

# A Decentralized Connectivity Strategy for Mobile Router Swarms<sup>\*</sup>

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## Abstract:

In this paper we create a mesh of mobile robots that move in a decentralized fashion (swarming) in a two-dimensional space while maintaining communication constraints. The motion of the agents is dictated by four factors: i) a spring-mass interaction between agents; ii) a repulsive force from the obstacles; iii) an attractive force from the base station; and iv) an attractive field to regions with high probability to find users. Dijkstra's algorithm is implemented for optimal routing and a Bit Error Rate minimization is performed for communication optimization. The network is seen as a switched system in which virtual springs interactions create and delete the sensing links between agents. Stability analysis in the sense of Lyapunov is presented. Simulation results validate the applicability of the proposed method.

## 1. INTRODUCTION

In recent years, communication-aware motion planning has attracted considerable attention in the robotics community. Especially in multi-agent systems, the communication in the network becomes fundamentally important, due to the uncertainty of wireless channels. Homeland security, search and rescue operations, disaster relief operations (Fig. 1), and wireless surveillance networks are just a few examples of scenarios in which the use of robotics depends on stable and robust communication.

The coordination of the agents of a network can happen either in a centralized or a decentralized fashion. In our previous work, Bezzo and Fierro [2010], we presented a centralized framework based on *disjunctive programming* to tether a chain of mobile routers and keep a base station connected to a user that explores an unknown environment. The main drawback is that every decision is made by a central controller which makes the method computationally costly. This disadvantage disappears if we use a decentralized approach in which each mobile sensor makes decisions based on the information from its neighbors. However many other challenges rise when dealing with this type of systems because of lack of decision center.

On the other hand, radio communication is uncertain and characterized by randomness. Every environment is different and the presence of obstacles, both fixed and mobile, affects the quality of the propagated and received signal. There are several parameters to characterize communication links, such as the *Signal to Noise Ratio* (SNR), the *Capacity* of the channel, and the *Bit Error Rate* (BER), Goldsmith [2005].

In this paper we evaluate the same problem as in our previous work, Bezzo and Fierro [2010], which is motivated by the DARPA LANDroid program, McClure et al. [2009]. The goal is to maintain a base station connected with some users that move in an environment populated with obstacles. With this work we are able to have a distributed scenario in which all mobile routers make decisions based on their neighbors and communication constraints.

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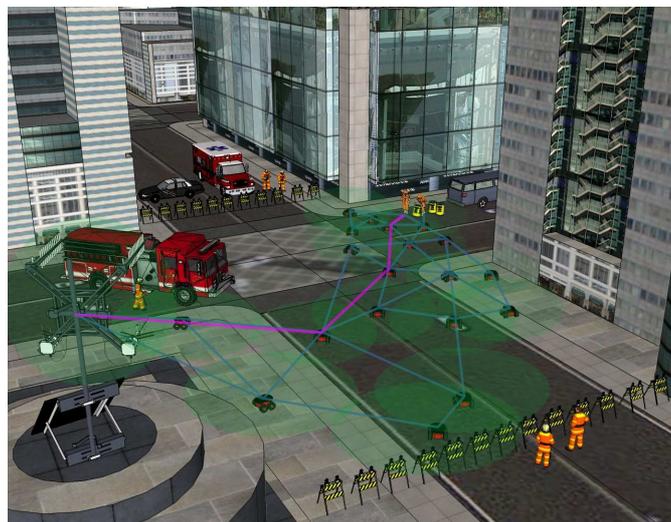


Fig. 1. A virtual environment with mobile routers. The lines between the robots represent sensed paths while the communication is depicted with circular patches.

### 1.1 Related Work

Multiple mobile robotic systems and wireless communications are not new topics and have been, individually studied for several years. Recently, however, roboticists people have recognized the need to consider realistic communication models when designing multi-robot systems, Mostofi [2009], Ghaffarkhah and Mostofi [2010], Lindhé and Johansson [2010], Fink and Kumar [2010], Gil et al. [2010].

