Coordination of Mobile Agents in Flocks with Time-varying Interactions

Shanying Zhu, Cailian Chen and Xinping Guan

Abstract: In this paper, we develop a novel self-propelled particle model to describe the emergent behavior of a group of mobile agents. All agents coordinate with their neighbors through local social forces accounting for velocity alignment and collision avoidance. We allow the interaction ranges to adapt to the group density. This range adaptation results in topology changes as well as discontinuities in those social forces. We apply differential inclusion technique to analyze the convergence properties of the proposed control method. The analytical and numerical results show that all the agents eventually align their velocities and avoid collisions with each other no matter how fast the topology changes.

Keywords: Co-operative control, discontinuities, time-varying systems

1. INTRODUCTION

The collective motion of animals such as fishes, birds and insects are examples of large scale self-organization observed in nature. In many cases, cohesive groups are formed, hundreds or thousands of agents move together in the same direction acting as a whole producing various astonishing aggregate patterns, say, lines and V-like formations. In order to reveal mechanisms of the complex emergent behaviors, a lot of so-called self-propelled particle models have been proposed and analyzed (Vicsek et al. (1995); Couzin et al. (2002); Mogilner et al. (2003); Jadbabaie et al. (2003); Olfati-Saber (2006); Cucker and Smale (2007)). The basic idea behind these models is that complex collective behavior arises from simple interactions among neighboring agents. Three widely adopted simple rules include repulsion, attraction and alignment (Reynolds (1987); Couzin et al. (2002)), i.e., while they avoid collision with each other, agents at the same time are attracted to others against dispersing too widely and align to others up to an intermediate distance. More recently, collective behavior and self-organization have also attracted attention from engineers with the aim of controlling mobile robots in the context of cooperative control and formation control and so on (Cortés et al. (2004); Dimarogonas and Johansson (2008)).

Most of the proposed models rely on the apriori assumption that agents interact with all those in a certain metric distance. This means that the interaction range of each agent is constant in spite of the group density. This assumption is reasonable for engineering applications of mobile robots equipped with on-board wireless sensors, since radio signal powers decay with distance and reliable wireless communication can be ensured only within a certain threshold. However, from the biological point of view, it is generically not the case. In a recent field study Ballerini et al. (2008a), the authors show that interaction is ruled by topological distance rather than metric distance. Each agent adapts its interaction range according to the local density. It reduces when the local density is high, and increases while it is low. The interaction range adaption to the local density is also adopted by Hemelrijk and Hildenbrandt (2008).

Despite considerable research interests in collective behavior modeling either numerical or empirical, few results on rigorous mathematical analysis of these models with topological interaction range are seen in open publications. This is partly due to the nonlinear nature of the biological interactions. In recent work Cucker and Smale (2007) and Ha and Liu (2009), stability analysis of a group of mobile agents with all-to-all interactions was analyzed. However, this scenario might not be realistic. Since birds or fishes unlikely sense those behind their neighbors, the phenomena of all-to-all interactions are rare in biological systems. Extension of the above model to account for collision avoidance can be found in Cucker and Dong (2010). Motivated by Cucker and Smale (2007), Shen (2007) extended the results to the case of agents under hierarchical leadership. The agents are ordered into different ranks. Lower-rank agents are only led by some agents of higher ranks, which makes the interaction more local. However, the author still considered time-invariant interactions. In nature, the bird flocks and fish schools often change their connection patterns in order to avoid predators and intercept strange intruders. In these scenarios, the time-varying interactions are more reasonable. In Tanner et al. (2007) and Zavlanos et al. (2010), flocking of mobile agents with time-varying
topology were considered. And stability analysis were examined in terms of differential inclusions and switched systems, respectively. However, the authors assumed that the interaction ranges remained constant during the system evolution, which might not be reasonable in natural systems as pointed out previously.

In this paper, we focus on the mathematical analysis of one particle model of mobile agents detailed in Section 2 with adaptive interaction range, while ensuring collision avoidance. The main contribution of this paper is to propose a novel self-propelled particle model with time-varying interactions. The convergence analysis of the proposed model is also provided by using theory of differential inclusions. We emphasize that in this paper no constraints are imposed on the switching signal. And the interaction range might not be constant during the evolution of the group, it is a function of time (see the simulation part, Section 4).

