Cooperative Allocation and Guidance for Air Defence Application

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Abstract: This project proposes a centralized algorithm to design cooperative allocation strategies and guidance laws for air defense applications. Scenarios in naval and ground context have been defined for performance analysis by comparison to a benchmark target allocation policy. The cooperative target allocation algorithm is based on the following features: No Escape Zones (differential game NEZ) computation to characterize the defending missile capturability characteristics; In Flight (re) Allocation (IFA algorithm, late committal guidance) capability to deal with target priority management and pop up threats; capability to generate and counter alternative target assumptions based on concurrent beliefs of future target behaviors, i.e. Salvo Enhanced No Escape Zone (SENEZ) algorithm. The target trajectory generation has been performed using goal oriented trajectory extrapolation techniques. The target allocation procedure is based on minimax strategy computation in matrix games.

Keywords: Game Theory, Differential Games, Minimax Techniques, Guidance Systems, Co-operative Control, Prediction Methods, Missiles

1. Introduction

This research programme has focused on the problem of naval-based air defence systems which must defend against attacks from multiple targets. Technology developments in the field of modular data links may allow the creation of a multi-link communication network to be established between anti-air missiles and the launch platform. The future prospect of such ad hoc networks makes it possible to consider cooperative strategies for missile guidance. Many existing guidance schemes are developed on the basis of one-on-one engagements which are then optimized for many-on-many scenarios ([Jang 2005], [Ge 2006]). A priori allocation rules and natural missile dispersion can allow a salvo of missiles to engage a swarm of targets; however, this does not always avoid some targets leaking through the salvo, whilst other targets may experience overkill.

Cooperative guidance combines a number of guidance technology strands and these have been studied as part of the research programme underline.

- Prediction of the target behaviour;
- A mid-course guidance to place the missile in position to acquire and engage the target;
- Allocation / re-allocation processes based on estimated target behaviour and no escape zones;
- Terminal homing guidance to achieve an intercept.

In the terminal phase, guidance has been achieved by handover to the LDG guidance law [Shinar 2002]. Two approaches to missile allocation have been considered [Shin 2010a]. This article focus on the second one exploiting the no escape zones (NEZ, [Isaacs 1967]) defined by a linear differential game (LDG) guidance law which either acts to define an allocation before launch (ABL) plan or refine an earlier plan to produce an in-flight allocation (IFA) plan.

A statement of the problem is given in section 2 SENEZ Concept. In section 6 Matrix Game Target Allocation Algorithm details of pre-flight and in-flight allocation planning are described. Missile guidance, both mid-course and terminal, is discussed in section 7 Guidance Logics. The simulation results from a Simulink 6DOF model are reviewed in section 9 SENEZ Results. Finally, in section 10 Conclusion and 11 SENEZ Perspective, there are the study conclusions and some remarks concerning the exploitation of these cooperative guidance technologies.

2. SENEZ Concept

There are occasions when the weapon system policy for defending against threats involves firing two or more missiles at the same target. Without any action taken, the missiles will naturally disperse en-route to the target, each arriving at the point of homing with a slightly different geometry. In such a case, there will be a significant overlap of the NEZ. A salvo enhanced no escape zone (SENEZ) was introduced to optimized this type of engagement, with the cooperating missiles increasing their chances of at least one missile intercepting the target.

Figure 1: Multi shoot example in SENEZ firing policy
In the naval or ground application, it is often the case that a number of assets may be situated in close vicinity to each other. In this situation, it may be difficult to predict which asset an inbound threat is targeting. In the case of air-to-air engagements, there are various break manoeuvres which a target aircraft could execute to avoid an interceptor. These paths can be partitioned into a small number of bundles determined by the number of missiles in the salvo.

By selecting well chosen geometric paths it should be possible to direct the defending missiles in such a way that each partition of the possible target trajectory bundles falls within the no escape zone (NEZ) of at least one missile. Consider a naval case of a two missile salvo, and a threat that is initially heading straight towards the launch vessel; there is a possibility that the threat may break left or right at some point. One defending missile can be directed to cover the break right and straight-on possibilities; the second missile would defend against the break left and straight-on possibilities. By guiding to bundle partitions prior to the start of homing, the NEZ of the firing is enhanced. At least one of the missiles will be able to intercept the target. This SENEZ firing policy differs from the more standard ‘shoot-look-shoot’ policy which considers the sequential firing of missiles where a kill assessment is performed before firing each new missile launch.