Similar to our previous work, the authors in Tekdas et al. [2010] propose a geometric type approach to maintain a single user connected with a base station in a polygonal type environment with circular obstacles. In Mostofi et al. [2010] a probabilistic framework is proposed for online assessment of wireless link quality. Authors in Dixon and Frew [2009] use the notion of channel capacity to improve the communication quality established by a chain of robots in the presence of RF disturbance. In Yuan and

Mostofi [2010], a robotic router network is optimized in the sense of end-to-end bit error rate. A stop-time policy is proposed in Lindhé and Johansson [2010] to control a robot to adjust its speed based on wireless link quality. Authors in Gil et al. [2010] use aerial vehicles as relays to build the communication infrastructure for a team of ground vehicles by designing a gradient based controller that incorporates the *Signal to Interference Ratio* (SIR). Similar work in Griffin et al. [2010] shows that a group of UAVs, by exploiting antenna diversity, follows the gradient of the signal available in a certain environment.

Standing more from a control point of view, authors in Shucker et al. [2008] utilize a spring-mass system analogy together with graph theory in order to demonstrate stability of a swarm of robots that move in a decentralized fashion to cover an area while maintaining connectivity.

Our work is motivated by the scheme developed in Shucker et al. [2008]. However, we consider a connectivity problem using mobile routers. Specifically, we build a spring-mass system in which: 1) sensing among agents is represented by virtual links that are created and removed according to specific switching rules, and 2) communication constraints are taken into consideration by using a routing algorithm and BER position optimization, Yuan and Mostofi [2010].

The remainder of this paper is organized as follows. In Section 2 we present the robot, obstacle, user and base station models. In Section 3 there is a detailed description of the controller, communication constraints and stability analysis. In Section 4 we summarize the algorithm used to implement our framework. In Section 5 we show simulation results and finally, conclusions are drawn in Section 6 .

## 2. PRELIMINARIES

In this section, we present the theory and tools that are behind the implementation of the decentralized controller which is described in more details in Section 3.

### 2.1 Robot model

In our paper the mesh of robots is built following the spring-mass virtual physics. We base our model from the work in Shucker et al. [2008] where the robots are treated as point-mass particles. More specifically, the sensing capabilities are simulated by springs interactions among the agents of the network. This approach produces a uniform deployment of the robots in a closed environment.

The dynamics of the  $i^{\text{th}}$  spring-mass robot, with  $i \in \{1, \dots, \mathcal{N}_r\}$ , follows the model:

$$\ddot{X}_i = \left[ \sum_{j \in \mathcal{S}_i} k_{ij} (l_{ij} - l_{ij}^0) \hat{\mathbf{d}}_{ij} \right] - \gamma_i \dot{X}_i, \quad (1)$$

with  $i = 1, \dots, \mathcal{N}_r$  and  $i \neq j$ ,

where  $X_i = (x_i, y_i)^T$  is the position vector of the  $i^{\text{th}}$  router relative to a fixed Euclidean frame, and  $\dot{X}_i$ ,  $\ddot{X}_i$  denote the velocity and acceleration (control input), respectively.  $\mathcal{S}_i$  is the set of neighbor robots, users or base station connected to the  $i^{\text{th}}$  router. Since we are using a spring-mass model, these sensing links between agents are virtual springs.  $l_{ij}$  is the length of the spring between robot  $i$  and  $j$  and  $l_{ij}^0$  is the rest spring length;  $\hat{\mathbf{d}}_{ij}$  is the unit vector indicating the direction of the force of the virtual spring between the robots and finally  $k_{ij}$  and  $\gamma_i$  are the spring constant between robots  $i$  and  $j$  and the damping

coefficient, respectively. Here, we assume  $k_{ij} = k_{ji}$ ,  $l_{ij}^0 = l_{ji}^0$  and  $\gamma_i > 0$ .

