New design of robust industrial accumulators for elastic webs

David KUHM∗,∗∗∗ Dominique KNITTEL∗,∗∗

∗ Web Handling Research Group, University of Strasbourg, 17, rue du Maréchal Lefebvre, 67100 Strasbourg, France (Corresponding author: knittel@unistra.fr).
** Laboratoire de Génie de la Conception INSA Strasbourg, 24, boulevard de la Victoire, 67084 Strasbourg, France
*** Laboratoire de Physique et Mécanique Textiles ENSISA, 11, rue Alfred Werner, 68093 Mulhouse, France

Abstract: This paper concerns the modelling of an accumulator used in an industrial elastic web processing plant (paper, fabric, polymer, metal ...). Accumulators are used to allow unwind or rewind roll changes while the rest of the line remains at a constant web velocity. Detailed nonlinear models of industrial accumulators are first given. The first one is a pneumatic actuated accumulator, the second one is motor actuated. These models are derived from the physical relationships that describe web tension and velocity dynamics in each web span of the accumulator. In a second part, control strategies are detailed. Industrial PI controllers, tuned with evolutionary algorithm on our realistic non-linear model, are presented, allowing good robustness against mechanical parameter variations. In this part the motor actuated accumulator performances are enhanced by including a dancer.

Keywords: Roll to roll systems, Accumulator, Modelling, Control, Optimization, Simulation.

1. INTRODUCTION

In many plants, having a continuous process line increases the productivity. This process line continuity can be performed by an accumulator. Accumulators (Fig. 1) are used to store web in the regular production phase. An accumulator in composed of a lot of free rollers. The carriage moves to store or restore the desired web length. This carriage can be moved by different type of actuators such as hydraulic systems, pneumatic systems or motors. Most of the production time, the carriage is maintained at its nominal position and the web just follows the path made by the free rollers. Just before the unwind roller is empty, the carriage is moved up in order to increase the web length stored in the accumulator. When the unwind roller is empty, the unwinder is stopped to change the roll. During this production step, the web is released by moving down the accumulator carriage in order to maintain a constant web velocity in the processing section, this allows nonstop production. After the unwind roller change, the carriage is moved back to its nominal position. The wound roll changing phase has to be as fast as possible and is limited by the accumulated web length and velocity. It is easy to control the web tensions inside an accumulator during the regular production phase (when the carriage remains at its nominal position) but in the transient phases (during the accumulator carriage motion), web tensions variations often occurs (due to inertia and friction of the free rollers) and can generate web folds or breaks. Different works have been published in web tension control of roll-to-roll systems. For example Zalian and Jones (1995), Brandenburg (1973), Benlatreche et al. (2008), Knittel et al. (2006, 2003), Gassmann et al. (2009), Giannoccaro et al. (2010) present the modelling and control of a continuous processing line composed only with tractors and free rollers, whereas Knittel et al. (2004) describe the unwinding and the winding process. Only few results are published in modelling or control of accumulators. These results can be found in Koç et al. (1999a,b), Pagilla et al. (2001, 2003), Kühm et al. (2009) and Knittel and Kühm (2010). The next part of this paper presents the detailed physically based models of accumulators including web span weight and pneumatic cylinder representation. Two different accumulators are presented: a pneumatic actuated and a motor actuated one. Finally, the last part of this contribution presents the accumulator performances analysis and highlights the effect of the Young’s modulus of the web. This part also presents control schemes and simulation results for both accumulators. The motor actuated accumulator performances are enhanced by including a dancer.
2. ACCUMULATOR MODELLING

2.1 Nonlinear model of the web longitudinal dynamics

Equations describing web tension behavior between two consecutive rollers and the velocity of each roll enable to build the nonlinear model of a web transport system (Koç (2000), Koç et al. (2002)). Our accumulator is modeled by a succession of web span tension and velocity between each roll and by the accumulator carriage motion.

Web tension calculation including web weight (Fig. 2) represents the web span between two consecutive rollers.

