An optimization framework for load and power distribution in wind farms: Low wind speed

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Abstract: In this paper, an optimization approach has been developed for wind farms such that the reference signals for each wind turbine is produced and load reduction on each of them is taken into account. The proposed approach is based on a spatially discrete model of the farm which provides wake information and delivers an approximation of wind speed all over the farm. The farm controller can affect the generated wake of each wind turbine either by changing the pitch angle and power reference or rotor speed reference signals, and this is the motivation for the proposed optimization method for a farm. The problem of determining the reference signals for wind turbine controllers is considered separately for low and high wind speed regions. However, because of the complexity of the high wind speed scenario formulated with this approach, only the problem in low wind speed has been scrutinized. Accordingly, in low wind speed, the reference signals for rotor speed are adjusted, taking the trade-off between power maximization and load minimization into account.

1. INTRODUCTION

Seeing wind farms grow in size and number, a demand for optimized performance and longer life time for each wind turbine arises. To extend the lifetime of wind turbines in a farm, load reduction methods should be included in controller design Hammerum et al. [2007]. A challenge in the controller design for wind farms is to provide a model to prognosticate the effect of the wake formed behind a wind turbine on the other wind turbines. The challenge is due to the significant decrement in mean wind speed and an increment in turbulence. The increase in turbulence intensity in the wakes behind wind turbines increases fatigue substantially Sørensen et al. [2008]. However, the wake effects generated by upwind turbines can be influenced by changing the reference signals for each wind turbine controller, e.g., pitch angle, rotor speed and power reference. In other words, the wind farm controller can affect the wake of each wind turbine by sending either the pitch angle and power reference signals or rotor speed reference signals to the wind turbine controllers; and This is the motivation for the proposed wind farm optimization.

There are numerous research in modeling and control of wind farms. An overall wind farm control that maximizes energy capture has been proposed in Steinbuch et al. [1988]. Zhao et al. [2006] proposed an optimization method to maximize the capacity of farms based on the limitations of the physical system such as voltage, voltage stability, generator power. Hansen et al. [2006] presented a concept with both centralized control and control for each individual wind turbine. In their approach, the controllers at turbine level ensure that relevant reference commands provided by the centralized controller are followed.

Notwithstanding all these control methods and many research efforts on fatigue load reduction in single turbines van der Hooft et al. [2003], Lescher et al. [2007], Sutherland [2000], Hammerum et al. [2007], results on the combined optimization of power and fatigue load are lacking. Consequently, the aim of the present work is to develop a wind farm controller in low wind speed to extract the maximum power from the farm, while it reduces the structural loads. The problem is that the controller should compute rotor speed reference signals for each individual wind turbine controller. This has been formulated in terms of a Pontryagin maximum principle, and the challenges of finding the optimal solution has been discussed. Moreover, the applied wind farm dynamical model Soleimanzadeh and Wisniewski [2010a,b] delivers an approximation of wind speed in the vicinity of each wind turbine that is suitable for optimization.

2. WIND FARM MODEL

The basic idea of the model presented in Soleimanzadeh and Wisniewski [2010a,b] is to calculate approximately the wind speed in the vicinity of each wind turbine in a farm.

The key element in the model is to solve a linear flow equation for the whole wind farm. The wind farm is divided into non-overlapping cells and spatial discretization of the flow equation is performed using finite volume techniques. In this model, wind turbines are modeled by means of their thrust coefficient, using the actuator disk approach. The model is considered to be in the far wake region, and the ambient shear flow has been mostly neglected. Moreover, the profile of velocity deficit is assumed to be axisymmetric. Finally, the dynamic equations of the wind
farm has been written in the following form, expressed in Soleimanzadeh and Wisniewski [2010b].

\[
\frac{dx_1(t)}{dt} = f_1(x_1(t), ..., x_n(t), u_1(t), ..., u_m(t)),
\]

where \( C_Q(\beta, \lambda) \) is the torque coefficient and \( C_Q = \frac{C_P}{\lambda} \), where \( C_P(\beta, \lambda) \) is the power coefficient.

which has been summarized as:

\[
\dot{X} = A(t, u)X + B(t, u).
\]

In (3), \( X \) is a vector of mean wind speed over the farm in \( \mathbb{R}^n \), where \( n \) is the number of cells all over the wind farm. The bigger the number of cells the better approximation of wind flow. The domain is the subset of \( \mathbb{Z}^2 \), corresponding to the \((i, j)\)-index of cells. The matrix \( A(t, u) \) is a linear function of \( u \), and it is a block diagonal matrix in \( \mathbb{R}^{n \times n} \); \( u \) is a function of the thrust coefficient in \( \mathbb{R}^m \), where \( m \) is the number of wind turbines. The thrust coefficient is a function of the tip speed ratio and pitch angle and they will be defined as reference signals for wind turbine controller. Thus, it is considered as system input.