### 3. Goal Oriented Target Prediction

Different approaches have been studied to predict target positions [Shin 2011]. Results detailed in the following are based on the version implementing the goal oriented approach; which is based on the hypothesis that the target will guide to a goal.

The target trajectories have been classified into three categories: threats coming from the left (with respect to the objective), from the front and from the right (Figure 2).

![Figure 2: Trajectory classification using three classes](image)

We generate these three assumption target trajectories defining one waypoint per trajectory class. We compute the trajectories that lead to the threat object passing by the waypoints using Trajectory Shaping Guidance (TSG) [Zarchan 2007]. The basic TSG is similar to PN (Proportional Navigation) with a constraint on the final line-of-sight (LOS) angle in addition. This means that near impact, the LOS angle \( \lambda \) equals a desired value \( \lambda_F \). A 3D version of this law is applied from the threat’s initial position to the waypoint. When the waypoint is reached, a switch is made from TSG to standard PN to guide on the objective. The LOS final angle of the TSG law is chosen to bring the threat aiming directly at the objective when it reaches the waypoint. Figure 3 illustrates how assumption target trajectories have been generated.

![Figure 3: 2D target trajectory generation using waypoints, TSG and PN as terminal homing guidance](image)

A set of three waypoints per target is defined using polar parameters (angle \( \Psi_{wpt} \) and radius \( R_{wpt} \)). All waypoints belong to a circle of radius \( R_{wpt} \) centered on the supposed objective. Waypoints are then spread with \( \Psi_{wpt} \) as an angular gap, using the initial objective-threat line as a symmetry line defined at RADAR detection. In this way there is one trajectory per hypothesis, as seen in the Figure 4.

![Figure 4: Waypoints geometry for target trajectory generation](image)

Waypoints are defined for each target depending on its position at the time it is detected. To avoid a high disturbance of the defending missiles guidance, it is assumed that these waypoints do not change as the engagement evolves.

Some hypotheses will become progressively less likely to be true and others appear to be a good approximation of reality. In due course, some hypotheses will become unachievable and will be discarded during the cost computation process.

![Figure 5: Evolution of the engagement; waypoints do not move; waypoints trajectories become impossible](image)

### 4. Predicted NEZ

The SENEZ target allocation algorithm is in charge of evaluating all missile-target-hypothesis engagements ([Le
This means the algorithm must be able to tell for each case if successful interceptions are possible and to give a cost on a scale that enables comparisons.

Usage of the following letters is now reserved:

- $W$ is the number of waypoints considered;
- $N$ is the number of defensive missile that can be allocated to a target (i.e. that are not already locked on a target, or destroyed);
- $P$ is the number of active and detected threats.

We will now use the following notation to name engagements (i.e. guidance hypotheses):

$$M_i T_j H_k$$

This means we are talking of the engagement of Missile $i$ $(1 \leq i \leq N)$ against Target $j$ $(1 \leq j \leq P)$, assuming it is behaving as described by Hypothesis $k$ $(1 \leq k \leq W)$.

Another useful notation is on the other hand:

$$T_j H_k$$

This is used to name what the target does (in this case target $j$ is following hypothesis $k$). Based on the assumption that the target and missile may guide in three different ways $H_1$, $H_2$ and $H_3$, a three by three matrix leading to nine costs can be presented in the following manner (Table 1).

<table>
<thead>
<tr>
<th>Guidance Hypothesis</th>
<th>What the target does</th>
<th>Cost 1</th>
<th>Cost 2</th>
<th>Cost 3</th>
<th>Cost 4</th>
<th>Cost 5</th>
<th>Cost 6</th>
<th>Cost 7</th>
<th>Cost 8</th>
<th>Cost 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_i T_j H_k$</td>
<td>$T_j H_1$</td>
<td>$c_{11}$</td>
<td>$c_{12}$</td>
<td>$c_{13}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_j H_2$</td>
<td>$c_{21}$</td>
<td>$c_{22}$</td>
<td>$c_{23}$</td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>$T_j H_3$</td>
<td>$c_{31}$</td>
<td>$c_{32}$</td>
<td>$c_{33}$</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 1: Costs over target trajectory alternatives and defending missile beliefs

As the number of missiles and targets increase, the size of the matrix will grow accordingly.

