Uniform Global Asymptotic Stability of an Adaptive Output Feedback Tracking Controller for Robot Manipulators

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Abstract: Two important practical aspects of robot manipulators control are the difficulty of obtaining accurate velocity measurements and the uncertainty of parameters of the dynamical model. To deal with these problems, several adaptive output feedback tracking controllers have been proposed. In this paper, it is proved that, as long as the regression matrix is persistently exciting, uniform global asymptotical stability can be achieved through an adaptive output feedback controller.

Keywords: Adaptive Control, Robot Control, Lyapunov Stability, Output Feedback Control.

1. INTRODUCTION

It is well known that measurement of joint velocities in a robot manipulator frequently produces noisy signals, so much that its use in a controller may not be feasible (see Daly and Schwarz (2006)). To overcome this problem, some output feedback controllers have been proposed. These controllers only require position measurements, while joint velocities are estimated through either an observer or filter.

In Berghuis and Nijmeijer (1993); Burkov (1993); Kelly (1993), output feedback controllers with a gravity compensation were proposed, achieving global asymptotical stability for the regulation case. In Arimoto et al (1994) an output feedback controller with desired gravity compensation was proposed. In Ortega et al (1995) an output feedback controller which compensates gravity uncertainty effects was designed; however, the asymptotical stability result is local.

For the tracking case, most results obtained are semiglobal. For instance, in Berghuis and Nijmeijer (1993); Lim et al (1996) output feedback controllers with exact model knowledge were proposed obtaining a local exponential stability result, while in Nicosia and Tomei (1990) the local asymptotical stability result is proven.

In Loria and Nijmeijer (1998) an output feedback tracking controller with bounded inputs was proposed, proving local asymptotical stability; in Santibanez and Kelly (2001) it was proved that, in presence of viscous friction and a proper bound of the desired joint speed, the asymptotical stability is global. Other variations on this controller are proposed in Moreno et al (2008a,b), proving local asymptotical stability via singular perturbations theory. In Zavala-Rio et al (2009) a generalization of the controller proposed in Santibanez and Kelly (2001) is designed.

Another practical consideration in robot manipulator control is uncertainty in the robot parameters. In particular, when some parameters of the robot dynamical model are unknown, an adaptive controller can be used. In an adaptive controller, an estimate of the model parameters is computed through an update law.

The first adaptive controller for robot manipulators with a rigorous stability proof was reported in Craig et al (1987). This controller required the knowledge of bounds on the robot parameters and measurement of joint accelerations. Other adaptive controllers were reported in Slotine and Li (1996); Sadegh and Horowitz (1987); Middleton and Goodwin (1988); Kelly et al (1989). An excellent tutorial is presented in Ortega et al (1989). An adaptive version of the controller PD with precompensation is reported in Santibanez and Kelly (1999).

A rigorous proof of uniform global asymptotical stability for adaptive tracking control of robot manipulators is presented in Loria et al (2005).

As for adaptive output feedback controllers, in Zhang et al (2000) global convergence to zero of the joint position error is proved. Because it was considered that such a
controller could not be implemented without velocity measurements, a redesign was proposed in Zergeroglu et al (2000), considering only position measurements. In Moreno et al (2010) an adaptive version of the output feedback controller proposed in Loria and Nijmeijer (1998) was presented, proving global convergence of tracking errors to zero with large enough viscous friction, while local exponential stability is proven in case that viscous friction is not large enough.

As far as the authors are aware, no proof of uniform global asymptotical stability has been presented for an adaptive output feedback controller. In this paper, it is proved that under the condition that the viscous friction is larger than the upper bound of the desired joint velocities, and assuming the regression matrix to be persistently exciting, uniform global asymptotical stability is achieved for the controller proposed in Moreno et al (2010).

The paper is structured as follows: Section II presents some preliminaries, including the robot dynamical model, the control objective, properties on the dynamical model and an important Theorem on UGAS of a type of nonlinear system. Section III presents the main result of the paper, proving UGAS of an adaptive output feedback tracking controller, and Section IV concludes the paper.