Since we take into consideration damping effects, the mesh of virtual springs has similar behavior as a real spring system in which dissipative forces act against the movement of the springs ensuring velocity to eventually reach zero.

### 2.2 Obstacle model

In order to prevent collisions with obstacles, we will define a workspace repulsive potential field, Spong et al. [2006]. A common way to proceed is to build a potential  $W_{O,i}$  whose value approaches infinity as the robot approaches the obstacle, and goes to zero if the robot is at a distance greater than  $\rho_0$  from the obstacle. Formally:

$$W_{O,i} = \begin{cases} \frac{1}{2} \eta_i \left( \frac{1}{\rho(X_i)} - \frac{1}{\rho_0} \right)^2 & \text{if } \rho(X_i) \leq \rho_0 \\ 0 & \text{if } \rho(X_i) > \rho_0 \end{cases}, \quad (2)$$

with  $i = 1, \dots, \mathcal{N}_r$ ,

where  $\rho(X_i)$  is the shortest distance between  $X_i$  and any obstacle in the workspace and  $\eta_i$  is a positive constant.

Then, the repulsive force is equal to the negative gradient of  $W_{O,i}$ . If  $\rho(X_i) \leq \rho_0$ , it is given by:

$$F_{O,i} = \eta_i \left( \frac{1}{\rho(X_i)} - \frac{1}{\rho_0} \right) \frac{1}{\rho(X_i)^2} \nabla \rho(X_i), \quad (3)$$

where  $\nabla \rho(X_i)$  is the gradient of the minimum distance between the  $i^{\text{th}}$  robot and the closest obstacle. It is essential to note that the expression in (3) exists only if (2) is differentiable. We can always assume differentiability if we consider the obstacle covered by a set of disks, Bezzo and Fierro [2011].

### 2.3 User and Base Station Potential Functions

Let  $\sigma : \mathbb{R}^n \rightarrow \mathbb{R}$  be the base station attractive potential function. With a proper design of  $\sigma(x)$ , the mobile routers can be attracted to areas with good signal quality and repulsed from areas with poor signal quality. The case  $\sigma(x)$  constant represents an isotropic scenario or in other words an equipotential area in the environment, Gazi and Passino [2004].

We consider here a quadratic type of equation to prevent the mobile routers from moving too far from the base station. The quadratic equation is expressed as follows:

$$\sigma(X_i) = \frac{A_\sigma}{2} \|X_i - c_\sigma\|^2, \quad (4)$$

where  $A_\sigma \in \mathbb{R}^+$ , and  $c_\sigma \in \mathbb{R}^2$  represents the position of the base station. Its gradient is given by:

$$\nabla_{X_i} \sigma(X_i) = A_\sigma (X_i - c_\sigma). \quad (5)$$

Note that equation (4) can either have a maximum or a minimum depending on the sign of  $A_\sigma$ . However, in this paper we consider  $A_\sigma > 0$  because it represents a more realistic scenario where the maximum of the SNR is at the transmitter position and SNR decays, while staying positive, as we move away from the base station.

Following this line of thought, we assume that the routers know where the user may be *a priori*. Therefore we can

define an attractive potential towards a defined region of the workspace.

For simplicity sake, we use the following exponential weight function to represent the user attractive function:

$$\nu(X_i) = B_\nu - \frac{A_\nu}{2} e^{-\frac{\|X_i - c_\nu\|^2}{\ell_\nu}}, \quad (6)$$

where  $A_\nu \in \mathbb{R}^+$ ,  $\ell_\nu \in \mathbb{R}^+$ ,  $c_\nu \in \mathbb{R}^2$  and  $B_\nu \geq A_\nu/2$ . In this paper  $c_\nu$  is the center of the user attractive function and the term  $\ell_\nu$  controls the shape of the exponential function. The gradient of (6) becomes:

$$\nabla_{X_i} \nu(X_i) = \frac{A_\nu}{\ell_\nu} (X_i - c_\nu) e^{-\frac{\|X_i - c_\nu\|^2}{\ell_\nu}}, \quad (7)$$

where  $A_\nu > 0$  because we want an attractive field towards the center of the user function.