5. Cost Computations

Costs are computed through trajectory extrapolations and NEZ considerations. Target trajectories are extrapolated as explained previously using TSG and PN. Missile trajectories are extrapolated using PN guidance on a Predicted Interception Point (PIP); however, other mid-course guidance laws such as DGGL [Shin 2010a] can be considered. Coordinates of this point are computed using the time to go:

$$t_p = \frac{R_{MT}}{V_c}$$

Where $R_{MT}$ is the missile-target distance and $V_c$ the closing velocity. Then, for any time $t$ of the trajectory:

$$XYZ_{pip}(t) = XYZ_{t}(t + t_p)$$

Where $XYZ_t$ are the target coordinates, in inertial frame. The PIP is assumed to have both its velocity and acceleration equal to zero. For every sample time of the target’s trajectory, the PIP coordinates are calculated, then the PN command of the missile and finally integrating this command generates the missile states at next sample time. For initial extrapolations, i.e. when missiles are not already in flight, it is assumed that their velocity vector is aimed directly at the waypoint of the hypothesis chosen. This is also used in the model when actually shooting missiles. PN on PIP objective makes use of the assumed knowledge of the target’s behaviour and allows the SENEZ target allocation algorithm to launch several defending missiles against the same “real” threat following different mid course paths. The SENEZ principle is indeed to shot multiple missiles to anticipate target’s behaviour such as doglegs, and new target detections. Once missile trajectories have been computed, the costs are evaluated. The NEZ concept is applied as well as a modelling of the field of view of the missile’s seeker. Two zones are defined; the first zone determines if a target can be locked by the seeker (information); the second zone determines if the target can be intercepted (attainability). The cost is simply the relative time when the target enters the intersection of both zones. If it never happens, the cost value is infinite. If the threat is already in both zones at the first sample time, the cost is zero.

When guiding on a hypothesis such as $M1T1H1$, it is supposed that the seeker always “looks” at the predicted position of threat $T1$, hypothesis $H1$. This gives at every sample time the aiming direction of the seeker. This seeker direction is tested against all other hypotheses to check if a target is within the field of view at this sample time. If positive, an interception test using the NEZ evaluates whether interception is possible. As soon as a target enters the field of view and becomes reachable for a hypothesis, the cost is updated to the trajectory’s current time. The cost computation concludes when all costs, i.e. of all hypotheses, have been computed, or when the last trajectory sample has been reached.

This cost logic has been chosen because of the following:

- It takes into account what the missile can or cannot lock on (seeker cone).
- It takes into account the missile’s ability to reach the threats (NEZ).
- In most cases, it can be assumed that low costs imply short interception times.

6. Matrix Game Target Allocation Algorithm

After costs have been computed, the algorithm has to find the best possible allocation plan. This means we need to construct allocation plans and combine costs. Consider the following illustrative example. One threat $T_1$ attacks one objective, with three possible hypotheses $H_1$, $H_2$ and $H_3$. Two missiles $M_1$ and $M_2$ are allocated to this target. First, it is necessary to determine the possible combinations, excluding options where the two missiles cover the same target hypothesis. We also compute the cost matrix of each missile as described in the previous sections 4 Predicted NEZ / 5 Cost Computation. Remember that low cost values imply hopefully early interceptions. Infinite values mean interception is not possible.

Using combinations of min max operators we construct the whole problem’s cost matrix (Table 2) and advice the best one (mini max game equilibrium, [Basar 1982]). The best allocation plan $(C^{*}_{i,j})$ is the plan that minimizes the cost value whatever is the target trajectory.

$$\min_{i,j}(C_{i,j}) = \min_{i,j}\left\{\max_{k}\left\{\min\left\{C_{M,T^iH_j,T/H_i}, C_{M,T^jH_j/T/H_i}\right\}\right\}\right\}$$
Where \(i, j\) are target waypoint beliefs defining the defending missile strategies (mid course trajectories) and \(k\) is the waypoint number defining the threat strategies (trajectories).

