Abstract: Missing position-error-signal sampling data in hard disk drives (HDDs) results in their servo systems having irregular sampling rates. Due to the natural periodicity of HDDs, which is related to the disk rotation, HDD servo systems with missing samples can be modeled as linear periodically time-varying (LPTV) systems. Based on previous $H_\infty$ control synthesis results for general LPTV systems having irregular sampling rates, the unavailability of PES for HDD servo systems can still achieve the robust performance of a desired error rejection function for disturbance suppressions. Implementation results on multiple real hard disk drives validate the predicted robust performance.

1. INTRODUCTION

Similar to the data packet dropout in networked control systems (Lin et al. [2003]), the undesired inaccessibility of feedback signals may occur in hard disk drive (HDD) servo systems. Consequently, sampling intervals for these HDD servo systems may not remain equidistant. In this paper, the unavailability of feedback signals at a given sampling instance is referred to as a “missing sample”. For example, false PES demodulation, due to incorrect servo address mark (SAM) detection (Ehrlich [2005]) or damaged servo patterns in several servo sectors, makes the feedback PES unavailable in those servo sectors, resulting in an irregular sampling rate. In addition, irregular sampling rates also frequently occur during the self-servo track writing (SSTW) process (Brunnett et al. [2007]). For example, during some SSTW processes, the time of writing the final concentric servo patterns may coincide with the time of reading the feedback position error signal from previously written servo patterns. This conflict, caused by the fact that an HDD servo system can not read and write at the same time, is referred to as a “collision” of reading the PES with writing the final servo pattern. Such a collision makes the feedback signal unavailable, resulting in an irregular sampling rate as well. Thus, it is important to address the issue of designing servo systems for HDDs under missing PES samples. Unlike the randomness of data packet dropout (Xiong and Lam [2007]), the location of damaged servo sectors and the collision in the self-servo track writing process is consistent on each single servo track. In other words, the unavailability of PES for HDD servo systems takes place at some fixed and pre-determined locations for a given track. Furthermore, by considering that the natural periodicity of HDDs is related to the disk rotation, the servo systems can be represented as linear periodically time-varying systems with period equal to the number of servo sectors.

Furthermore, since there tend to be large variations in HDD dynamics due to the variations in manufacturing and assembly, it is not sufficient to achieve adequate servo performance for a single plant. Therefore, the designed controllers must guarantee a pre-specified level of performance for a large set of HDDs. By utilizing classical loop-shaping ideas which are familiar to most practicing engineers, the $H_\infty$ loop-shaping control design (Skogestad and Postlethwaite [2005]) has been well studied to deal with the robust performance of the desired loop shape. Thus, the optimal $H_\infty$ control is quite attractive for HDD servo systems. With such control, a set of HDDs with plant variations and a regular sampling rate is able to achieve a desired error rejection function for disturbance attenuation (Hirata et al. [1992]). The purpose of this paper is to extend these results for HDDs with missing PES samples.

In this paper, we consider the optimal $H_\infty$ track-following control design for HDD servo systems with missing PES samples. First, these servo systems with missing samples are modeled as LPTV systems, by considering that the natural periodicity is related to the disk rotation. Then, an optimal $H_\infty$ track-following controller can be synthesized using the control synthesis methodologies for general LPTV systems presented in Nie et al. [2010]. The resulting optimal $H_\infty$ controller can be directly obtained by solving discrete Riccati equations. With the developed control synthesis algorithm, the HDD servo systems with missing PES samples can still achieve the robust performance of an adequate error rejection function for disturbance suppressions.
However, because the total number of PES samples in a HDD revolution is large and the resulting controller is periodically time-varying, a significant amount of memory is required to store all of the control parameters for the synthesized controller. Thus, in order to reduce the memory storage requirements, a simplified controller implementation is developed in this paper, which utilizes a reduced number of periodic control parameters. In order to demonstrate the effectiveness of the developed control design algorithm, the resulting optimal \( H_c \) tracking control has been evaluated through both simulation and implementation study on a set of actual hard disk drives. The simulation study presented in this paper validates the effectiveness of the proposed control and the feasibility of the control simplification. Moreover, implementation results on the ten tested hard disk drives validate the obtained robust performance and furthermore illustrate that the proposed control is implementable in a set of real HDDs.