2. PROBLEM FORMULATION

Consider a group of $N$ mobile agents with the dynamics of each agent described by a double integrator

\[
\begin{align*}
\dot{x}_i(t) &= \hat{v}_i(t), \\
\dot{v}_i(t) &= u_i(t), \quad i = 1, 2, \ldots, N,
\end{align*}
\]  

(1)

where $x_i, \hat{v}_i \in \mathbb{R}^n$ are the position and velocity of agent $i$, respectively, and $u_i \in \mathbb{R}^n$ is the acceleration.

In designing our self-propelled particle model, we incorporate the following two rules to govern the inter-agent interactions: (i) Alignment of Velocities. Each agent aligns to others within a density-dependent interaction range; (ii) Short-range Repulsion. Agents avoid others that are close by. In the starling flocks, the empirical investigation argues that birds tend not to get closer than a certain minimum distance, and they are surrounded by a hard sphere Ballerini et al. (2008b). In light of these considerations, it is straightforward to model interaction network as an undirected dynamic graph $G(t) = (\mathcal{V}, \mathcal{E}(t))$ where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of vertices representing the agents and $\mathcal{E}(t) \subset \mathcal{V} \times \mathcal{V}$ is the set of time-varying edges referring to the interaction links between agents $i$ and $j$, satisfying the following conditions:

1. if $\|\hat{x}_{ij}(t)\| \leq r$, then $(i, j) \in \mathcal{E}(t)$;
2. if $(i, j) \notin \mathcal{E}(t)$ and $r_i < \|\hat{x}_{ij}(t)\| \leq R(t)$, then $(i, j)$ is a new link to be added to $\mathcal{E}(t)$;
3. if $\|\hat{x}_{ij}(t)\| > R(t)$, then $(i, j) \notin \mathcal{E}(t)$;

where $\hat{x}_{ij}(t) \triangleq \hat{x}_i(t) - \hat{x}_j(t)$ and $R(t) > 0$ are the radii of interaction range and hard sphere of agents, respectively. Notice that the interaction range $R(t)$ changes with time, it is not constant as in Olfati-Saber et al. (2006); Tanner et al. (2007) and Zavlanos et al. (2010). Any agents that can interact with each other directly at time $t$ are called neighbors. Therefore, the set of neighbors of agent $i$ is defined as $\mathcal{N}_i(t) = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}(t), j \neq i\}$. Note that the neighbors of each agent are time-varying. If the information flows between any two vertices via a sequence of different vertices, then we say that graph $G(t)$ is connected.

Numerical and empirical investigations support the idea that the behavior of agents results from local coordination based upon the relative positions and velocities with each other. As far as velocity alignment is concerned, different from Couzin et al. (2002, 2005), here the information flow suffers from path loss. And the amplitude of information decreases with distance between any two agents $i$ and $j$ according to the rule

\[ a_{ij}(\hat{x}) = \frac{H}{(1 + \|\hat{x}_{ij}\|^2)^\eta}, \quad H > 0, \tag{2} \]

where $\eta > 0$ is the path loss coefficient, which resembles radio-wave propagation loss in wireless channels Barbarossa and Scutari (2007). In this way, agent $i$ attempt to align its velocity with its neighbors determined by the following force

\[ \psi_i = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(\hat{x}) \frac{\hat{v}_{ji}}{\|\hat{v}_{ji}\|}, \]

where $\hat{v}_{ij} \triangleq \hat{v}_i - \hat{v}_j$. If no neighbors are sensed or the velocity equals to that of its neighbors, then the agent keeps its current velocity. Therefore, $\psi_i$ can be written into

\[ \psi_i = \sum_{j \in \mathcal{N}_i(t)} a_{ij}(\hat{x}) \text{SGN}(\hat{v}_{ji}), \]

where

\[ \text{SGN}(\hat{v}_{ji}) = \begin{cases} \frac{\hat{v}_{ji}}{\|\hat{v}_{ji}\|}, & \hat{v}_{ji} \neq 0, \\ 0, & \hat{v}_{ji} = 0. \end{cases} \]