Throughout this paper, we use the notation \( \lambda_{\min}(A(x)) \) and \( \lambda_{\max}(A(x)) \), to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix \( A(x) \), for any \( x \in \mathbb{R}^n \). Also, we define \( \lambda_{\min}(A) \) as the greatest lower bound (infimum) of \( \lambda_{\min}(A(x)) \), for all \( x \in \mathbb{R}^n \). Similarly, we define \( \lambda_{\max}(A) \) as the least upper bound (supremum) of \( \lambda_{\max}(A(x)) \), for all \( x \in \mathbb{R}^n \). The norm of vector \( x \) is defined as \( \|x\| = \sqrt{x^T x} \) and that of a matrix \( A(x) \) is defined as the corresponding induced norm \( \|A(x)\| = \sqrt{\lambda_{\max}(A(x)^T A(x))} \).

We denote by \( \mathbb{R}_+ \) the space of nonnegative real numbers.

2. PRELIMINARIES

2.1 Robot dynamics

The dynamics of a \( n \)-link serial rigid robot manipulator, considering viscous friction, can be expressed as (see Spong et al (2006)):

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F_c + g(q) = \tau, \tag{1}
\]

where \( q \) is the \( n \times 1 \) vector of joint positions, \( \dot{q} \) is the \( n \times 1 \) vector of joint velocities, \( \ddot{q} \) is the \( n \times 1 \) vector of joint accelerations, \( M(q) \) is the \( n \times n \) symmetric positive definite inertia matrix, \( C(q, \dot{q}) \) is the \( n \times n \) matrix of centripetal and Coriolis torques, \( F_c \) is the \( n \times n \) diagonal positive definite matrix of viscous friction coefficients, \( \tau \) is the \( n \times 1 \) vector of applied torques, and \( g(q) \) is the \( n \times 1 \) vector of gravitational torques, obtained as the gradient of the robot potential energy \( U(q) \), i.e.

\[
g(q) = \frac{\partial U(q)}{\partial q}. \tag{2}
\]

We assume that the links are joined together with revolute joints. This assumption is instrumental in Properties 2-5.

2.2 Control objective

Assume that only the robot joint positions vector \( q(t) \in \mathbb{R}^n \) is available for measurement and the robot parameters are unknown. Then, the adaptive output feedback tracking control problem consists in designing a control law to compute the applied torques vector \( \tau \in \mathbb{R}^n \) together with a parameter estimation update law so that the limit

\[
\lim_{t \to \infty} \ddot{q}(t) = 0, \tag{3}
\]

is satisfied, where

\[
\ddot{q}(t) = q_d(t) - q(t) \tag{4}
\]

is the tracking error and \( q_d(t) \in \mathbb{R}^n \) is the desired joint positions vector.

We assume that the desired time-varying trajectory \( q_d(t) \) is three times differentiable and is bounded for all \( t \geq 0 \) in the sense

\[
\|q_d(t)\| \leq \mu_1 \tag{5}
\]

\[
\|\dot{q}_d(t)\| \leq \mu_2 \tag{6}
\]

where \( \mu_1 \) and \( \mu_2 \) are known positive constants.

2.3 Properties of the dynamic model

Some important properties of robot dynamics (1) are (see Spong et al (2006)):

Property 1. Using Christoffel symbols, the matrix \( C(q, \dot{q}) \) and the time derivative \( \dot{M}(q) \) of the inertia matrix satisfy (see Koditschek (1984); Spong et al (2006)):

\[
\ddot{q}^T \left[ \frac{1}{2} \dot{M}(q) - C(q, \dot{q}) \right] \dot{q} = 0 \forall q, \dot{q} \in \mathbb{R}^n
\]

and

\[
\dot{M}(q) = C(q, \dot{q}) + C(q, \dot{q})^T \forall q, \dot{q} \in \mathbb{R}^n.
\]

Property 2. There exists a positive constant \( k_e \) such that for all \( x, y \in \mathbb{R}^n \):

\[
\|C(x, y)z\| \leq k_e \|y\| \|z\|.
\]

Property 3. The gravitational torque vector \( g(q) \) is bounded for all \( q \in \mathbb{R}^n \) (see Craig et al (1987)). This means that there exist constants \( \gamma_i \geq 0 \) such that

\[
|g_i(q)| \leq \gamma_i, i = 1, 2, ..., n.
\]

for all \( q \in \mathbb{R}^n \), where \( g_i(q) \) stands for the \( i \)-th element of vector \( g(q) \). Equivalently, there exists a positive constant \( k' \) such that

\[
\|g(q)\| \leq k' \forall q \in \mathbb{R}^n.
\]

Property 4. There exists a positive constant \( k_y \) such that

\[
\|g(x) - g(y)\| \leq k_y \|x - y\|
\]

for all \( x, y \in \mathbb{R}^n \).