Web velocity calculation

Web tension calculation including web weight (Fig. 2) represents the web span between two consecutive rollers. Assuming no sliding between web and rollers, the equation of continuity applied to a web transport system between two consecutive rollers is given by relation (1). This relation enables to calculate the web span strain $\varepsilon_k^w$ induced by web strains and velocities.

$$\frac{d}{dt} \left( \frac{L_k}{1+\varepsilon_k^w} \right) = -\frac{V_{k+1}}{1+\varepsilon_k^w} + \frac{V_k}{1+\varepsilon_{k-1}}$$

where $L_k$ is the web length between the $k^{th}$ and the $(k+1)^{th}$ rollers. It can vary because of accumulator carriage motion. $V_k$ is the velocity of the $k^{th}$ roll and $\varepsilon_k^w$ represents the strain of the $k^{th}$ web span induces by the web strain and velocity (between the $k^{th}$ and the $(k+1)^{th}$ rollers).

The strain $\varepsilon_k^w$ in the web span resulting from the gravity is obtained from equation (2):

$$\varepsilon_k^w = \frac{1}{L_k} \int_0^{L_k} \frac{\rho g L_k}{2E} \, dL = \frac{\rho g L_k}{2E}$$

The equivalent strain $\varepsilon_k$ in the web span is then deduced by combining equations (1) and (3):

$$\varepsilon_k = \varepsilon_k^w + \varepsilon_k^w$$

Web tension $T_k$ is related to web strain $\varepsilon_k$ by the Hooke’s law.

Web velocity calculation

Assuming no sliding between web and rollers, the web velocity $V_k$ is equal to the peripheral velocity of the roll. The velocity dynamics of the $k^{th}$ roll, is calculated by applying the torque balance on it:

$$\frac{d}{dt} (J_k \Omega_k) = C_{mk} - C_{rk} - C_{fk}$$

where $\Omega_k = \frac{V_k}{R_k}$ is the angular velocity of the $k^{th}$ roll, $J_k$ is roll inertia and $R_k$ is roll radius. $C_{mk}$ is motor torque for a driven roll. $C_{rk}$ stands for the torque introduced by the web and $C_{fk}$ is the friction torque between the roll and its shaft. Equations (4) and (5) are used and adapted for each web span of the studied accumulator.

2.2 Nonlinear model of the accumulator carriage motion

Two accumulator carriage actuators are often used in industry. This part presents the nonlinear model of a pneumatic actuator, including static frictions and also presents the nonlinear model of an accumulator carriage actuated by an electric motor.

Displacement of the accumulator carriage

In the phase of wound roll change, the output speed $V_{out \, accu}$ is maintained at nominal speed $V_{ref}$ while the input speed $V_{in \, accu}$ decreases to 0. In order to maintain the output web tension and velocity constant during this change, it is necessary to move adequately the accumulator carriage. In our study, the accumulator is composed of $n+1$ rollers and therefore has $n$ web spans. The accumulator carriage displacement speed reference, during the wound roll change, is given by relation (6). This speed reference enables to find the reference carriage position ($X_{pref}$ on Fig. 7 or $L_{ref}$ on Fig. 8).

$$V_{accu} = \frac{1}{n} (V_{in \, accu} - V_{out \, accu})$$

Unwinding speed during the accumulation phase

During the web accumulation phase, the unwinder speed has to be increased to keep a constant web tension in the accumulator. The unwinding speed reference is deduced from the equation (6):

$$V_{in \, accu} = n V_{accu} + V_{out \, accu}$$

Pneumatic jack representation

Accumulator motion can be ensured by a pneumatic jack (Fig. 3). Some works have been done in modelling hydraulic systems or in modelling pneumatic systems. Examples can be found in Hamiti et al. (1996), Zhihong and Bone (2008). Models of hydraulic systems including valves and cylinder can be found in Jelali and Kroll (2002). These representations have been adapted to our pneumatic jack. In our air cylinder, only chamber A is connected to the valve, chamber B is directly connected to a high flow outlet. Pressure in chamber B is thus supposed equal to the tank pressure.
Pressure/flow equation for a valve. A valve comport lot of non linearities such as dead band, saturations, hysteresis, friction forces. The air flow across the valve is represented by relations (8) and (9). (Jelali and Kroll (2002))

\[ Q_A = C_{v1}\cdot sign(x_v)\cdot\text{sign}(P_S - P_A) \cdot \sqrt{|P_S - P_A|} - C_{v2}\cdot sign(-x_v)\cdot\text{sign}(P_A - P_T) \cdot \sqrt{|P_A - P_T|} \]  

\[ Q_B = C_{v3}\cdot sign(-x_v)\cdot\text{sign}(P_S - P_B) \cdot \sqrt{|P_S - P_B|} - C_{v4}\cdot sign(x_v)\cdot\text{sign}(P_B - P_T) \cdot \sqrt{|P_B - P_T|} \]  

where \( x_v \) determines air flow direction. \( C_{v_i} \) are the valve orifice coefficients. 