As for collision avoidance, agent $i$ perceives a force $\phi_i$ to move away from too close neighbors in a hard sphere with radius $r$. In this hard sphere, the repulsion force increases as the distance between agent $i$ and its neighbors decreases and tends to infinity when the distance is zero. While outside this hard sphere, the repulsion force vanishes. One of such repulsion forces is given by

\[ \phi_i = \sum_{j \in \mathcal{N}_i(t)} \phi_{ij}(\hat{x}) \hat{x}_{ji}, \]

where

\[ \phi_{ij}(\hat{x}) = \begin{cases} \frac{1}{\|\hat{x}_{ij}\|^2} + \frac{1}{r(2\theta + 1)} g(\hat{x}_{ij}), & 0 < \|\hat{x}_{ij}\| \leq r, \\ 0, & \|\hat{x}_{ij}\| > r. \end{cases} \tag{3} \]

in which $\theta > 1$ is a constant and $g(\hat{x}_{ij}) = \theta(\|\hat{x}_{ij}\|^2 - r^2) - r^2$.

Thus, the acceleration $\hat{u}_i$ implemented on agent $i$ is the sum of these two forces

\[ \hat{u}_i = \psi_i + \phi_i. \tag{4} \]

3. CONVERGENCE ANALYSIS OF FLOCKING WITH TIME-VARYING INTERACTIONS

Since the acceleration $\hat{u}_i$ in (4) depends not only on the time-varying $\mathcal{N}_i(t)$, but also on the velocity differences between agent $i$ and its neighbors. The presence of topology switching and SGN($\hat{v}_{ji}$) will introduce discontinuities to the right hand side of (1). Let the network $G(t)$ switch at time instants $t_1, t_2, \ldots$, and define a switching signal $\sigma(t) : [0, \infty) \to \mathcal{P}$, where $\mathcal{P}$ is a finite index set for all different topologies generated by $N$ agents. Here, unlike Jadbabae et al. (2003); Olfati-Saber and Murray (2004) and Zavlanos et al. (2010), we do not impose any hypothesis on the switching signal $\sigma(t)$. Denote the set of discontinuous
points of \( u_i \) by \( S \). Then \( S \) consists of two parts \( S_1 \) and \( S_2 \), where \( S_1 \) is the set of discontinuous points due to topological switching and \( S_2 = \bigcup_{i=1}^{N} \bigcup_{j \in N_i} \{ (t, x) : \dot{x}_i = \dot{v}_i \} \) due to \( \text{SGN}(v_{ji}) \). For dynamic system (1) with acceleration (4), the classical notion of solution is not applicable any more. Note that \( S \) has measure zero. Thus Filippov solution is an appropriate choice and is widely used in the literature Filippov (1988); Paden and Sastry (1987) and Shevitz and Paden (1994).

In order to study the stability of dynamic system (1) and (4), it is convenient to employ a new coordinate system and investigate the moving frame as suggested in Olfati-Saber (2006). Let us consider the center of mass of the group

\[
x_c = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i, \quad v_c = \frac{1}{N} \sum_{i=1}^{N} \dot{v}_i.
\]

Since (1) with (4) is translational invariant, the fluctuations around the center of mass \( x_i = \dot{x}_i - x_c, \dot{v}_i = \dot{v}_i - v_c \) satisfy the same form of dynamics as (1)

\[
\begin{align*}
\dot{x}_c(t) &= v_c(t), \\
\dot{v}_c(t) &= u_c(t),
\end{align*}
\]

where the acceleration (4) now can be expressed as

\[
u_i(t) = \sum_{j \in N_i} a_{ij}(x) \text{SGN}(v_{ji}(t)) + \sum_{j \in N_i} \phi_{ij}(x)v_{ji}(t).\]

By stacking the position and velocity vectors, we can express (5) in terms of differential inclusions as follows

\[
\begin{align*}
\dot{x}(t) &= v(t), \\
\dot{v}(t) &\in K |u|(t),
\end{align*}
\]

where \( u \) is the stack vector of all accelerations \( u_i, \) \( i = 1, 2, \ldots, N, \) and \( K \) is the Filippov set-valued map Filippov (1988). We do not make any assumption on the uniqueness of solutions to (7). In the following, the stability analysis of (7) will be investigated by using theory of differential inclusions Filippov (1988) and nonsmooth analysis Clarke (1990).

Before moving onto the main result, we first give some lemmas that will be useful in the sequel.