The two diagrams Figure 6 and Figure 7 summarize the defending guidance phases (mid-course, homing phase) and explain how the Simulink Common Model operates.

![Figure 6](image1.png)

**Figure 6:** During mid course, the guidance logics block extrapolates targets states and PIP coordinates. It also determines if the seeker locks on one of the targets.

![Figure 7](image2.png)

**Figure 7:** In homing phase, “guidance logics” sends true states of locked target. The seeker block applies noises for measurement computation. Kalman filter estimates target’s states. Finally a DGL1 [Shima 2003] command is applied.

8. Scenario description

Several scenarios for air defence in the ground and naval context have been defined. A target allocation benchmark policy, with neither re-allocation, nor SENEZ features, has been defined for comparison purpose. Scenario 3 (Figure 8) deals with ground defence where Air Defence Units (ADUs) are located around (red circle) the objective to be protected (blue diamond). A threat aircraft launches a single missile and then escapes the radar zone. The aircraft and missile are supersonic.

![Figure 8](image3.png)

**Figure 8:** Benchmark trajectories in scenario 3

The benchmark policy consists in launching a defending missile as soon as a threat appears in the radar detection range. The benchmark algorithm starts by launching one missile on the merged target. When both
targets split, a second missile is shot. This second defending missile will intercept the attacking missile. Due to the sharp escape manoeuvre of the aircraft the first defending missile misses the aircraft. After missing the aircraft, the benchmark algorithm launches a third missile to chase the escaping aircraft. This last missile never reaches its target.

9. SENEZ Results

When the aircraft crosses the RADAR range the SENEZ algorithm launches two defending missiles (Figure 9). In ground scenarios, several ADUs are considered, the algorithm automatically deciding by geometric considerations which ADU to use when launching defending missiles. For simplicity, in naval and ground scenarios only one location is considered as the final target goal (ground objective to protect, blue diamond). Simple waypoints are used to generate target trajectory assumptions, even if it is possible to extend the concept to more sophisticated target trajectory assumptions.

Figure 9 explains what happens when using the SENEZ algorithm and what the improvements with respect to the benchmark policy are. The defending missiles are in green and in cyan colours. The aircraft trajectory is in the red line turning on the right side. The magenta line is the trajectory of the missile launched by the aircraft. The defending missiles intercept when the threat trajectories switch from plain to dot lines. The dot lines describe what happens when using the benchmark policy in place of the SENEZ algorithm. The dot lines in black are the target trajectory assumptions continuously refined during the engagement. A straight line assumption was considered by the algorithm, however defended missiles assigned to the right and to the left threats are enough to cover the three waypoint assumptions elaborated when the initial threat appears. The SENEZ algorithm intercepts the attacking missile at longer distance than the benchmark algorithm, around a 1km improvement. Moreover, SENEZ only launches two defending missiles and also intercepts the launching aircraft which the benchmark algorithm fails to do. The fact that SENEZ directs missiles to the left and right sides, plus the fact that SENEZ launches earlier than the benchmark explains the SENEZ performance improvement.

Figure 9: SENEZ target allocation algorithm on Scenario 3

Monte-Carlo runs have been executed for all the scenarios, comparing interception times obtained with the benchmark model to those obtained with the SENEZ. Disturbances for these runs were as follow:

- Seeker noise;
- Initial position of the targets (disturbance with standard deviation equal to 50m);
- Initial Euler angles of the target (disturbance with standard deviation equal to 2.5°).

Performance analyses have also been executed on various other scenarios for ground and naval applications contexts. Moreover, parametric studies have been conducted on the following aspects:

- Waypoint placements;
- The drag coefficient of the defensive missiles;
- The radius and range of the seekers;
- Plus some variations on the scenario definition as time of appearance of the second target in scenario 3.