### 3. COORDINATION OF MOBILE ROUTERS VIA SWITCHED SYSTEMS

In this work we are interested in the problem of maintaining connectivity between a base station  $b$  and some users  $\mathcal{T}_j$  that move within an obstacle-populated environment. A swarm of mobile robots is available to help to achieve this goal by covering the area of interest while avoiding obstacles in a fully decentralized fashion.

We consider two different graphs as follows: a *visibility graph*,  $\mathcal{G}_v$ , coordinates the movements of the nodes by using the spring mass algorithm (1), while a *communication graph*,  $\mathcal{G}_c$ , coordinates the communication between the routers.

#### 3.1 Problem Formulation

Formally the swarming connectivity problem can be stated as follows:

**Problem 3.1. Swarming of mobile router networks:** Given a set  $\mathcal{M}$  of  $\mathcal{N}_r$  mobile routers  $X_i$  and users  $\mathcal{T}_j$  moving at a maximum speed  $\dot{\mathcal{T}}_j < \dot{X}_i$  within a specified area  $\mathcal{W}$  with an unknown trajectory  $g_{\mathcal{T}}(t) \in \mathbb{R}^2$ , find a set of feasible control policies  $\mathbf{u}_i$  for each router such that the network can provide efficient connectivity service between the base station  $b$  and the users.

Fig. 1 shows an example of the situation we are describing in problem 3.1 in which a swarm of robots covers an area populated with obstacles and at the same time keeps connectivity with a couple of users.

#### 3.2 Visibility graph

Depending on how the neighborhood  $\mathcal{S}_i$  is defined, the spring-mass model (1) builds meshes of robots with different behaviors. In order to have a complete coverage of the workspace, we add the following constraints.

1) Between any two nodes (robots, base station or user)  $i$  and  $j$ , we form a spring if and only if there is no robot  $k$  inside the circle of diameter  $\widehat{ij}$ , Bullo et al. [2009]. We define a variable  $\tau$  that is 1 if there is a spring between two robots and 0 vice versa. Formally the constraint is represented as follows:

$$\tau_{ij} = \begin{cases} 1 & \text{if } \widehat{ikj} \leq \pi/2 \\ 0 & \text{if } \widehat{ikj} > \pi/2 \end{cases}, \quad (8)$$

with  $i, j, k = 1, \dots, \mathcal{N}_r$  and  $i \neq j \neq k$ ,

where  $\widehat{ikj}$  is the interior angle of the three robots configuration.

2) We assume that in the environment there is one cluster of robots that is initially connected to the base station.

Also, the sensor installed in each robot has a visibility region, meaning that a spring-like connection is built between two nodes only if a router senses a target within its sensing range. We follow rule (8) and limit the sensing range to a threshold equal to  $\varphi$ .

In other words, we can express this constraint as follows:

$$\delta_{ij} = \begin{cases} 1 & \text{if } \|X_i - \mathcal{T}_j\| \leq \varphi \\ 0 & \text{if } \|X_i - \mathcal{T}_j\| > \varphi \end{cases}, \quad (9)$$

where  $\delta_{ij}$  is a binary variable that is equal to 1 if the  $i^{\text{th}}$  mobile router detects the  $j^{\text{th}}$  target.

As can be seen,  $\tau_{ij}$  and  $\delta_{ij}$  affect the value of  $\mathcal{S}_i$  in (1).

#### 3.3 Communication Graph

The graph in Section 3.2 creates problems if used also for communication purposes, because the visibility graph deletes links that could be used for communication.

Therefore we superimpose another graph on the top of the visibility graph built in the previous section, in which a wireless connection between any two nodes  $i$  and  $j$  exists if and only if  $X_j \in N_\xi(X_i)$ , where  $N_\xi(X_i)$  represents a neighborhood of  $X_i$  with radius  $\xi$ .