**Lemma 1.** For any \( x \in \Omega = \{ x \in \mathbb{R}^N : \sum_{i=1}^{N} x_i = 0 \} \), denote \( \| x \|_{\Omega} = \sqrt{\sum_{i=1}^{N} \sum_{j \in N_i} \| x_{ji} \|^2} \). If the interaction graph formed by the neighboring sets \( N_i, i = 1, 2, \ldots, N \) is connected, then we have

\[
\nu \cdot |x| \leq \| x \|_{\Omega} \leq \mu \| x \|,
\]

where constants \( \nu > 0, 0 < \mu \leq \sqrt{2N} \).

**Proof.** The proof is omitted due to space limitation. \( \square \)

Denote \( \mathcal{D} = \{ x \in \mathbb{R}^N : \| x_{ji} \| > 0, \forall (i, j) \in \mathcal{E} \} \) and define a Lyapunov-like function \( V : \mathcal{D} \times \mathbb{R}^N \to [0, \infty) \) by

\[
V(x, v) = \| v \|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i} \int_0^\infty \phi_{ij}(s) |x_{ji}|^2 ds.
\]

Note that the associated topology changes do not introduce discontinuities to \( V \), even if graph \( \mathcal{G}(t) \) is time-varying.

One feature of the solutions to differential inclusion (7) and (6), which is essential to collision avoidance for all agents in the flock, is summarized in the next proposition.

**Proposition 2.** Let \( (x, v) \) be a solution to the differential inclusion (7) and (6) with initial condition \( (x(0), v(0)) \) satisfying \( x_i(0) \neq x_j(0), \forall (i, j) \in \mathcal{E}(0) \). Then \( \forall t \geq 0 \), we have

\[
\| x(t) \| \leq \| x(0) \| + \sqrt{V(0)} t, \quad \| v(t) \| \leq \sqrt{V(0)}
\]

and there exists a constant \( b > 0 \) such that \( \| x_{ji}(t) \| \geq b \), for \( i \neq j \).

**Proof.** Without loss of generality, we assume that there are infinite number of switching instants. Otherwise, if there is only a finite number of switches, and the final switch at time \( t_m \), we can artificially define \( t_{m+j} = t_m + j \gamma, j = 1, 2, \ldots \), where \( \gamma > 0 \) is an arbitrary constant.

Consider the interval \( [t_k, t_{k+1}] \), \( k = 0, 1, \ldots \) with \( t_0 = 0 \), then \( \mathcal{G}(t) \) is a fixed graph between switching instants \( t_k \) and \( t_{k+1} \). And the gradient of \( V \) corresponding to \( x \) in interval \( [t_k, t_{k+1}] \) is given by

\[
\nabla_x V = -2 \sum_{j \in N_i(t_k)} \phi_{ij}(x) x_{ji}.
\]

Define the set-valued derivative \( \dot{V} \) of \( V \) with respect to (7) as in Bacciotti and Ceragioli (1999)

\[
\dot{V} = \{ d : \exists w \in \mathcal{K} [v_{ii}] \text{ such that } (e^T w = d, \forall e \in \partial V(x, v)) \},
\]

where \( \partial V(x, v) \) is the Clarke generalized gradient of \( V \) at \( (x, v) \) (see Clarke (1990)). Since \( V(x, v) \) is smooth, \( \dot{V} = \nabla V^T \mathcal{K} [v_{ii}] \).

Furthermore, the symmetry nature of graph \( \mathcal{G}(t) \) and the nonnegativity of \( \phi_{ij}(x) \) imply that

\[
\sum_{i=1}^{N} \sum_{j \in N_i(t_k)} a_{ij}(x) v_{ji}^2 v_j = 2 \sum_{i=1}^{N} \sum_{j \in N_i(t_k)} a_{ij}(x) v_{ji}^2 v_j.
\]