Attention is also paid to finding waypoint placements that would be convenient for all ground to air scenarios, or all surface-to-air scenarios. The optimal placement of the waypoints highly depends on the scenario. This tends to prove there would be an advantage in increasing the number of waypoints / missiles corresponding to an increased number of SENEZ hypotheses.

Potential benefits were first illustrated on all the scenarios considered against targets performing highly demanding evasive manoeuvres as well as apparent single targets that resolve into two splitting targets. The trajectories obtained gave a better idea of the SENEZ behaviour. However, the way that target hypotheses are issued proved to be critical. This has been demonstrated by the parametric studies as placement of the waypoints changed greatly the results from one scenario to another. The sensitivity to parameters such as drag and seeker features has also been investigated. Results obtained during these parametric studies seem to show the initial number of waypoints / hypotheses per target chosen three might be too low.

Statistical studies have also been conducted. While providing improved performances in terms of time of last interception in most cases, the standard deviation greatly increased in some scenarios due to misses among the first salvos. These misses may be due to the simplified Kalman estimator used in our model, to the choice of the mid-course guidance chain compared with one-on-one. This project has developed various component technologies supporting the concept of cooperative guidance.

For the terminal phase, differential game guidance laws were applied where the no escape zone was used to characterize the ability of the missile to capture the target. The focus of this article is concentrated on the way in which some of these technologies are combined to provide an
enhanced capability when salvos are launched to deal with target threats, the SENEZ concept. Allocation algorithms have been extended to consider the future possible behaviour of the target; the technique can determine how many missiles to fire and provide the initialization for the missiles in the salvo.

Initial results have demonstrated the potential of the SENEZ concept where in some cases this technique has produced results that were better than the baseline allocation algorithm. Although the potential has been demonstrated it remains to examine the full robustness of the approach in terms of range of scenarios and optimization of parameter setting.

11. SENEZ Perspectives

SENEZ guidance attempts to embed the future possible target behaviour into the guidance strategy by using goal oriented predictions of partitioned threat trajectories to drive missile allocation and guidance commands. As such the SENEZ approach offers an alternative to mid-course guidance schemes which guide the intercepting missile or missiles towards a weighted track. The general application of SENEZ would lead to a major change in weapon $C^2$ philosophy for naval applications which may not be justifiable.

The SENEZ engagement plan requires that a missile be fired at each partitioned set of trajectories. This is different from many existing naval firing policies which would fire a single missile to the target at long range and would delay firing another missile until later when, if there were sufficient time, a kill assessment would be undertaken before firing a second round. Depending on the evolution of target behaviour, current $C^2$ algorithms may fire a second missile before the potential interception by the first missile. So existing systems tend to follow a more sequential approach, the naval platform needing to preserve missile stocks so that salvo firings are limited; unlike air platform the naval platform cannot withdraw rapidly from an engagement. The proposed engagement plan is purely geometric in formation as opposed to current schemes which use probabilities that the target is making for a particular goal. This latter type of engagement plan will generally result in fewer missiles being launched. In the SENEZ scheme, a missile salvo will be fired more often because the potential target trajectories are all equally likely. For instance, when the target is at long range, it is likely that its choice of asset to attack is equiprobable, whereas at the inner range boundary, it is most likely that the target is straight-flying towards its intended target.

Despite these potentially negative assessments of the SENEZ concept, there will be occasions when current $C^2$ algorithms will determine that it is necessary to launch a salvo against a particular threat. For instance, a particularly high value asset such as an aircraft carrier may be targeted and a high probability of successful interception is required. In such circumstances there could be merit in the SENEZ approach. Essentially, in the naval setting SENEZ may be considered as a possible enhancement for the salvo firing determined by the engagement planning function in existing $C^2$ systems.

For air-to-air systems the scope for considering a SENEZ form of guidance may be greater. It is often policy for aircraft to fire two missiles at an opposing aircraft engaged at medium range. With a two aircraft patrol, the leader and the wing aircraft will each fire a missile at the target, there is an opportunity to shape the guidance so that possible break manoeuvres are covered. With separate platforms firing the missiles it would be necessary for inter-platform communication so that each missile could be allocated to a unique trajectory partition.

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