1} \]
\[ - d_{2m+2} d_2 \]
\[ + \sum_{i=1}^{m} \theta_i E_{2i} \beta (b_2 + \tilde{f}_1) - \beta \sum_{i=1}^{m} \theta_i E_{2i-1} \]
\[ p_2 = -d \beta + \beta \sum_{i=1}^{m} \theta_i E_{2i} \]
\[ p_3 = \bar{d} - \sum_{i=1}^{m} \theta_i E_{2i} \]
Note that \( p_2 \) and \( p_1 \) are unknown constant vectors, while \( d \) is a known constant vector. By virtue of (3) and, since \( \beta_m \leq \beta \leq \beta_M, b \in \omega_1 \) and \( w \in \Omega_w \), it is possible to determine functions \( p_1M(e) \) and \( c_1M(e) \) such that for any \( e \in \mathbb{R} \), for any \( w \in \Omega_w \), for any \( b \in \Omega_b \) and for \( \beta_m \leq \beta \leq \beta_M: \| p_1(x_1, y_r, w) \| \leq p_1M(e), \| c_1(x_1, y_r, w) \| \leq c_1M(e) \).
Introduce the observer
\[ \dot{\chi} = D \chi - \bar{d} \phi_1 \]
(25)
with error dynamics (\( \dot{\chi} = \chi - \tilde{\chi} \))
\[ \dot{\chi} = D \chi + p_1 e + p_2 \eta_1 + p_3 e \]
(26)
Now, if \( \rho = 1 \) we set \( \phi_1 = u \) with
\[ u = -\chi_1 - \mu^T \bar{\theta} - ke - \alpha(e) e \]
(27)
in which \( k > 0 \) and \( \alpha \) will be later defined, whereas the adaptation law is chosen as
\[ \dot{\bar{\theta}} = g \text{Proj}(\mu e, \hat{\theta}) \]
(28)
with \( g > 0 \) and \( \text{Proj}(\cdot, \cdot) \) being the smooth projection operator defined as (see Pomet and Praly (1992))
\[ h_5(e) = 10\gamma_2^2 \max_{b \in \Omega_1} \|P_1\|^2 \]
\[ h_6(e) = \delta \|P_2\|^2 p_2^2 M(e) . \] (34)

Recalling property (iv) of Proj, from (32) and (33) we obtain

\[ \dot{V} \leq -e^2(k - h_1 - h_2 - \frac{1}{4}) - e^2(\alpha - c_1 - h_5 - h_6) - \|\eta\|^2(1 - 0.2 - \delta h_3) - 0.1\delta \|\bar{\chi}\|^2 - e^2(1 + \delta h_4) - \|\bar{\theta}\|^2 + \|\bar{\tilde{\theta}}\|^2 \] (35)

so that if \( k, \alpha \) and \( \delta \) are chosen as

\[ 0 < \delta < \frac{0.8}{h_3}, \]
\[ k > \frac{1}{4} + h_1 + h_2, \]
\[ \alpha \geq h_5 + h_6 + c_{1M} \] (36)

the inequality (35) implies

\[ \dot{V} \leq -c_\nu V + (1 + \delta h_4)e^2 + \|\bar{\tilde{\theta}}\|^2 \] (37)

where \( c_\nu > 0 \) is a suitable real. Equation (37) in turn, since \( e(t) \) and \( \|\bar{\tilde{\theta}}\| \) are bounded (recall property (i) of Proj), implies that \( e(t), \|\eta(t)\|, \|\bar{\chi}(t)\| \) are bounded. From (17), \( \|\bar{x}(t)\| \) is also bounded. From (37) it follows that for a suitable \( \epsilon_{\nu b} > 0 \),

\[ V(t) \leq V(0)e^{-\epsilon_{\nu b} t} + \frac{c_{\nu b}}{\epsilon_{\nu b}} \sup_{\xi \in \xi_0} \left\| e(\tau) - \bar{\tilde{\theta}}(\tau) \right\|^2 . \] (38)

The regressor vector \( \mu(t) \) in (28) is generated by the filters (21), whose input is \( \phi_1(t) \). We note that \( \mu_i(t) \) may be equivalently generated by the following filters with proper initial conditions

\[ \bar{\xi}_i[1] = D\bar{x}_1 - \beta E_{21}(b_{21}e + \eta_1) + \beta e(E_{21} - E_{21}\bar{f}_1) \]
\[ \mu_i[1] = [1, 0, \cdots, 0] \bar{\xi}_i[1] \]
\[ \bar{\xi}_2[2] = D\bar{x}_2 + E_{21}\phi_{1r}, \phi_{1r} \equiv \phi_{1r}(t) \]
\[ \mu_i[2] = [1, 0, \cdots, 0] \bar{\xi}_2[2] \]
\[ \mu_i = \mu_i[1] + \mu_i[2], \quad 1 \leq i \leq m \] (39)

by means of the relations

\[ \bar{\xi}_i[1] + \bar{\xi}_2[2] = \xi_i - \beta E_{21} e, \quad 1 \leq i \leq m \] (40)

so that \( \bar{\xi}_i(t), \mu_i(t) \) and \( \phi_i(t) \), from (27), are bounded. If \( \phi_{1r}(t) \) is a sufficiently rich signal of order \( 2m \), then \( [\mu_1[2], \cdots, \mu_m[2]]^T \) is a persistently exciting vector (see Sastry and Bodson (1989)). This fact implies that (see Marino et al. (2001)) the solution of the matrix differential equation

\[ \dot{Q} = -Q + [\mu[2][2]\mu[2][2]]^T, \quad Q(0) = e^{-T_p}k_p I \] (41)

with \( T_p \) and \( k_p \) positive reals satisfying

\[ \int_0^{t+T_p} \mu[2]([\tau][\tau]d\tau \geq k_p I, \quad \forall t \geq 0 \] (42)

is such that

\[ \sup_{t \geq 0} \left\| \mu[2][2]\mu[2][2] \right\| \geq k_p e^{-2T_p} I, \quad \forall t \geq 0 . \] (43)