It follows from Theorem 1 of Paden and Sastry (1987) that

\[
\dot{V} = 2 \mathcal{K} [v_{ii} \psi],
\]

\[
= 2 \mathcal{K} \left( \sum_{i=1}^{N} \sum_{j \in N_i(t_k)} a_{ij}(x) v_{ji}^T \text{SGN}(v_{ji}) \right),
\]

\[
\leq - \sum_{i=1}^{N} \sum_{j \in N_i(t_k)} a_{ij}(x) v_{ji}^T \mathcal{K} [\text{SGN}(v_{ji})],
\]

\[
= - \sum_{i=1}^{N} \sum_{j \in N_i(t_k)} a_{ij}(x) \| v_{ji} \|,
\]

where \( \psi = (\psi_1^T, \psi_2^T, \ldots, \psi_N^T) \). By Lemma 1 of Bacciotti and Ceragioli (1999), one has \( \frac{d}{dt} V(x, v) \in \dot{V} \), almost everywhere. This along with (11) implies that, for almost all \( t \in [t_k, t_{k+1}] \),

\[
\frac{d}{dt} V(x, v) \leq \max \dot{V} \leq 0.
\]

Hence, \( V(t) \) is a non-increasing function of \( t \) in each interval \( [t_k, t_{k+1}] \), \( k = 0, 1, \ldots \). In particular, we have \( V(t) \leq V(0), t \in [0, t_1] \), namely,
\[ \|v(t)\|^2 + \sum_{i=1}^{N} \sum_{j \in N(i)} \int_{0}^{\infty} \phi_{ij}(s)ds \leq V(0). \]  

Note that \( \theta > 1 \) and \( \phi_{ij}(x) \to \infty \), as \( \|x_{ij}(t)\| \to 0 \). It follows from the fact that \( V(0) \) is finite, there must exist a constant \( 0 < b \leq r \) such that \( \|x_{ij}(t)\| \geq b \) for all \( i \neq j \) and \( t \in [0, t_1) \).

Moreover, we claim that \( V(t_1) \leq V(0) \). It suffices to deal with the case that there are new links between agents added to \( \mathcal{E}(t_1) \) compared with the initial configuration \( \mathcal{E}(0) \). In fact, the addition of new links between agents \( i \) and \( j \) can occur only in the region where \( r < \|x_{ij}(t)\| \leq R(t) \). However, \( \phi_{ij}(x) = 0 \) in this case, thus it gives

\[ V(t_1) = \|v(t_1)\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N(i)} \int_{0}^{\infty} \phi_{ij}(s)ds \leq V(0) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N(i)} \int_{0}^{\|x_{ij}(t_1)\|^2} \phi_{ij}(s)ds \leq V(0). \]

Applying recursively the above arguments, we can obtain that \( V(t_k) \leq V(t_0) \), for \( k = 0, 1, \ldots \). And we have \( \|x_{ij}(t)\| \geq b, i \neq j, \|v(t)\| \leq \sqrt{V(0)} \), for all \( t \geq 0 \). Furthermore, from (7), it is easy to derive that

\[ \frac{d}{dt}\|x(t)\| = \frac{x(t)^T v(t)}{\sqrt{x(t)^T x(t)}} \leq \|v(t)\|, \]

which by integration implies \( \|x(t)\| \leq \|x(0)\| + \sqrt{V(0)} t, \forall t \geq 0 \). This completes the proof. \( \square \)

Based on Proposition 2, it is straightforward to construct an invariant set formed by the Lyapunov-like function \( V(x, v) \). Its invariant property is established in the following lemma.

**Lemma 3.** For any constant \( c > 0 \), the level set \( \Sigma_c = \{ (x, v) \in \mathcal{D} \times \mathbb{R}^N : V(x, v) < c \} \) is strongly invariant for differential inclusion (7) and (6), i. e. for each \( (\zeta_0, \xi_0) \in \Sigma_c \), all maximal solutions to (7) and (6) starting from \( (\zeta_0, \xi_0) \) remains in \( \Sigma_c \) thereafter.

Based on Proposition 2, Lemma 3 and a nonsmooth extension of LaSalle’s invariance principle to differential inclusions (Bacciotti and Ceragioli (1999)), we can state our main result about the asymptotic convergence of the solutions to (1) and (4).