In order to write the wind farm dynamic model in a compact form, the (3) will be re-written as follows:

\[
\hat{x} = \zeta_t(x, u) + B_t(u),
\]

where

\[
\zeta_t(x, u) = \begin{pmatrix} u^T A_1(t)x \\ u^T A_2(t)x \\ \vdots \\ u^T A_n(t)x \end{pmatrix},
\]

while, \( u \in \mathbb{R}^m \) and \( A_i \in \mathbb{R}^{m \times n} \) and \( x \in \mathbb{R}^n \). Furthermore

\[
\begin{pmatrix} u^T A_1(t) \\ u^T A_2(t) \\ \vdots \\ u^T A_n(t) \end{pmatrix} x = \begin{pmatrix} x^T A_1(t) \\ x^T A_2(t) \\ \vdots \\ x^T A_n(t) \end{pmatrix} u.
\]

Above mentioned matrix structure is used in optimal control design of the farm.

This model provides an approximate knowledge of what is happening downstream wind turbines; therefore, it is seen useful to estimate fatigue loads or total power production of the farm.

3. CONTROL STRATEGY

Based on the structural load analysis, pitching the blades to reduce the power has a small influence on loads. Whereas, reducing the speed considerably decreases the structural load. Therefore, a trade-off should be made between the reduction of fluctuations and power capture Thomsen [1967], Sørensen et al. [2004].

The control strategy is to consider the wind farm control problem separately for low and high wind speed. In above rated wind speed, the farm controller provides power reference and pitch angle reference signals for each individual wind turbine controller. In below rated wind speed, the farm control has two alternatives. First one is tracking the demanded power when it is less than the available power of all the farm; and second one is to extract maximum possible power from each wind turbine. If the demanded power from the farm is less than available power, the strategy is similar to the one in high wind speed that wind turbine controllers should track the power reference and pitch reference signals provided by the wind farm controller. Providing the reference signals is such that the total required power from the farm is tracked and structural loads on each individual wind turbine is minimized Soleimanzadeh and Wisniewski [2011].

The second option, which is the subject of interest in this paper, is when the maximum possible power should be extracted from the wind farm while minimizing the total turbine loading. Therefore, in the rest of the paper, in below rated wind speed, wind turbines are to produce as much power as possible, so the pitch angle is kept almost constant. Thus, the wind farm controller provides the rotor speed reference signals for individual wind turbine controllers.

The aerodynamic loads below rated wind speed are generally lower than those above it. However, one of our objectives in this paper is to reduce the tower fluctuations (fore-aft and side-to-side), which will significantly reduce fatigue loads. The reason for this selection is that the tower fore-aft motion is strongly coupled with the blade flap motion. Moreover, the tower side-to-side motion is strongly coupled with the blade edge and drive train torsion Suryanarayanan and Dixit [2007].

The tower motion dynamics can be approximated by a second order system of differential equations. Assuming that there is no coupling between tower fore-aft and tower side-to-side dynamics, following equations are devised van der Hooft et al. [2003]:

\[
M \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dot{x}_{FA} + D \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dot{x}_{SS} + K \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_{FA} = \begin{bmatrix} F_T \end{bmatrix},
\]

where \( x_{FA} \) is the tower fore-aft displacement and \( x_{SS} \) is the tower side-to-side displacement. \( M, D \) and \( K \) are the mass, damping and stiffness. \( F_T(\ldots) \) is the thrust force, and is equal to:

\[
F_T(C_T, V) = \frac{1}{2} \rho \pi R^2 V^2 C_T,
\]

where \( \rho, R \), and \( V \) are respectively density, rotor radius and wind speed. \( C_T(\beta, \lambda) \) is the thrust coefficient and \( \beta \) and \( \lambda \) are the pitch angle and the tip speed ratio. \( \lambda = \Omega R/V \) and \( \Omega \) is the rotor speed. \( f_t(\ldots) \) is the tangential force, and is equal to

\[
f_t(C_Q, V) = \frac{1}{2} \rho \pi R^2 V^2 C_Q,
\]

where \( C_Q(\beta, \lambda) \) is the torque coefficient and \( C_Q = C_P/\lambda \), where \( C_P(\beta, \lambda) \) is the power coefficient.
As previously mentioned, one of our goals is to reduce fatigue by increasing tower damping. Bearing this in mind, we increase the tower damping factor $D_t$ in dynamic equations to a desired level.