This information is used to find the shortest route between base station and user, and finally to optimize the BER.

#### 3.4 Routing and BER Optimization

Given the assumption above, we can apply the well known Dijkstra's algorithm to find the shortest-paths tree between any two nodes in the network, Bullo et al. [2009]. Specifically we are interested in finding the shortest route between base station and user, once the user is discovered in the visibility graph. This algorithm is computationally fast with worst case performance  $\mathcal{O}(|E| + |V| \log |V|)$ , where  $|E|$  is the total number of edges and  $|V|$  the total number of vertices of the communication graph.

Once a route between the base station and a user is found, we proceed with a BER optimization approach, proposed in Yuan and Mostofi [2010], in order to optimize the positions of the robots along that route. Next, we describe the basic framework of the approach.

Consider the path loss component of the wireless channel. Then the received SNR can be modeled as follows:

$$\lambda = \frac{\alpha}{l^n}, \quad (10)$$

where  $l$  is the distance separation between the robots, and  $\alpha$  is function of system parameters such as antenna gain, transmit power and frequency of operation. The parameter  $n$  is the path loss exponent, which depends on the environment (usually  $2 \sim 6$ ) Goldsmith [2005]. Assume that M-QAM modulation (which is a commonly used modulation type, Proakis and Salehi [1995]) is utilized in all the agents, then the BER can be characterized as follows:

$$P_b \approx 0.2 e^{-1.5 \frac{\lambda}{M-1}}, \quad (11)$$

where  $M$  is the modulation type.

Without loss of generality, we label all the agents (including user, routers and base station) from 1 to  $m$  in the order of the direction of the route, where  $m$  denotes total number of agents. Then the probability of correct reception  $P_c$  from the user to the base station can be approximated by the following expression:

$$P_c \approx \prod_{i=2}^m \left( 1 - 0.2 e^{-1.5 \frac{\lambda_{i-1,i}}{M-1}} \right), \quad (12)$$

where  $\lambda_{i-1,i}$  represents the received SNR from the  $(i-1)^{\text{th}}$  to  $i^{\text{th}}$  agent. The approximation of  $P_c$  is based on only considering correct receptions, *i.e.*, if a bit gets flipped a number of times, but it's correctly received at the end, we do not consider such a case as a correct reception. Since BER is usually very small (smaller than  $10^{-3}$ ), this approximation is very tight.

Based on (12), we have the following optimization framework to improve the performance:

$$\begin{aligned} & \text{maximize } \mathcal{J} = \sum_{i=2}^m \ln \left( 1 - 0.2 e^{-1.5 \frac{\lambda_{i-1,i}}{M-1}} \right) \\ & \text{subject to } x_i \in \mathcal{W}, \quad \forall i \in \{2, \dots, m-1\}, \end{aligned} \quad (13)$$

where  $\mathcal{W}$  represents the workspace. Notice that  $\lambda_{i-1,i} = \alpha_{i-1,i} / \|X_{i-1} - X_i\|^n$  and  $\mathcal{J} < 0$ . Then, we can optimize the performance by using the following external force:

$$u_i = \nabla_{X_i} \mathcal{J}. \quad (14)$$

### 3.5 Controller

The overall control law for each mobile router, given equation (1) for the spring interaction between agents, the obstacle avoidance expression (3), the attractive force from the base station (5), the attractive field from the user function (7) and the BER minimization for optimal route (14), becomes:

$$\begin{aligned} \ddot{X}_i = & \left[ \sum_{j \in \mathcal{S}_i} k_{ij} (l_{ij} - l_{ij}^0) \hat{\mathbf{d}}_{ij} \right] - \gamma_i \dot{X}_i \\ & + \eta_i \left( \frac{1}{\rho(X_i)} - \frac{1}{\rho_0} \right) \frac{1}{\rho(X_i)^2} \nabla \rho(X_i) \\ & - \nabla_{X_i} \sigma(X_i) - \nabla_{X_i} \nu(X_i) + \nabla_{X_i} \mathcal{J}. \end{aligned} \quad (15)$$

One interesting feature of using this type of control law is that the system is able to reconfigure and heal itself (for a detailed discussion, the reader is referred to Bezzo and Fierro [2011]).