**Theorem 4.** Consider a group of \( N \) agents with dynamics (1) steered by the control input (4) and the fluctuation dynamics (5). Suppose that the initial conditions satisfy \( \hat{x}_j(0) \neq \hat{x}_i(0), \forall (i, j) \in \mathcal{E}(0) \), and the network topology \( \mathcal{G}(t) \) is connected for all \( t \geq 0 \). If the path loss exponent \( \eta \) satisfies one of the following conditions: i) \( \eta \leq 0.5 \); ii) \( \eta > 0.5 \) and

\[ V(x(0), v(0)) < \frac{H \nu}{4\eta - 2} (1 + 2\|x(0)\|^2)^{0.5 - \eta}, \]

where \( \phi_{ij}(\hat{x}) \) is defined in (3) and \( \nu \) in (8), then there exist constants \( b, B > 0 \) such that \( \forall t \geq 0, b \leq \|\hat{x}_i(t) - \hat{x}_j(t)\| \leq B \) holds. Moreover, all agents move with the same velocity eventually, i. e. \( \hat{v}_i(t) \to \hat{v}_j(t), \forall i \neq j \), as \( t \to +\infty \).

**Proof.** From Lemma 3, we know that the level set \( \Sigma_{V(0)} \) is strongly invariant for differential inclusion (7) and (6). Consequently, in order to employing the nonsmooth LaSalle’s invariance principle to determine the asymptotic convergence properties of the solutions to differential inclusion (7) and (6), the compactness of \( \Sigma_{V(0)} \) should be established. Since \( \|v(t)\| \leq \sqrt{V(0)} \), it suffices to provide the bound for \( x(t) \). Note that the state-dependent interaction radius \( R(t) \) may be unbounded, the technique by means of network connectivity used in Tanner et al. (2007) can be no longer applied in the present case. Our proof is based on ideas from Ha and Liu (2009); Cucker and Dong (2010).

Suppose on the contrary that \( \|x(t)\| \) is unbounded, then there exist an integer \( k^* \geq 0 \) and \( \hat{\tau} \in [t_{k^*}, t_{k^* + 1}) \) such that for any constant \( \mathcal{M} \), one has \( \|x(t)\| \geq \mathcal{M}, \forall \hat{\tau} \leq \tau \leq \tau^* \), where \( \tau^* < t_{k^* + 1} \).

Under the assumption that \( \mathcal{G}(t) \) is connected all the time, we use Lemma 1 and the fact \( max_{i,j} \|x_{ij}\|^2 \leq 2 \|x\|^2 \) to find that

\[ \sum_{i=1}^{N} \sum_{j \in N(i)} a_{ij}(x)v_{ij} \geq \frac{H}{(1 + 2\|x\|^2)\eta} \sum_{j \in N(i)} v_{ij} \geq \frac{H \nu}{(1 + 2\|x\|^2)\eta} \|x\|. \]

Combining the above inequalities with (11) and (13) yields

\[ \frac{d}{dt}V(t) \leq \max \dot{V} \leq - \frac{H \nu}{(1 + 2\|x\|^2)\eta} \|x\|. \]

In consequence, for \( t_{k^*} \leq \hat{\tau} \leq t < t_{k^* + 1} \), by integration one has

\[ V(t) - V(\hat{\tau}) \leq - \int_{\hat{\tau}}^{t} \frac{H \nu}{(1 + 2\|x\|^2)\eta} \|x\| d\tau, \]

which further implies

\[ \int_{0}^{\tau^*} \frac{1}{(1 + 2\|x\|^2)\eta} d\|x\| \leq \frac{1}{H \nu} V(0). \]

By changing the variable \( z = \|x\| \), we can get that

\[ \Theta(M) \triangleq \int_{\|x(0)\|}^{M} \frac{1}{(1 + 2z^2)\eta} dz \leq \frac{1}{H \nu} V(0). \]

As for \( \Theta(M) \), it evaluates to

\[ \Theta(M) = \frac{1}{\sqrt{2} \nu} \ln \left( \sqrt{2} z + \sqrt{1 + 2z^2} \right) \bigg|_{\|x(0)\|}^{M}, \text{ if } \eta = 0.5, \]

and
\[ \Theta(M) \geq \int_{\|x(0)\|}^{M} \frac{z}{(1 + 2z^2)^{\eta+0.5}} \, dz = \frac{1}{2 - 4\eta} (1 + 2z^2)^{0.5-\eta} \bigg|_{\|x(0)\|}^{M} , \text{ if } \eta \neq 0.5. \] (20)