This paper is organized as follows. Section 2 discusses the modeling of HDD servo systems with missing PES samples. In Section 3, the optimal \( H_c \) control algorithm is provided. The simulation and experiment results for the developed controller are presented in Section 4. Conclusions are given in Section 5.

2. MODELING OF HDD SERVO SYSTEMS WITH MISSING PES SAMPLES

As mentioned in the previous section, an irregular sampling rate can be caused by missing a sample when the feedback PES is unavailable. Similarly to Abramovitch et al. [1998], HDD servo systems with missing PES samples can be modeled by the block diagram shown in Fig. 1. Here, we consider the output multiplicative uncertainty with the nominal plant of the voice coil motor (VCM) \( G_v^n \) and the plant uncertainty weighting function \( W_\Delta \) as

\[
G_v = (1 + W_\Delta \Delta) G_v^n \| \Delta \|_\infty \leq 1. \tag{1}
\]

![Fig. 1. Modeling of HDD servo systems with missing PES samples](image)

For control synthesis convenience, notch filters (Semba et al. [2001]) are incorporated into the VCM plant \( G_v \). In order to obtain the nominal VCM plant, the actual VCM plant frequency response for one single drive was measured on the disk area where there are no missing PES samples. Then, the nominal VCM model \( G_v^n(s) \) was identified to match the experiment frequency response, as described in (Nie and Horowitz [2009]). Note that in order to reduce the controller order, the nominal VCM plant is just identified as a 7th order model.

The uncertainty weighting function \( W_\Delta \), considered in the control design is shown in Fig. 2. The selected uncertainty weighting function demonstrates that the real plant could have an unstructured uncertainty of a ±26% gain variation at low frequency and a ±112% gain variation at high frequency respectively from the nominal plant.

![Fig. 2. Plant uncertainty weighting function \( W_\Delta \)](image)

In Fig. 1, \( d \) is the overall contribution of all disturbances (Nie and Horowitz [2009]) including torque disturbance, windage, non-repeatable disk motions and measurement noise to PES. In addition, \( K \) is the controller to be designed by using the feedback signal \( y(k) \) in the case of irregular sample rates. Moreover, \( y \) is switched to PES when the PES is available, while \( y \) is switched to zero when the PES is unavailable (Nagamune et al. [2005]). Then, the servo system with missing PES samples has the following state-space realization:

\[
G_o \sim \begin{bmatrix} x(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} A_o & B_o^D \\ C_o^D(k) & D_o^{D}(k) \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \\ y(k) \end{bmatrix}. \tag{2}
\]

Here, all of matrices except \( C_o^D(k) \) and \( D_o^{D}(k) \) in the above state-space realization are constant and

\[
[C_o^D(k), D_o^{D}(k)] = \begin{cases} [C_{o,21}^D(k), D_{o,21}^{D}] & \text{if PES}(k) \text{ is available} \\ [0,0] & \text{otherwise} \end{cases}. \]

Since the location of damaged servo sectors and the collision in the self-servo track writing process is fixed for each servo track, the unavailability of PES for HDD servo systems takes place at some fixed and pre-determined locations (Brunnet et al. [2007]) on the disk for each track. Throughout this paper, the set of servo sectors on which the PES is unavailable is defined as \( M_{\text{miss}} = \{ i : \text{PES is unavailable on servo sector } i \} \).

Furthermore, by considering that the natural periodicity of HDDs is related to the disk rotation, the servo systems can be represented by LPTV systems with the state-space realization in (2) whose elements are periodic, for example, \( [C_o^D(k), D_o^{D}(k)] = [C_o^D(k + N), D_o^{D}(k + N)] \). Here, \( N \) represents the number of servo sectors.