Property 5. The so-called residual dynamics is defined by

\[
h(\ddot{q}, \dot{q}) = [M(q_d) - M(q_d - \ddot{q})]\ddot{q}_d
\]

\[
+ [C(q_d, \dot{q}_d) - C(q_d - \ddot{q}_d, \dot{q}_d - \dot{\ddot{q}}_d)]\dot{q}_d
\]

\[
+ g(q_d) - g(q_d - \ddot{q})\].
The residual dynamics satisfies the inequality
\[ \| \mathbf{h}(\tilde{q}, \dot{\tilde{q}}) \| \leq k_c1 \mu_1 \| \dot{\tilde{q}} \| + \delta \sigma \tanh(\alpha \sigma) \| \tanh(\sigma \tilde{q}) \|, \] (7)
where \( \sigma > 0 \), the constant \( \mu_1 \) in (5), and
\[ \delta = k_y + k_M \mu_1 + k_2 \mu_2^2, \]
\[ \alpha = 2k_1 + k_2 \mu_2^2 + k_1 \mu_1 \delta, \] (9)
where
\[ k_M \geq n^2 \left[ \max_{i,j,k} \| \frac{\partial M_{ij}(q)}{\partial q_k} \| \right] \]
\[ k_{c2} \geq n^2 \left[ \max_{i,j,k} \| \frac{\partial c_{ijk}(q)}{\partial q_k} \| \right] \]
\[ k_1 \geq \sup_{q \in \mathbb{R}^n} \| g(q) \| \]
\[ k_2 \geq \lambda_{\max} \{ M(q) \}, \] (13)
for all \( q \in \mathbb{R}^n \), where \( M_{ij}(q) \) is the \( ij \)-element of matrix \( M(q) \) and \( c_{ijk}(q) \) is the \( ijk \)-Christoffel symbol.

Property 6. The robot model (1) can be linearly parameterized as
\[ M(q)\dot{q} + C(q, \dot{q})\dot{q} + F \dot{q} + g(q) = Y(q, \dot{q}, \ddot{q}) \theta \] (14)
for all \( \dot{q}, \ddot{q} \in \mathbb{R}^n \), where \( Y(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n \times m} \) is the regression matrix and \( \theta \in \mathbb{R}^m \) is the vector of the unknown parameters of the robot, which are assumed to be constant.

2.4 UGAS of a type of nonlinear systems

We start by recalling the definitions of PE (persistence of excitation) and Uδ-PE functions given in Loria et al. (2002).

Definition 1. The locally integrable function \( \Phi : \mathbb{R}_+ \to \mathbb{R}^{n \times m} \) is said to be persistently exciting (PE) if there exist \( \mu > 0 \) and \( T > 0 \) such that
\[ \int_t^{t+T} \Phi(\tau) \Phi(\tau)^T d\tau \geq \mu I, \forall t \in \mathbb{R}_+. \] (15)

Let \( x \in \mathbb{R}^n \) be partitioned as \( x = [x_1, x_2] \), where \( x_1 \in \mathbb{R}^{n_1} \) and \( x_2 \in \mathbb{R}^{n_2} \). Define the column vector \( \phi : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_{+} \) such that \( t \to \phi(t, x) \) is locally integrable. Define also \( \mathcal{D}_1 = \{ x \in \mathbb{R}^n : x_1 \neq 0 \} \).

Definition 2. The function \( \phi \) is said to be uniformly \( \delta \)-persistently exciting (Uδ-PE) with respect to \( x_1 \) if for each \( x \in \mathcal{D}_1 \) there exist \( \delta > 0 \), \( T > 0 \) and \( \mu > 0 \) such that for all \( t \in \mathbb{R}_+ \),
\[ \| z - x \| \leq \delta \implies \int_t^{t+T} \| \phi(\tau, z) \| d\tau \geq \mu. \] (16)

The property of Uδ-PE defined above roughly means that for every \( x \neq 0 \) the function \( \Phi(t) = \phi(t, x) \) is PE in the sense of Definition 1 and \( \mu \) and \( T \) are the same for all neighboring points of \( x \). For uniformly continuous functions, we do not need to check the condition on neighboring points. More precisely we have the following.