In what follows, we consider two cases:

i) \( \eta \leq 0.5. \) In this case, since \( \ln(\sqrt{2z} + \sqrt{1 + 2z^2}) \) and \( (1 + 2z^2)^m, m > 0 \) are all monotonically increasing functions of \( z, \) the right hand sides of (19) and (20) can both be sufficiently large, if we pick \( M \) to be large enough. This clearly is impossible.

ii) \( \eta > 0.5. \) It follows from (18) and (20) that, for arbitrary \( \varepsilon > 0, \)

\[ \left(1 + 2\|x(0)\|^2\right)^{0.5-\eta} \leq \frac{4\eta}{H^\nu} - \frac{2}{H^\nu} V(0) + \varepsilon, \]

which again results in a contradiction with (14).

Therefore, we can find a constant \( B > 0 \) such that \( \|x(t)\| \leq B/2. \) Furthermore, it can be derived that \( \|\hat{x}_i - \hat{x}_j\| \leq B, \) \( \forall i \neq j, \) for all \( t \geq 0. \)

Now it follows from Theorem 3 of Bacciotti and Ceragioli (1999) that all solutions to (7) and (6) starting from \( \Sigma_{V(0)} \) approaches the largest weakly invariant set contained in

\[ \left\{ (x, v) \in \mathbb{R}^{Nn} \times \mathbb{R}^{Nn} : 0 \in V(0) \right\} \bigcap \Sigma_{V(0)}. \]

In view of (11) and by recalling that graph \( G(t) \) is connected all the time, one can see that \( 0 \in V \) if and only if \( v_1 = v_2 = \cdots = v_N. \) However, \( \sum_{i=1}^{N} v_i = 0. \) Then \( v_i = 0, i = 1, 2, \ldots, N. \) This means that the velocities of all agents \( \hat{v}_i, i = 1, 2, \ldots, \) asymptotically match with the mean velocity \( v_c = \frac{1}{N} \sum_{i=1}^{N} \hat{v}_i. \) This completes the proof. \( \square \)

4. SIMULATION RESULTS

In this section, we will present some numerical simulations in order to illustrate the theoretical results obtained in the previous sections.

The simulations are performed with a group of 10 mobile agents moving in the flat plane with dynamics (1), which are initialized with (i) random initial positions in the square \([-15, 15] \times [-15, 15]; \) (ii) random initial velocities chosen from \([-1, 2] \times [-1, 2]; \) and (iii) randomly generated graph \( G(0) \) with interaction range \( \hat{R}(0) = 7.4m \) such that \( G(0) \) is connected and all agents are separated by at least 0.1m, see Fig. 1 (a). The radius of the hard sphere of each agent is \( r = 0.2m. \) The following parameters are used in the simulations: \( H = 5, \eta = 0.25, \theta = 2. \) We run the simulations 100s and choose suitable scales to show our simulation results.

Fig. 1 presents the initial positions and velocities of all 10 agents. Fig. 2 (a) depicts their position trajectories. To measure the degree of collision avoidance of the group, we make use of an indicator, namely the minimum distance of nearest neighbors defined by \( \min_{i \neq j} \| x_i - x_j \|. \) As shown in Fig. 2 (b), all the agents are separated at least by 0.9m, thereby forming a collision-free flock. Alignment of velocities can be seen in Fig. 3. Although frequent topology changes produce transients, see Fig. 4, the overall convergence is evident and the convergence time is as short as around 1.5s. Note that the interaction range now is time-varying, see Fig. 4, this is an exclusive feature of our self-propelled particle model which is supported by the field study Ballerini et al. (2008a) These figures demonstrate the observed behavior of groups of mobile agents and validate our mathematical results in previous sections.

For the case that \( \eta = 0.75 > 0.5, \) the simulation results of 3-D flocking of 50 mobile agents are also given in Fig. 5 and Fig. 6, from which we can observe the similar phenomena.
5. CONCLUSION

We have developed a self-propelled particle model in this paper to mimic the collective behavior of a group of mobile agents. The proposed model is fully distributed, since all the agents are steered by only local social forces. Different from many existing results, we only give some mild assumptions on the network connectivity and initial configuration. The acceleration is subtly designed to guarantee the stability and convergence. Theory of differential inclusions is applied to deal with the case of time-varying interaction ranges. Simulation results have validated the theoretical analysis.

REFERENCES