Dynamic representation of the servo-valve. The dynamic of a servo valve is given by (Jelali and Kroll (2002)): 

\[ \frac{1}{\omega_v^2} \ddot{x}_v + 2D_x \dot{x}_v + x_v + f_{hs}\cdot sign(\dot{x}_v) = K_v\cdot u_v \]  

where \( x_v \) denotes the valve position, \( K_v \) is the valve gain, \( w_v \) is the valve natural frequency, \( D_x \) is the valve damping coefficient and \( f_{hs} \) is the valve hysteresis. These coefficients are often given by valves manufacturers.

Pressure dynamics in cylinder chamber. The air cylinder also presents non lineairities such as pressure dependent bulk modulus, saturations, friction forces. Air flow inside each cylinder chamber is represented according to relations (13) and (14) (Jelali and Kroll (2002)):

\[ Q_A = Q_{Li} = \dot{V}_A + \frac{V_A}{E'(P_A)} \dot{P}_A \]  

\[ Q_B = Q_{Li} = \dot{V}_B + \frac{V_B}{E'(P_B)} \dot{P}_B \]  

\[ V_A = V_{P1A} + \left( \frac{S}{2} + x_p \right) A_P \]  

\[ V_B = V_{P1B} + \left( \frac{S}{2} - x_p \right) A_P \]  

\[ \dot{V}_A = A_P \dot{x}_p \]  

\[ \dot{V}_B = -\alpha A_P \dot{x}_p \]  

with \( V_A \) and \( V_B \) the volume of each chamber, including the valve connecting line, \( Q_{Li} \) represents the internal flow leakage between the two chambers, \( V_{P1A} \) and \( V_{P1B} \) are the pipeline volumes, \( S \) is the piston stroke, \( A_P \) is the piston surface, \( \alpha \) is the ratio between chamber A and chamber B piston surface. \( E'(P_A) \) and \( E'(P_B) \) are the effective bulk modulus depending on the pressure. \( \dot{x}_p \), the piston position, allows to deduce the web span length. Finally from relations (13) to (18) the pressure dynamics into each cylinder chamber is then given by:

\[ \dot{P}_A = E'(P_A) \left( Q_A - A_P \dot{x}_p - Q_{Li} \right) \]  

\[ \dot{P}_B = E'(P_B) \left( Q_B + \alpha A_P \dot{x}_p + Q_{Li} \right) \]  

Piston motion and friction representation. In order to provide the piston motion dynamics, a special attention has to be paid to static friction force \( F_s \). A novel approach is used in this work to include it properly in the simulator. Static friction sign depends obviously on the sign of the piston velocity. But the key point that has to be handled is when piston velocity is zero. One has to make sure that the piston remains immobile until the force generated by pressure on the piston surface is sufficient to exceed the static friction force. Theses conditions are summarized in (22). From Fig. 3, the equation of the piston motion is then deduced from the second Newton law applied to the piston.

\[ \begin{cases} m_t \ddot{x}_p = (P_A - \alpha P_B) A_P - F_{web} - F_{fs}(\dot{x}_p) - F_{fs} - m_t g & \text{if } B_1 \text{ or } B_2 \text{ satisfied} \\ m_t \ddot{x}_p = (P_A - \alpha P_B) A_P - F_{web} - F_{fs}(\dot{x}_p) + F_{fs} & \text{if } B_3 \text{ or } B_4 \text{ satisfied} \\ m_t \ddot{x}_p = 0 & \text{else} \end{cases} \]  

where \( F_{web} \) is the force applied to the carriage by the web, \( F_{fs} \) is carriage dynamic friction force, \( F_{fs} \) is carriage static friction force, \( m_t \) is carriage mass and \( g \) is gravity. Friction of the cylinder piston are represented using a combination of dynamic and static friction. The sign of the static friction force and the dead band are obtained whether the condition \( B_1 \) to \( B_4 \) given in (22) are satisfied or not.

\[ \begin{cases} B_1 : PS - F_{web} - m_t g > F_{fs} \\ B_2 : \dot{x}_p > \varepsilon x_p \\ B_3 : PS - F_{web} - m_t g < -F_{fs} \\ B_4 : \dot{x}_p < -\varepsilon x_p \end{cases} \]  