Lemma 1. If \( \phi(t, x) \) is continuous uniformly in \( t \), then \( \phi(t, x) \) is Uδ-PE with respect to \( x_1 \) if and only if for each \( x \in \mathcal{D}_1 \) there exist \( T > 0 \) and \( \mu > 0 \) such that, for all \( t \in \mathbb{R}_+ \),
\[ \int_t^{t+T} \| \phi(\tau, x) \| d\tau \geq \mu. \] (17)

In particular, a function of the form
\[ \phi(t, x) = (t^2 + 1) x \] (18)
is Uδ-PE with respect to \( x \) if and only if \( \phi \) is PE (see Loria et al. (2005)).

We can now recall an useful Theorem on uniform global asymptotical stability of nonautonomous systems, presented in Loria et al. (2005). This theorem applies to systems of the form
\[ \dot{x} = f(t, x), \] (19)
with:
\[ \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = f(t, x) = \begin{pmatrix} f_1(t, x_1) + f_2(t, x_2) \\ f_3(t, x_1) \end{pmatrix}, \] (20)
where \( x = [x_1^T, x_2^T]^T \), \( x_1 \in \mathbb{R}^{n_1}, x_2 \in \mathbb{R}^{n_2} \). For \( f \) uniformly continuous in the region \( x_1 = 0 \). We define
\[ f_0(t, x_2) = f_2(t, x) |_{x_1 = 0}, \] (21)
and notice that necessarily, \( f_0(t, 0) = 0 \). Suppose the following assumptions are satisfied:

Assumption 1. There exists a continuously differentiable function \( V : \mathbb{R}_+ \times \mathbb{R}^{n_1} \to \mathbb{R}_+ \), which is positive definite, decrescent, radially unbounded and has a negative semidefinite time-derivative. More precisely, assume that there exist continuous, positive definite, radially unbounded functions \( V_1, V_2 : \mathbb{R}_+ \to \mathbb{R}_+ \) and \( U : \mathbb{R}^{n_1} \to \mathbb{R}_+ \) continuous positive definite, such that
\[ V_1(x) \leq V(t, x) \leq V_2(x) \] (22)
\[ \dot{V}(t, x) \leq -U(x_1) \] (23)
for all \( (t, x) \in \mathbb{R}_+ \times \mathbb{R}^{n_1} \).

Assumption 2. The function \( f_2(t, x) \) is continuously differentiable and, moreover, it is uniformly bounded in \( t \) on each compact set of the state \( x_2 \). More precisely, for each \( r_2 > 0 \) there exist \( f_M > 0 \) and continuous nondecreasing functions \( p_i : \mathbb{R}_+ \to \mathbb{R}_+ \) with \( i = 1, 2 \) such that \( p_i(0) = 0 \) and for all \( (t, x) \in \mathbb{R}_+ \times \mathbb{R}^{n_2} \):
\[ \begin{aligned}
\max_{\| x_2 \| \leq r_2} \{ \| f_0(t, x_2) \|, \| \frac{\partial f_0}{\partial t} \|, \| \frac{\partial f_0}{\partial x_2} \| \} & \leq f_M, \\
\max_{\| x_2 \| \leq r_2} \| f_2(t, x) - f_2(t, x_2) \| & \leq p_1(\| x_1 \|), \\
\max_{\| x_2 \| \leq r_2} \{ \| f_1(t, x_1) \|, \| f_3(t, x_1) \| \} & \leq p_2(\| x_1 \|).
\end{aligned} \] (24)
(25)
(26)

We are now ready to cite the theorem that we will employ to prove uniform global asymptotical stability of a nonlinear time-varying system of the form (20).
Theorem 1. (See Loria et al (2002)) The system (19),(20) under Assumptions 1 and 2 is UGAS if and only if the function $f_\theta(t,x_2)$ is Ud-PE with respect to $x_2$.

Remark 1. In Loria et al (2002), condition (22) is expressed as
\[ \alpha_1(\|x\|) \leq V(t,x) \leq \alpha_2(\|x\|), \]
with $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$. However, condition (22) implies the existence of $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ such that (27) is fulfilled (see Lemma 4.3, Khalil (2002)).