### 3.6 Switched systems: Stability Analysis

The constraints introduced in Section 3.1 produce a connectivity graph that is often referred to *Gabriel Graph*, Shucker et al. [2008], Bullo et al. [2009]. While the formation of this graph has a lot of practical advantages, the continuous construction and destruction of springs create problems in terms of proving stability (Bhasin and Liu [2005], Hespanha and Morse [1999], Shucker et al. [2008]). In fact the spring-mass virtual system represents a switched system since there is a coupling between continuous dynamics and discrete events.

Following Shucker et al. [2008], by using an appropriate Lyapunov function and Barbalat's lemma, we first prove

static (*i.e.*, fixed graph topology) stability, then it is followed by the analysis of dynamic (*i.e.*, switching graph topology) stability.

*Theorem 1.* The virtual spring-mass system (15) with switching topology is stable (*i.e.*, it reaches a rest state with a constant potential).

**Proof.** *Static Stability.*

Let the total energy function of our system be:

$$\begin{aligned} V = & \sum_{i=1}^{N_r} \left[ \frac{1}{2} \sum_{j \in \mathcal{S}_i} P_{ij} + \frac{1}{2} \dot{X}_i^T \dot{X}_i + \frac{1}{2} \eta_i \left( \frac{1}{\rho(X_i)} - \frac{1}{\rho_0} \right)^2 \right] \\ & + \sum_{i=1}^{N_r} [\sigma(X_i) + \nu(X_i)] - \mathcal{J}, \end{aligned} \quad (16)$$

where  $P_{ij} = k_{ij} (l_{ij} - l_{ij}^0)^2$  and it is a conservative term.

By taking the first order derivative of the total energy and substituting for  $\ddot{X}$  with the expression in (15), it is easy to see that the derivative becomes:

$$\frac{dV}{dt} = - \sum_{i=1}^{N_r} \left( \gamma_i \dot{X}_i^T \dot{X}_i \right). \quad (17)$$

Since  $\gamma_i > 0$ , the right hand side of equation (17), which represents the total energy dissipated by damping, is clearly negative semi-definite.

Finally taking the derivative of (17), we obtain:

$$\frac{d^2V}{dt^2} = -2 \sum_{i=1}^{N_r} \left( \gamma_i \dot{X}_i^T \ddot{X}_i \right). \quad (18)$$

The expression in (18) is finite as long as the mobile routers speed and the difference  $(l_{ij} - l_{ij}^0)$  are finite.

Therefore, by Barbalat's lemma, the total control law in (15) guarantees that the system is stable in absence of switching, or if we consider the intervals between the switches, Shucker [2006].

*Dynamic Stability.* In order to prove dynamic stability for this type of systems, we use a similar argument of Shucker [2006] in which the authors introduce an *energy reserve*  $\Delta E$  to cancel the switching effects and prove stability.

Following this approach, we see that, in an interval of time  $\Delta t$  big enough to cover a switch between two topologies, the rate of variation of  $V$  is:

$$\frac{\Delta V}{\Delta t} = \sum_{i=1}^{N_r} \left[ \frac{1}{2} \sum_{j \in \Delta \mathcal{S}_i} k_{ij} (l_{ij} - l_{ij}^0)^2 - \gamma_i (\Delta^t X_i)^T (\Delta^t X_i) \right], \quad (19)$$

where the first term reflects the changes due to switching in the system and  $\Delta^t X_i = \frac{\Delta X_i}{\Delta t}$ .