Consider the function

\[ W = V + \lambda_0\|Q\bar{\theta} - \mu[2]\beta e\|^2 + \sum_{i=1}^m \zeta_i^T(1)P_2\zeta_i(1) \] (44)

where \( \lambda_0 \) and \( l_1 \) are suitable positive reals. From (35) and (36), we have

\[ \dot{V} \leq -c_{\nu v} \left\| e - \bar{\theta} \right\|^2 + c_{\nu 2} e^2 \]

with \( c_{\nu 1} \) and \( c_{\nu 2} \) suitable positive reals, which along with (29), (41) and (44), for sufficiently small \( \lambda_0 \) with respect to \( c_{\nu 1} \), and sufficiently small \( \lambda_0 \) with respect to \( l_1 \), imply \( \dot{V} \leq -c_{\nu v} W + c_{\nu 2} e^2 \), for suitable positive \( c_{\nu 1} \) and \( c_{\nu 2} \). If \( \mu \geq 1 \), we set \( \phi_1 = \delta^1 + \xi_1^1 \) with \( \delta^1 = -\bar{\chi}_1 - \mu^2 \bar{\theta} - ke - \alpha(e) e \)

and proceed as in Marino and Tomei (2008b) to design iteratively the control input \( u(t) \). \( \Box \)

3. EXAMPLE

Consider the second order nonlinear system

\[ \dot{x}_1 = x_2 + a_1 x_1^{2} + \frac{1}{\beta} u \]
\[ \dot{x}_2 = \frac{1}{\beta} b_2 u \]
\[ e = x_1 + w \] (45)

in which \( w(t) = \sin(\omega t) \). The parameters \( 0.2 \leq \omega \leq 10 \), \(-1 \leq a_1 \leq 2, 1 \leq a_2 \leq 2, 0.5 \leq b_2 \leq 3, 0.2 \leq \beta \leq 5 \) are real unknown constants with known bounds. We assume that the estimate of \( u = \phi_{1r} \) is given by an unbiased linear combination of two distinct sinusoids. Following the design outlined in Section 2 (with slight modifications due to the fact that the spectrum of \( \bar{R} \) is in this case \{ \pm \omega_1, \pm \omega_2 \}, i.e. zero is not included), the resulting control is given by

\[ \dot{\bar{\xi}}_1 = D\bar{x}_1 + [0 \quad u \quad 0 \quad 0]^T \]
\[ \dot{\mu}_1 = \mu_{11} \]
\[ \dot{\bar{\xi}}_2 = D\bar{x}_2 + [0 \quad 0 \quad 0 \quad u]^T \]
\[ \dot{\mu}_2 = \mu_{21} \]
\[ \dot{\bar{\theta}}_1 = g_1 \mu_1 e \]
\[ \dot{\bar{\theta}}_2 = g_2 \mu_2 e \]
\[ u = -ke - k_1 e^3 - \bar{\chi}_1 - \mu_1 \bar{\theta}_1 - \mu_2 \bar{\theta}_2 \] (46)

with

\[ D = \begin{bmatrix} -d_1 & 1 & 0 & 0 \\ -d_3 & 0 & 1 & 0 \\ -d_4 & 0 & 0 & 1 \\ -d_5 & 0 & 0 & 0 \end{bmatrix} , \quad \bar{d} = [d_2 \ d_3 \ d_4 \ d_5]^T . \]

Some numerical simulations have been carried out for system (45) controlled by (46). The unknown parameters of the system have been set as: \( a_1 = 1, a_2 = 2, b_2 = 0.5, \)

\[ 6787 \]
Fig. 1. Output error, input error and parameters estimates: $\omega = 1$

$\beta = 0.5$, while the parameters of the controller have been chosen as: $g = 1000$, $d_2 = 6$, $d_3 = 13$, $d_4 = 12$, $d_5 = 4$. Since the values of the control gains $k$ and $k_1$ are obtained through conservative inequalities, the actual values used in the simulations have been chosen as $k = 5$, $k_1 = 2$, and are smaller than the ones needed to satisfy the inequalities (36). All initial conditions of the system and of the controller have been set to zero. Two values for $\omega$ have been considered: $\omega = 1$, $\omega = 0.5$. The results are illustrated in Figs. 1 and 2 where the time histories are reported of the output regulation error $e(t)$, of the input reference error $u(t) - u_r(t)$ and of the parameters estimates $\hat{\theta}_1(t), \hat{\theta}_2(t)$, for $\omega = 1$ and $\omega = 0.5$, respectively.

4. CONCLUSIONS

A global robust adaptive regulator by output error feedback is designed for uncertain nonlinear systems in output feedback form, with exosystem containing uncertain parameters. Under the assumption that a solution to the regulator problem exists, but with no a priori assumption on the required control input (e.g. no ‘immersion’ assumption), an adaptive regulator is designed which contains a robust nonlinear stabilizing part and an adaptive internal model. Asymptotic regulation is achieved when the used adaptive internal model can generate the required control signal; otherwise, the regulation error tends to a residual ball whose radius decreases as the reference input modeling error decreases.

REFERENCES