3. MAIN RESULT

The adaptive output feedback tracking controller, proposed in Moreno et al (2010), is given by:
\[ \tau = Y(q_d, \dot{q}_d, \ddot{q}_d)\dot{\theta} + K_p \tanh(\dot{\theta}) + K_p \tanh(\sigma \dot{q}), \]
where $\dot{\theta} = q_d - q$ denotes the link position tracking error vector, $K_p$ and $K_q$ are $n \times n$ diagonal positive definite matrices, $\sigma$ is a positive constant, and the reference trajectory $q_d(t)$ is chosen such that the transpose of the regressor matrix $Y(q_d(t), \dot{q}_d(t), \ddot{q}_d(t))^T$, defined in Property 6, is PE in the sense of Definition 1.

The function $\tanh$ is defined as the hyperbolic tangent function in vectorial form, that is, $\tanh(y) = [\tanh(y_1) \tanh(y_2) \cdots \tanh(y_n)]^T$, with $y_i$ being the $i$-th element of vector $y$.

The signal $\dot{\theta}(t)$ in (28) is obtained from the following nonlinear filter
\[ \dot{\theta} = -\Gamma_0 Y^T(q_d, \dot{q}_d, \ddot{q}_d) \dot{\theta} - \varepsilon Y^T(q_d, \dot{q}_d, \ddot{q}_d) \tanh(\sigma \dot{q}) dt, \]
with $\varepsilon \in \mathbb{R}^n$, $A$ and $B$ are $n \times n$ diagonal positive definite matrices.

The estimated parameter vector $\dot{\theta}$ is computed through the update law
\[ \dot{\theta} = \Gamma_0 Y^T(q_d, \dot{q}_d, \ddot{q}_d) \dot{\theta} - \varepsilon Y^T(q_d, \dot{q}_d, \ddot{q}_d) \tanh(\sigma \dot{q}) dt, \]
with $\Gamma_0$ a diagonal positive definite matrix and $\varepsilon$ a positive constant suitably selected.

The system (28),(29),(30),(31),(1) is expressed by the closed loop equation:
\[
\begin{bmatrix}
\dot{\dot{q}} \\
\dot{\theta} \\
\dot{\theta}
\end{bmatrix}
=
\begin{bmatrix}
M(q)^{-1}[-C(q,\dot{q})\dot{q} - F_v \dot{q} - K_p \tanh(\dot{\theta})] \\
-\Gamma_0 Y(q_d, \dot{q}_d, \ddot{q}_d)^T [\dot{\theta} + \varepsilon \tanh(\sigma \dot{q})] \\
-\Gamma_0 Y(q_d, \dot{q}_d, \ddot{q}_d)^T [\dot{\theta} + \varepsilon \tanh(\sigma \dot{q})]
\end{bmatrix}
\]
where $h(\dot{q}, \ddot{q})$ is the so called residual dynamics defined in Property 5.

Define the constants
\[
\begin{align*}
\gamma_1 &= \frac{\delta_\alpha}{\tanh(\delta_\alpha)}, \\
\gamma_2 &= 2k_{cl} \mu_1 + \lambda_{\max} \{F_v\}, \\
\gamma_3 &= k_{cl} \sqrt{n} + \sigma \lambda_{\max} \{M(q)\} \lambda_{\max} \{F_v\}.
\end{align*}
\]
Assumption 3. Assume that the damping introduced by the viscous friction coefficients $F_v$ is large enough such that it satisfies
\[ \lambda_{\min} \{F_v\} > k_{cl} \mu_1. \]
Assumption 4. The matrix of proportional gains $K_p$ is large enough so that it achieves
\[ \lambda_{\min} \{K_p\} > \min \{\lambda_{\min}(A), \lambda_{\min}(B^{-1}A)\}, \]
Assumption 5. The constant $\varepsilon$ from the adaptive law (31) is selected such that it satisfies
\[
\gamma_2 \gamma_3 > \varepsilon < \gamma_1 \gamma_3.
\]

Our stability result on the origin of (32) is summarized in the following proposition.

Proposition 1. The origin $(\hat{q}, \hat{\dot{q}}, \hat{\dot{\theta}}) = 0$ of (32), under Assumptions 3, 4 and 5 is USG if and only if the matrix $Y(q_d(t), \dot{q}_d(t), \ddot{q}_d(t))^T$ is PE in the sense of Definition 1.