3. OPTIMAL \( H_c \) CONTROL FOR HDD SERVO SYSTEMS WITH MISSING PES SAMPLES

3.1 Optimal \( H_c \) control formulation

In order to deal with the HDD plant variations and missing PES samples, we will extend well-known \( H_c \) LTI error rejection loop-shaping design techniques for HDDs with regular sampling rates to irregular sampling rates. To do so, the \( H_c \) norm of linear time-invariant systems must be generalized as \( \ell_2 \) induced norm for LPTV systems. For an LPTV system \( H \) with input \( w \)
and output $z$, its $\ell_2$ induced norm (Peters and Iglesia [1997]) is defined as

$$\|H\|_{\ell_2\rightarrow\ell_2} = \left( \sup_{w \in \ell_2 \setminus \{0\}} \frac{\sum_{k=0}^{\infty} z^T(k)z(k)}{\sum_{k=0}^{\infty} w^T(k)w(k)} \right)^{1/2}.$$  \hfill (3)

Fig. 3. The HDD servo system with the nominal VCM plant

For the optimal $H_\infty$ control of LPTV systems, we need to design a controller $K$ satisfying the following conditions (Skogestad and Postlethwaite [2005]) for the nominal VCM plant $G_p^n$:

$$\|T_v \cdot W_p \cdot T_e \cdot W_\Delta\|_{\ell_2\rightarrow\ell_2} < 1$$  \hfill (4)

where $T_v$ is the sensitivity function (i.e. error rejection transfer function) from $d$ to $PES$, as shown in Fig. 3, while $T_e$ is the complementary sensitivity function from $d$ to the plant output $y_k$. The $W_p$ and $W_\Delta$ blocks in (4) are the loop-shaping performance weighting function and the plant uncertainty weighting function respectively. In order to attenuate disturbances, the performance weighting function $W_p$ is designed as a low-pass filter and thus $W_p^{-1}$ is a high-pass filter. The inverse of the frequency response for the designed performance weighting function is shown in Fig. 4. Then with the proposed control synthesis methodology, all of the disk drives are expected to possess the better disturbance attenuation than the one described by the inverse of the performance weighting function.

![Frequency response of the performance weighting function inverse](image)

Fig. 4. The frequency response of the performance weighting function inverse

It is known that Equation (4) guarantees the robust performance that the magnitude of the obtained error rejection function (for single-input-single-output (SISO) systems) is always smaller than $|W_p(e^{i\omega})|^{-1}$ for the uncertain plant characterized in (1).

Similar to the optimal $H_\infty$ control of HDD servo systems with multi-rate sampling and actuation in Nie et al. [2010], we consider the block diagram shown in Fig. 5 for the control design of the servo systems with an irregular sampling rate. In the figure, $W_u$ is the control input weighting value to be tuned so that the inequalities in (4) hold. As discussed in Nie et al. [2010], this control design formulation is different from the standard $H_\infty$ control problem formulation (Skogestad and Postlethwaite [2005]) due to the introduction of the fictitious disturbance $w_2$ and the fact that the performance weighting function was moved to the disturbance input side. As a result, the constraint $D_2(k)D_1^T(k) \succeq 0$ for all $k$ in the developed control synthesis technique in Nie et al. [2010] is satisfied. As will be shown subsequently, the introduction of $w_2$ does not affect the optimal $H_\infty$ controller and the optimal closed-loop $\ell_2$ induced norm at all. Moreover, by changing the position of the performance weighting function, the feedback signal $y$ is affected by the “colored” disturbance $(W_p w_1)$ more directly when missing samples occur.