We can build a new potential function  $V_{new} = V + E$ , where  $E$  is solution of the following equation:

$$\frac{\Delta E}{\Delta t} = \frac{1}{2} \sum_{i=1}^{N_r} \left[ \gamma_i (\Delta^t X_i)^T (\Delta^t X_i) - \sum_{j \in \Delta \mathcal{S}_i} k_{ij} (l_{ij} - l_{ij}^0)^2 \right], \quad (20)$$

which depends on the changes in  $\mathcal{S}$ .

Taking the expressions (19) and (20) back to differential forms and substituting into the first derivative of  $V_{new}$ , we obtain the following expression:

$$\frac{dV_{new}}{dt} = \frac{dV}{dt} + \frac{dE}{dt} = -\frac{1}{2} \sum_{i=1}^{N_r} (\gamma_i \dot{X}_i^T \dot{X}_i), \quad (21)$$

which is negative semi-definite as desired. Finally by differentiating (21), we obtain:

$$\frac{d^2V_{new}}{dt^2} = - \sum_{i=1}^{N_r} (\gamma_i \ddot{X}_i^T \dot{X}_i). \quad (22)$$

Therefore the new function has the following properties:

- $V_{new}$  is positive-definite;
- $\dot{V}_{new}$  is negative semi-definite;
- $\ddot{V}_{new}$  is bounded and finite.

By Barbalat's lemma, we conclude that the system with the new energy function is also dynamically stable, Bezzo and Fierro [2011].

### 3.7 Decentralized Deployment of Mobile Routers

By assembling together everything we described in the previous sections, the overall problem is decentralized. Every robot, by sensing its neighbors, computes the algorithm in (15), subject to:

- the constraint on the angle (8);
- the constraint on the distance (9);
- the bounds on the input:

$$0 \leq |\ddot{X}_i| \leq U_{max};$$

and

- the bounds on the maximum velocity:

$$0 \leq |\dot{X}_i| \leq V_{max}.$$

Once the user is found, then:

- the Dijkstra's algorithm is run to find the shortest route, and
- the BER optimization (13) is performed over the specific route.

Because of the movement of the user and the routers, the optimal route between base station and user may change from time to time. However we can always use our algorithm to update the route and thus the BER position optimization framework.

Together with these constraints, we also have to make sure that the robots are not in singular positions, for example, perfectly aligned with one another. We also need to make sure that the number of robots are enough to cover the entire workspace, Bezzo and Fierro [2011].

## 4. ALGORITHM DESCRIPTION

Algorithm 1 illustrates how the simulation scenario has been implemented and summarizes the main events which occur during the simulation.

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### Algorithm 1 Swarming of Mobile Robot Networks.

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1: for all Robots  $i$  do
2:   while  $l_{ij} \neq l_{ij}^0$  do
3:     Check for singularities
4:     Compute the algorithm (15) subject to the con-
       straints (8) and (9)
5:     if a spring link is created between any robot and
       a target then
6:       Find the shortest route between base station
       and user (Dijkstra's algorithm)
7:       Run the BER position optimization (13)
8:     end if
9:     return  $\ddot{X}_i$ ,  $\dot{X}_i$  and  $X_i$ 
10:  end while
11: end for
    
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As demonstrated in Shucker [2006] the computational complexity for each robot is  $\mathcal{O}(N_s^2)$ , where  $N_s$  is the number of spring-links connected to each robot.

The algorithm stops when the users reach the final positions and the spring-mass system reaches equilibrium.

## 5. SIMULATION RESULTS

Fig. 2 shows a general simulation scenario with a cluster of 20 robots inside a convex environment with a rectangular obstacle in the middle. A base station is located inside the environment at the top right corner. The network of mobile routers knows a priori the locations where the probability to find a user is higher. In this simulation, we assume one user moving following a circular trajectory.

The mobile routers have communication capabilities depicted with concentric circles around them (Fig. 2) and they disperse in the given workspace following the controller described in Section 3.5.

The goal of the network is to explore the environment, find the user, build a connectivity network between the base station and the user, while avoiding obstacles.