Proof. If we define:
\[
x_1 = \begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix}, \quad x_2 = \dot{\theta}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},
\]
then (32) can be expressed in the form (20) as follows:
\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix} &= \begin{bmatrix} M(q)^{-1}[-C(q,\dot{q})\dot{q} - F_v \dot{q} - K_p \tanh(\dot{\theta})] \\
-\Gamma_0 Y(q_d, \dot{q}_d, \ddot{q}_d)^T [\dot{\theta} + \varepsilon \tanh(\sigma \dot{q})]
\end{bmatrix} + \begin{bmatrix} 0 \\ M(q)^{-1} Y(q_d, \dot{q}_d, \ddot{q}_d) \dot{\theta}
\end{bmatrix}, \\
\frac{d}{dt} x_1 &= -\Gamma_0 Y(q_d, \dot{q}_d, \ddot{q}_d)^T [\dot{\theta} + \varepsilon \tanh(\sigma \dot{q})], \\
\frac{d}{dt} x_2 &= -\Gamma_0 Y(q_d, \dot{q}_d, \ddot{q}_d)^T [\dot{\theta} + \varepsilon \tanh(\sigma \dot{q})].
\end{align*}
\]
In order to prove UGAS of the origin of the system (39),(40) we will use Theorem 1. The first step is to check that Assumption 1 is satisfied. Consider the Lyapunov function:

\[ V(t,x) = \frac{1}{2} q^T M(q) \dot{q} + \sum_{i=1}^{n} k_i b_i^{-1} \ln(\cosh(\tilde{\theta}_i)) \]

\[ + \sum_{i=1}^{n} k_i \sigma_i^{-1} \ln(\cosh(\sigma_i \tilde{q}_i)) + \frac{1}{2} \theta^T \Gamma_a^{-1} \theta \]

+ \epsilon \text{tanh}(\sigma \tilde{q})^T M(q) \dot{q}. \]

By bounding each of the terms of \( V(t,x) \), upper and lower bounds are given by

\[ V_1(x) \leq V(t,x) \leq V_2(x), \quad (41) \]

where

\[ V_1(x) = \left[ \sum_{i=1}^{n} k_i \sigma_i^{-1} \ln(\cosh(\sigma_i \tilde{q}_i)) \right] P \]

\[ + \left[ \sum_{i=1}^{n} k_i \sigma_i^{-1} \ln(\cosh(\sigma_i \tilde{q}_i)) \right] \frac{1}{2} \theta^T \Gamma_a^{-1} \theta, \]

\[ V_2(x) = \frac{1}{2} \lambda_{\max}(M) \frac{\|q\|^2}{\|q\|} + \sum_{i=1}^{n} k_i b_i^{-1} \ln(\cosh(\tilde{\theta}_i)) \]

\[ + \frac{1}{2} \theta^T \Gamma_a^{-1} \theta, \]

\[ + \epsilon \lambda_{\max}(M) \|\text{tanh}(\tilde{q})\| \|\tilde{\theta}\|, \]

where

\[ P = \begin{bmatrix} \sigma_i^{-1} \lambda_{\min}(K) - \epsilon \sqrt{2} \lambda_{\max}(M) \\ \frac{1}{2} \lambda_{\min}(M) \end{bmatrix}. \quad (42) \]

\[ P \] is a symmetric positive definite matrix under Assumption 3. The time derivative of \( V(t,x) \) is given by

\[ \dot{V}(t,x) = \epsilon \text{tanh}(\sigma \tilde{q})^T (-F \tilde{q} - K_\theta \text{tanh}(\tilde{\theta})) \]

\[ + \epsilon \text{tanh}(\sigma \tilde{q})^T C(\sigma \tilde{q}, \tilde{q}) \text{Sech}^2(\sigma \tilde{q}) \tilde{q} - \epsilon \frac{1}{2} \theta^T \Gamma_a^{-1} \theta \]

\[ + \epsilon \text{tanh}(\sigma \tilde{q})^T M(q) \text{Sech}^2(\sigma \tilde{q}) \tilde{q} - \epsilon \tilde{q}^T F \tilde{q} \]

\[ + \epsilon \dot{q}^T h(\tilde{q}, \dot{\tilde{q}}) - \text{tanh}(\tilde{\theta})^T K_\theta B^{-1} \dot{\text{tanh}}(\tilde{\theta}). \quad (43) \]