With these changes and utilizing the block diagram in Fig. 5, the optimal $H_\infty$ control problem is to find an optimal linear time-variation controller $K$ and a minimum $\gamma$ with the closed loop $\ell_2$ induced norm less than $\gamma$, i.e.

$$\min_{K, \gamma} \gamma \quad \text{s.t.} \quad \|T_{z_e \rightarrow w}\|_{\ell_2\rightarrow\ell_2} < \gamma$$  \hfill (5)

where $T_{z_e \rightarrow w}$ represents the transfer function matrix from $w = [w_1 \ w_2]^T$ to $z = [z_1 \ z_2 \ z_3]^T$. Suppose the LPTV system $G_1$ (with input $[w_1 \ w_2 \ u]^T$ and output $[z_1 \ z_2 \ z_3 \ y]^T$) shown in Fig. 5 can be represented as the following state-space realization:

$$G_1 \approx \begin{bmatrix} x(k+1) \\ z(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2(k) & D_{21}(k) & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \\ u(k) \end{bmatrix}$$  \hfill (6)

with

$$[C_2(k) \ D_{21}(k)] = \begin{cases} \begin{bmatrix} C_{2m} \\ D_{21m} \end{bmatrix} & \text{if PES}(k) \text{ is available} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \text{otherwise} \end{cases}$$

In (6), all of matrix entries except $C_2(k)$ and $D_{21}(k)$ are constant and $C_2(k)$ and $D_{21}(k)$ are periodic with period $N$.

For the control synthesis in the next section, we will use the following notation, $B = [B_1 \ B_2]$, $D_{1\bullet} = [D_{11} \ D_{12}]$, $C(k) = \begin{bmatrix} C_1 \\ C_2(k) \end{bmatrix}$, and $D_{2\bullet}(k) = \begin{bmatrix} D_{11} \\ D_{21}(k) \end{bmatrix}$.

3.2 Optimal $H_\infty$ track-following control

As mentioned in Nie et al. [2010], there may exist many controllers that satisfy the inequality in (5) for a given $\gamma$. Similar to Nie et al. [2010], we would like to obtain the minimum entropy controller (Peters and Iglesia [1997]) among all these
controllers. In order to synthesize the optimal $H_\infty$ controller, we first need to obtain the unique minimum entropy controller satisfying the $\ell_2$ induced norm constraint in (5) with a fixed $\gamma$, and then utilize a bi-section search method to find the minimum $\gamma$ and the corresponding optimal controller. In the minimum entropy control synthesis methodology that follows, for simplicity and without loss of generality, we will assume that $\gamma = 1$.

Utilizing the control design methodology presented in Nie et al. [2010], we obtain the following unique stabilizing minimum entropy time-varying controller $K$, which satisfies the constraint in (5) and is given by the following state-space realization:

\[
\begin{aligned}
\hat{x}(k+1) &= \hat{A}\hat{x}(k) + B_{2u}u(k) + F_1(k) (\hat{C}_2(k)\hat{x}(k) - y(k)) \\
u(k) &= -T_{22}^{-1}\hat{C}_{12}\hat{x}(k) + L_2(k) (\hat{C}_2(k)\hat{x}(k) - y(k)) \\
\end{aligned}
\]

(7)

Notice that for the irregular sampling rate HDD servo system shown in (6), all matrix entries in the state-space realization (6) are constant except $C_2(k)$ and $D_{21}(k)$. Consequently, the controller synthesis algorithm utilized to calculate the control parameters in (7) can be furthermore simplified as follows.

1. Solve the state feedback algebraic Riccati equation:

\[
X = A^TXA + C_1^TC_1 - M (R + B^TXB)^{-1}M^T
\]

where $M = A^TB + C_1^TD_1$.

2. Define $T = \begin{bmatrix} T_{11} & 0 \\ T_{12} & T_{22} \end{bmatrix}$ with $T_{11} \succ 0$ and $T_{22} \succ 0$.

3. Compute $T$ using:

\[
R + B^TXB = T^TJT
\]

where $R = D_{11}^TD_1 - \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, J = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$.