The network of mobile routers evolves from the first snapshot in Fig. 2(a), converging to the area where the user operates (Fig. 2(c)). Once the user is discovered and the optimal route between user and base station is built, then the BER optimization (13) is run to optimize the positions of the routers (Fig. 2(d)).

As we can see from the diagram in Fig. 3, once there is a route between user and base station the BER optimization algorithm (13) drive the three robots of the route to the positions that minimize the BER.

## 6. CONCLUSIONS

In this work we have presented a framework based on spring-mass virtual environment, artificial attractive and repulsive fields, and BER optimization. The springs between base station, robots and users represent interactive sensing links while the artificial fields guide the mobile routers to specific locations in the environment. The algorithm developed is decentralized since each mobile router makes decisions based on the surrounding neighbors and the informations from the environment. Gabriel Graph is used to guarantee a uniform deployment of the robots in the workspace.

Stability of the proposed switched system is presented for a case with a simple obstacle. Simulation results in

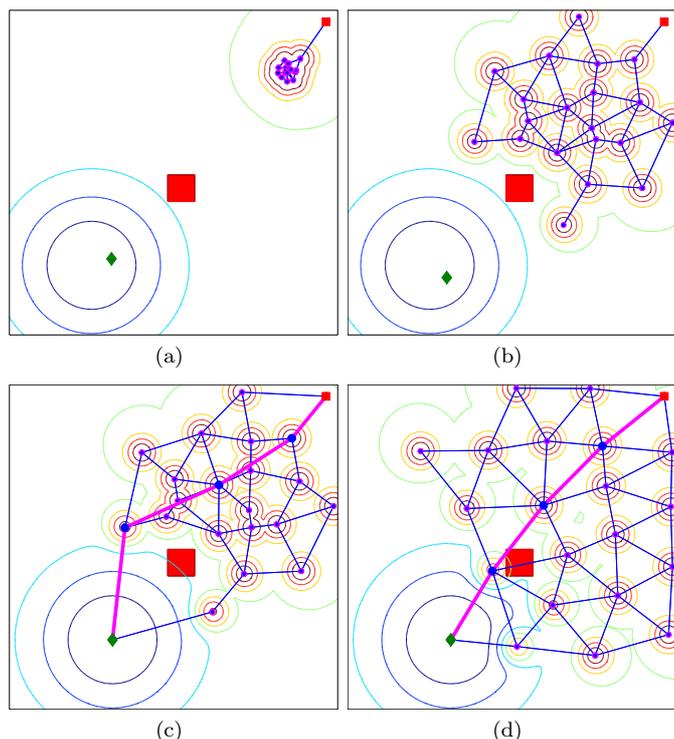


Fig. 2. Mobile sensors maintaining connectivity between a moving user (diamond) and a base station (square on the top right).

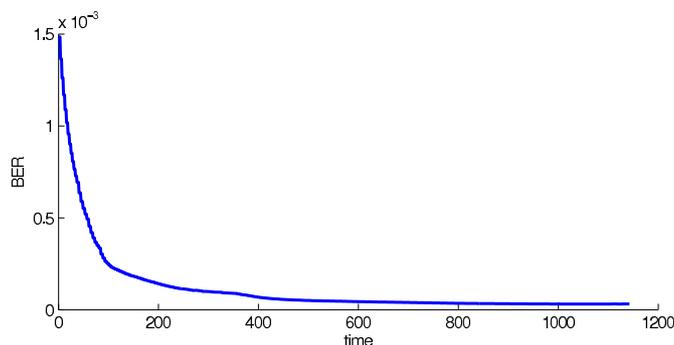


Fig. 3. BER between the base station and the user.

Section 5 demonstrate the applicability of the technique for scenarios in which the mesh of routers is able to stretch and keep the user connected with a base station while avoiding obstacles.

Future work will be centered on extending the proposed methodology in fading environments. Testing the algorithm with experiments in our laboratory, Cruz et al. [2007], is also in our agenda.

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