Jack pressure regulation. In order to achieve the expected force that has to be applied on the carriage, the pneumatic jack uses a pressure regulation. The control scheme is illustrated on Fig. 3. The force reference is deduced from the expected nominal web tension desired in the accumulator and has to compensate the carriage weight.

\[ T_{ref} = nT_{web \ ref} + m_t g \]  

Motor actuated accumulator including a dancer. Instead of using a pneumatic jack in order to move the accumulator carriage, in some accumulator, a motor can be used. A representation of the motor actuator is shown in Fig. 5. The carriage is maintained at its nominal position thanks to a brake during the regular production phase. During the motion phase of the carriage, the brake is deactivated and the motor moves adequately the accumulator carriage. At the opposite of the pneumatic accumulator, due to the presence of the brake, there is no device to attenuate web tension variations into the motor actuated accumulator. A novel approach is to include a pneumatic dancer into the motor actuated accumulator to attenuate web tension variations (Preliminary work has been done in Koç (2000)). An example of a motor actuated accumulator including a dancer is given in Fig. 4. The dancer jack can be modelled with from relations (8) to (22). The force applied by the
Fig. 4. Example of a motor actuated accumulator including a pneumatic dancer

dancer is deduced from relation (23). The equation of the carriage motion is deduced by applying a torque balance on the carriage motor (24). The carriage position \( x_c \) is related to \( w_m \) by the radius of the motor/chain gear \( R_m \). Friction forces are represented in the same way as for the pneumatic jack.

\[
J_m w_m = u_{dm} k_{dm} - R_m (F_{web} + F_{fv}(\dot{x}) \pm F_{fs} + m g)
\]

with \( J_m \) is the motor inertia, \( u_{dm} \) is the reference voltage, \( k_{dm} \) is the ratio from reference voltage to torque, \( F_{web} \) is the force applied to the carriage by the web and \( F_{fv} \) is the motor dynamic friction force. The accumulator carriage motor is position controlled in order to move the carriage to the desired position. The control scheme is given in Fig. 5.

Fig. 5. Generic motor representation including position control

2.3 State space representation of the accumulator

The linear model is deduced from the nonlinear one, and can be described by a state space representation (25). The states are the tension and velocity in each web span. The linear model is obtained by linearizing an accumulator including an “ideal” carriage actuator (neglecting static friction and dead band). The linear model is useful for Bode diagrams calculation. The state space model has tree inputs \( u + v \) and one output \( T_{out} \) the accumulator output tension. The input \( u \) is the carriage displacement velocity, \( V_d \) is the unwinder speed.

\[
\begin{align*}
X &= [T_d \ V_a \ T_a \ V_{a1} \ T_{a1} \ldots \ V_{n+1} \ T_{n+1}]^T \\
v &= [V_d] \quad u = \frac{dL}{dt}
\end{align*}
\]

3. ACCUMULATOR PERFORMANCES ANALYSIS AND CONTROL

3.1 Influence of the mechanical parameters

Depending on the type of accumulator, mainly two control strategies are applied: one using \( V_{in\ accu} \) as control signal (controller output), the other using \( L \). Using this control strategies let appears two transfer functions that can be studied in the accumulator: a first one between the accumulator output tension \( T_{out} \) and the accumulator input velocity \( V_{in\ accu} \) named \( W_1 \) and a second one between \( T_{out} \) and the web span length \( L \) named \( W_2 \). In the following part, the influence of the Young’s modulus variations around it’s nominal value is analyzed on simulated Bode diagrams.

Influence of elasticity modulus

Web elasticity influences the web dynamics (tension and velocity) in the transient phases. One can observe in Fig. 6 that the static gain and resonances (gains and frequencies) are depending on the Young’s modulus. The same observation have been made both transfer functions \( W_1 \) and \( W_2 \). Very often in the industry, Young’s modulus changes occur during the manufacturing process. Therefore the accumulator performances have to be guaranteed in spite of web elasticity variations. Consequently, the controllers have to be adjusted/calculated for each value of web elasticity, or robust for a given web elasticity range.

Fig. 6. Bode diagram of \( W_2 \) for different Young’s modulus

**Remarks** Other parameters variations can affect the accumulator performances (Kuhm et al. (2009)). Web span length in the accumulator has a significant influence on the web dynamics. Free rollers inertia also influences the web dynamics.