4. Get $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = (R + B^TXB)^{-1}M^T$.

5. Calculate the following matrices for the filtering Riccati equation:

\[
\begin{aligned}
\hat{A} &= A + B_1F_1, \hat{C}_2(k) = C_2(k) + D_{21}(k)F_1, \text{ and } \hat{C}_{12} = -T_{22}F_2. \\
\text{Let } D_{12} \text{ be an orthogonal matrix to } D_{11}. \\
\text{In addition, define a matrix } W \text{ such that } W^T(W - I - T_{11}^{-1}T_{11}^{-1})W \text{ has appropriate dimensions so that the following matrix multiplication is well defined:}
\end{aligned}
\]

\[
\begin{bmatrix} D_{11} & 0 \\ 0 & D_{12} \end{bmatrix} = D_{11}W + D_{12}T_{22}.
\]

6. Update forwards in time the filtering Riccati equation solution with zero initial condition:

\[
Y(k) = \hat{A}Y(k-1)\hat{A}^T + B_1B_1^T - \hat{M}(k).
\]

\[
\begin{bmatrix} \hat{R}(k) + \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix} & Y(k-1) \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix}^T \end{bmatrix}^{-1} = \hat{M}(k)
\]

where $Y(k) \succeq 0$ and

\[
\hat{M}(k) = \hat{A}Y(k-1) \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix}^T + B_1 \begin{bmatrix} D_{11} \\ D_{21}(k) \end{bmatrix}.
\]

7. Define $\hat{T}(k) = \begin{bmatrix} \hat{T}_{11}(k) & \hat{T}_{12}(k) \\ 0 & \hat{T}_{22}(k) \end{bmatrix}$, with $\hat{T}_{11} \succ 0$ and $\hat{T}_{22} \succ 0$.

8. Compute $\hat{T}(k)$ using:

\[
\begin{bmatrix} \hat{R}(k) + \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix} & Y(k-1) \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix}^T \end{bmatrix}^{-1} = \hat{T}(k)\hat{F}\hat{T}(k)
\]

where

\[
\hat{R}(k) = \begin{bmatrix} D_{11} & 0 \\ 0 & D_{21}(k) \end{bmatrix} \begin{bmatrix} D_{11} & D_{12}(k) \end{bmatrix}^T - \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \hat{F} = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}.
\]

9. Obtain

\[
\begin{bmatrix} \hat{F}_1(k) \\ \hat{F}_2(k) \end{bmatrix} = \left( \hat{R}(k) + \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix} Y(k-1) \begin{bmatrix} \hat{C}_{12} \\ \hat{C}_2(k) \end{bmatrix}^T \right)^{-1} \times \hat{M}^T(k)
\]

10. Calculate the filter gains:

\[
\begin{aligned}
L_0(k) &= T_{22}^{-1}(k)\hat{T}_{12}(k)\hat{T}_{22}^{-1}(k), \\
F_0(k) &= \hat{F}_1^T(k)T_{22}^{-1}(k) + \hat{F}_2^T(k).
\end{aligned}
\]

Since, as proved in Nie et al. [2010], the Riccati equation solution $Y(k)$ in step 6) is unique and periodic, $Y(k)$ computed "forwards" in time with a zero initial condition converges to a steady-state periodic solution with period $N$. It turns out that the filter gains $F_0(k)$ and $L_0(k)$ are also periodic with period $N$. In addition, it is straightforward to show that both $T_{12}(k)$ and $F_0(k)$ are equal to 0 at the instance $k$ when the PES is unavailable. Thus, both $F_0(k)$ and $L_0(k)$ are also equal to 0 at the time when PES(k) is unavailable, which verifies the claim that the fictitious disturbance $w_2$ does not affect the optimal $H_\infty$ controller. With the zero gains of $F_0(k)$ and $L_0(k)$ at the instance when a missing PES sample occurs, the time varying control parameter $C_2(k) = C_2(k) + D_{21}(k)F_1$ in (7) can be simply substituted by a constant parameter $C_{2m} = C_{2m} + [D_{21m} 0]F_1$ without affecting the controller effect. As a result, for the HDD servo systems with missing PES samples, all of the control parameters of the minimum entropy $H_\infty$ controller shown in (7) except $F_0(k)$ and $L_0(k)$ are constant.