The time derivative (43) may be upper bounded by

\[ \dot{V}(t,x) \leq -U(x_1), \quad (44) \]

where

\[ U(x_1) = \left[ \|\text{tanh}(\sigma \tilde{q})\| \right]^T \begin{bmatrix} Q_1 \|\text{tanh}(\sigma \tilde{q})\| \\ Q_2 \|\text{tanh}(\sigma \tilde{q})\| \end{bmatrix}, \]

and

\[ Q_1 = \begin{bmatrix} \frac{\epsilon}{2} \left[ \lambda_{\min}(K) - \gamma_1 \right] & -\frac{1}{2} \gamma_1 - \frac{1}{2} \gamma_2 \\ -\frac{1}{2} \gamma_1 - \frac{1}{2} \gamma_2 & \lambda_{\min}(F) - k \epsilon \mu_1 - \epsilon \gamma_3 \end{bmatrix}, \]

\[ Q_2 = \begin{bmatrix} \frac{\epsilon}{2} \left[ \lambda_{\min}(K) - \gamma_1 \right] & -\frac{\epsilon}{2} \lambda_{\max}(K) \\ -\frac{\epsilon}{2} \lambda_{\max}(K) & \lambda_{\min}(K_\theta B^{-1} A) \end{bmatrix}. \]

Under Assumption 5, \( Q_1 \) and \( Q_2 \) are positive definite matrices. Therefore, \( U(x_1) \) is a positive definite function, and Assumption 1 is satisfied.

We will now verify that Assumption 2 holds. The functions \( f_i \) for \( i = 1,2,3 \) from (39) and (40) are smooth and depend on \( t \) only through the reference trajectories and their derivatives which are assumed to be bounded for all \( t \). It is clear that

\[ f_0(t,x_2) = \begin{bmatrix} M(q_d) - Y(q_d, \dot{q}_d, \ddot{q}_d) \end{bmatrix}. \quad (45) \]

Then, by simple inspection of (45), and considering the boundedness of \( q_d(t) \) and their derivatives, we conclude that \( f_0 \) satisfies (24). By similar arguments, (26) holds (in particular, note that \( f_1 \) and \( f_2 \) are zero when \( x_1 = 0 \)). Finally, (25) is satisfied since \( f_2 \) and \( f_0 \) are linear in \( x_2 \); therefore \( \|f_2(t,x) - f_0(t,x_2)\| = \|P(t,x)\|_{x_2} \| \) has the form \( \|P(t,x_1)\|_{x_2} \| \), where \( P(t,x_1) \) is a smooth function uniformly bounded in \( t \) and which, moreover, is zero when \( x_1 = 0 \).

It only remains to show that \( f_0(t,x_2) \) is \( U \delta \)-PE with respect to \( x_2 \). Hence, we need to show that for each \( x_2 \neq 0 \) there exist \( \mu > 0 \) and \( T > 0 \) such that for all \( t \geq 0 \),

\[ \int_t^{t+T} x_2^T Y(q_d(\tau), \dot{q}_d(\tau), \ddot{q}_d(\tau))^T M(q_d(\tau))^{-1} x_2 d\tau \geq \mu. \quad (46) \]

Since \( M(q_d(\tau))^{-1} \) is full rank, then (46) holds if and only if the function \( \phi(t,x) = Y(q_d(t), \dot{q}_d(t), \ddot{q}_d(t))x_2 \) is \( U \delta \)-PE. Since \( \phi(t,x) \) is in the form (18), it is \( U \delta \)-PE if and only if \( Y(q_d(t), \dot{q}_d(t), \ddot{q}_d(t)) \) is PE in the sense of Definition 1. From Theorem 1, the origin of (32) is UGAS. Due to space constraints, simulation results are not presented in this paper. However, experimental results for the controller are presented in Moreno et al (2010).

4. CONCLUSIONS

In this paper, the adaptive output feedback tracking controller proposed in Moreno et al (2010) was revised.
Uniform global asymptotical stability of the controller was proved; as far as the authors know, this is the first proof of uniform global asymptotical stability of an adaptive output feedback tracking controller. The stability analysis was carried out via Lyapunov theory, complemented by a theorem proposed in Loria et al. (2002) on uniform global asymptotical stability of a certain type of nonlinear systems.

REFERENCES