3.2 Control strategy

Control structure

As mentioned previously in this paper, the accumulator can be controlled in two different ways: the controller can operate either on the web span length or the input velocity. An output web tension controller can be synthesized by using the input velocity of the accumulator as control signal. The second control strategy uses the web length (by moving the accumulator carriage) as control signal. In industrial applications, both control schemes are used.
Pneumatic actuated accumulator  For the pneumatic actuated accumulator, we use the input velocity of the accumulator as control signal. During the regular production phase, we use the input velocity as actuator and during the wound roll change there is no control, the web just moves the carriage by itself. The on/off continuous switching strategy of the controller is performed by weighting the controller output with a coefficient decreasing from 1 to 0. The control scheme is given in Fig. 7.

Motor actuated accumulator  For the motor actuated accumulator, we used a combination of the two strategies. During the regular production phase, we use the input velocity as actuator and during the wound roll change we use the web length as actuator. The switching strategy between the two controllers is performed by weighting the controllers output by a coefficient between 0 and 1. The control scheme is given in Fig. 8.

Controller optimization methods  The PI controllers are optimized using a multi-objective evolutionary algorithm. Evolutionary algorithm are used due to a lot of nonlinearities in our system. The controllers also have to be calculated for a time-varying web length. A first cost function, $J_1$ uses the ITAE criterion to ensure a good reference tracking. The second cost function $J_2$ has the following form:

$$J_2 = \theta_E - \theta_{E+\Delta E}$$

(27)

where $\theta_i$ is the IAE criterion applied to the shell of the system error signal for a Young’s modulus equal to $i$. Using the shell of the error signal instead of the error signal is a new method to compare performances if parameters variations occurs. It focus only on the amplitude of the tension variations, regardless the number and frequency of the oscillations. $J_2$ allows to find more robust controllers. The third and fourth cost function, $J_3$ and $J_4$, are the minimum and the maximum of the error signal and allow to limit high tension pikes.

3.3 Simulation results

One can observe on Fig. 9 that optimized PI controller leads to good carriage position and web tension regulation performances.

Fig. 7. Pneumatic actuated accumulator control scheme

Fig. 8. Motor actuated accumulator control scheme

Fig. 9. Pneumatic actuated accumulator simulation results

Using these regulation schemes (Fig. 3 and Fig. 7) allows to ensure moderate tension variation during the wound roll change phase (4, 5 and 6), despite large Young’s modulus variations.

Accumulator simulation sequence:

1. The accumulator is at its nominal web length, velocity and tension.
2. The accumulator carriage moves up to reach the maximum web length and the input velocity is increased to maintain nominal web tension.
3. The accumulator is charged and maintained at maximum web length.
4. Input velocity decreases and the accumulator carriage is moved down to restore web and maintain nominal web tension.
5. Input velocity is equal to zero, the accumulator carriage is moved down to restore web and maintain nominal web tension.
6. Input velocity increases to reach the nominal velocity, the accumulator carriage is moved down to restore web and maintain the nominal web tension.
7. The accumulator is at its nominal web velocity and tension, the accumulator is maintained at a constant position.

Fig. 10 presents simulation results for a motor actuated accumulator. The simulation sequence is the same as for the pneumatic one. As observed, there are more tension variations as for the pneumatic accumulator. These tension variations are due to the fact that in this accumulator there
Fig. 10. Motor actuated accumulator simulation results is no device to absorb tension variations. As mentioned in a precedent part, a dancer is included to the accumulator to attenuate tension variations. As expected, including a dancer into the motor actuated accumulator highly reduce the accumulator output web tension variations during the transitions phases. A performances comparison with or without a dancer during the accumulator starting phase is given on Fig. 11.

Fig. 11. Effect of a dancer in a motor actuated accumulator

4. CONCLUSION

This paper presents the physical equations used in the modelling of an industrial accumulator including a complete model of the air cylinder. The detailed nonlinear model has been programmed in Matlab/Simulink software environment. Two different accumulators are presented: a pneumatic actuated and a motor actuated one. The use of optimized PI controllers gives good performances even if Young’s modulus variations occurs. As expected, including a dancer into a motor actuated accumulator highly reduces web tension variations.

ACKNOWLEDGEMENT

The authors wish to thank Region Alsace for having partly funded this research and the Momenatic France company for their very helpful discussions.

REFERENCES