4. SIMULATION AND EXPERIMENT RESULTS

In order to evaluate our proposed optimal $H_\infty$ control design methodology in this paper, the algorithm will be applied to the track-following control of multiple hard disk drives with missing PES samples through both simulation and implementation studies. The ten hard disk drives considered here were provided by Western Digital Corporation. For these 2.5" disk drives, the number of servo sectors is $N = 274$ and the spindle rotation speed is 9000 RPM. For one of these drives, servo patterns on some servo sectors at the inside diameter (ID) have been damaged and thus the PES on these servo sectors is not available for the servo system. Specifically, we found that for this particular drive, the PES is unaccessible on the following 57 servo sectors: $\text{M}_{\text{miss}} = \{2, 6, 10, 14, 18, 27, 31, 35, 39, 43, 47, 52, 56, 60, 64, 68, 72, 81, 85, 89, 93, 97, 101, 110, 114, 118, 122, 126, 135, 139, 143, 147, 151, 155, 164, 168, 172, 176, 180, 184, 193, 197, 201, 205, 209, 218, 222, 226, 230, 234, 238, 247, 251, 255, 259, 263, 272\}$. Such a missing sample sequence will be utilized to synthesize the optimal $H_\infty$ track-following controller in this section. For the other nine disk drives, there are no damaged servo sectors. However, in order to test the synthesized controller’s ability of achieving robust performance and handling missing PES samples, the feedback PES has also been manually dropped at the specific servo sectors described by $\text{M}_{\text{miss}}$ for the nine undamaged disk drives.

4.1 Control design and simplification

It is well known that if the optimal $H_\infty$ controller yields a minimum $\gamma \leq 1$, then the controller is able to achieve the robust performance by satisfying the corresponding conditions illustrated in (4). In other words, the designed controller achieves the better disturbance attenuation than the inverse of performance
weighting function shown in Fig. 4 for all the plant variations with the multiplicative plant uncertainty weighting function in Fig. 2. With the determined performance weighting function and plant uncertainty weighting function, the weighting value \( W_\gamma \) is to be tuned so that the achieved \( \gamma \) is less than or equal to 1 and simultaneously the control actuation generated by the resulting controller \( K \) is appropriate under the hardware constraints of real HDD servo systems.

For the real hard disk drives described in the previous section, a weighting value of \( W_\gamma = 40 \) was selected and thus the resulting optimal \( H_\gamma \) control is able to achieve an optimal \( \ell_2 \) induced norm \( \gamma = 0.88 \). Moreover, the resulting gains \( F_i(k) \) and \( L_i(k) \) in (7) are zero when the feedback PES is unavailable at the instance \( k \). The designed gain \( L_i(k) \) for one HDD revolution is shown in Fig. 6.

![Fig. 6. The designed \( L_i(k) \)](image)

Since the control parameters \( F_i(k) \) and \( L_i(k) \) are time-varying, we have to save all 217 (= 274 – 57) sets of non-zero time-varying control parameters, requiring a significant amount of memory. Unfortunately, it is almost impossible to reserve so much memory for storing these control parameters in real HDDs. Fortunately, we found that the non-zero time-varying parameters \( F_i(k) \) and \( L_i(k) \) have very small variations, which motivates us to treat all non-zero time-varying control parameter values as constants. Specifically, the constant values \( F_i \) and \( L_i \) for the identified nominal \( F_i(k) \) and \( L_i(k) \) are obtained by taking the average of all non-zero \( F_i(k) \) and \( L_i(k) \) respectively. Then, the approximate control parameters \( F_i \) and \( L_i \) will be applied when the PES is available. Consequently, just one set of control parameters \( F_i \) and \( L_i \) has to be stored in memory. The approximate values for \( L_i(k) \) with \( L_i \) are also shown in Fig. 6. The simulation study, which will be presented in next section, shows that such control parameter approximation has a tolerable negative affect on the control performance.

### 4.2 Simulation study

In order to evaluate the robust performance of the designed controller, a total of 50 different VCM plants was collected. These various VCM plants were randomly generated based on the identified nominal VCM plant and the uncertainty weighting function shown in Fig. 2 by using Matlab function “usample”.

The simulation results of the Root Mean Square (RMS) 3\( \sigma \) values of PES for the nominal plant and the worst-case plant are illustrated in Table 1. Based on these time-domain simulation results, we are confident that the proposed optimal \( H_\gamma \) control indeed attains its predicted robust performance. Moreover, Table 1 also shows the simulation results by using the approximate control parameters. The results imply that the performance degradation caused by the control parameter approximation is so small that the control simplification presented in the previous section is viable. Thus, the simplified controller implementation with the control parameter approximation was utilized for the experimental study which will be illustrated in the following section.

#### 4.3 Control implementation study

The simulation results presented in the previous section illustrate that the proposed control design methodology in this paper is able to handle an irregular sampling rate and achieve robust performance. In this section, we discuss its implementation on multiple actual hard disk drives with missing PES samples. The designed controller was coded on the disk drives’ own processor by changing their firmware code. For the disk drive with damaged servo sectors, the designed \( H_\gamma \) control is tested and evaluated in the disk area where the missing samples occur with the missing sample sequence of \( M_{miss} \) as illustrated in the previous section. As a result, at the servo sector \( i \in M_{miss} \), the PES is unavailable. For the other nine undamaged disk drives, the feedback PES was also manually dropped at the servo sectors described by \( M_{miss} \) during the experimental study, in order to evaluate the designed controller.

Unlike linear time-invariant systems, a linear time-varying system with irregular sampling rates has no well-defined error rejection transfer function. However, in order to evaluate the implemented controller for the considered disk drives, we define the following disturbance-PES relationship called approximate error rejection transfer function. A sweep sinusoid excitation is injected into the position of the disturbance \( d \) shown in Fig. 1, which is the same as the way that the excitation is injected for measuring the error rejection transfer function for disk drives with a regular sampling rate. Then, the PES response caused by the excitation is collected. Since the PES is unaccessible at the time when missing samples happen, we approximately treat the previous available PES as the current unavailable PES. Afterwards, the discrete Fourier transform (DFT) is computed for the collected PES data and then the DFT component at the frequency of the sinusoid excitation is identified. Consequentially, the approximate frequency response from the disturbance \( d \) to PES is calculated by dividing the identified PES DFT component by the corresponding excitation DFT component. Finally, the approximate error rejection transfer functions were measured on the ten considered disk drives and are shown as the blue lines in Fig. 7.

The inverse of the performance weighting function is also shown in Fig. 7. The experiment results in Fig. 7 illustrate that all of the measured approximate error rejection transfer functions are below the inverse of the performance weighting function.
In hard disk drive servo systems, sometimes an irregular sampling rate is unavoidable, for example, when false PES de-modulation is caused by damaged servo sectors and when the unavailability of feedback signals is due to the collision in the self-servo track writing process. By considering that the position of the unavailable feedback signal is fixed and the natural periodicity of HDDs is related to the disk rotation, the servo systems with missing PES samples were represented as linear periodically time-varying systems. In addition, since there tend to be large variations in HDD dynamics due to variations in manufacturing and assembly, the designed controllers must guarantee a pre-specified level of performance for a large set of hard disk drives. Based on the optimal $H_\infty$ control synthesis methodologies presented in Nie et al. [2010] for general LPTV systems, an optimal $H_\infty$ track-following controller was synthesized for HDD servos with missing PES samples in this paper. The simulation and experimental study on multiple hard disk drives demonstrated the synthesized controller’s effectiveness in handling irregular sampling rates and achieving the robust performance of a desired error rejection transfer function for disturbance attenuation. Moreover, the simulation results were validated by the implementation results on the ten actual 2.5” hard disk drives. In the experimental study, around 20% improvement of the $3\sigma$ PES was obtained by the proposed control algorithm over the intuitionistic methodology for the ten tested disk drives.

REFERENCES