In order to maximize the conversion efficiency in the low wind speed region, the rotor speed is changed in proportion to the wind speed to maintain the optimal level; whereas, the pitch angle is kept constant. Therefore, an additional thrust can be produced by rotor speed variations to increase damping, $\Delta F_t(C_T, V) \propto \Delta C_T(\Omega)$. In this regard, the variation of $C_T(\Omega)$ is approximated by a linear function:

$$\Delta C_T(\Omega) = \frac{\partial C_T(\Omega)}{\partial \Omega} \Delta \Omega,$$

where $\Delta \Omega$ is the small variation of rotor speed which increases effective damping. In other words, $\Delta \Omega$ increases the damping factor $D_t$, and is proportional to $-\dot{x}_{FA}$:

$$\Delta \Omega \approx -\frac{D_{p_r}}{2\rho R^2 V^2 \partial C_T(\Omega)/\partial \Omega} \dot{x}_{FA},$$

where $D_{p_r}$ is additional damping factor. Therefore, in low wind speed:

$$\Delta F_t(C_T, V) = \frac{1}{2}\rho R^2 V^2 \frac{\partial C_T(\Omega)}{\partial \Omega} \Delta \Omega \approx -D_{p_r} \dot{x}_{FA}. (10)$$

In other words, tower damping $D_t$ in the tower model is added to another damping factor $D_{p_r}$ due to the rotor speed perturbations. As a result, the system model has a new set of parameters Bossanyi [2003].

The tangential force variations, which contributes to side-to-side motion, can be approximated as follows:

$$\Delta f_t(C_P, V) \propto \Delta (C_Q(\Omega) \propto (C_P/\lambda), (11)$$

$$\Delta C_Q(\Omega) \approx \frac{\partial (\Omega^{-1} C_P(\Omega)/V)}{\partial \Omega} \Delta \Omega$$

$$\approx -\left[\frac{C_P(\Omega)}{\Omega^2} + \Omega^{-1} \frac{\partial C_P(\Omega)}{\partial \Omega}\right] \Delta \Omega. (12)$$

Analogous to fore-aft movement, side-to-side additional damping is introduced as follows:

$$\Delta f_s(C_Q, V) = \frac{1}{2}\rho R^2 V^2 \frac{\partial (\Omega^{-1} C_P(\Omega))}{\partial \Omega} \Delta \Omega$$

$$\approx -D_{p_r} \dot{x}_{SS}. (14)$$

On the other hand, the extracted power from each wind turbine is expressed as (for $i^{th}$ turbine):

$$P_{W,t_i} = \frac{1}{2}\rho R^2 V^3 C_P(\Omega), (15)$$

where $C_P(\beta, \lambda)$ is the power coefficient, which is a function of pitch angle $\beta$, and tip speed ratio $\lambda$. In order to extract maximum power from wind farm, the total power should be maximized:

$$P_{total} = \sum_{i=1}^{N} P_{W,t_i}. (16)$$

The summands $P_{W,t_i}$ are given in (15), and $C_P(\beta, \lambda)$ is used as a part of the cost function for the controller. Since pitch is constant in low wind speed, $C_P(\beta, \lambda)$ only depends on $V$ and $\Omega$, thus we write $C_P(V, \Omega)$. Finally, $N$ is the number of wind turbines.

In order to solve the trade-off between load and power, the damping factors $\partial C_T(\Omega)/\partial \Omega$ and $\partial (\Omega^{-1} C_P(\Omega))/\partial \Omega$, based on (10) and (14), and the power coefficients $C_P(V, \Omega)$ based on (16), should be maximized.

4. PROBLEM FORMULATION IN LOW WIND SPEED

In this problem definition of variables in low wind speed are:

$$u_j = \Omega_j, \quad x_i = V_i, (17)$$

where $\Omega_j$ is the rotor speed of $j^{th}$ wind turbine, and $V_i$ is the wind speed in $i^{th}$ cell.

To complete the model, mathematical expressions for the functions $C_P(\ldots)$ and $C_Q(\ldots)$ are estimated based on the numerical tables of NREL 5MW wind turbine Jonkman et al. [2009]. The estimated nonlinear polynomials are expressed as follows, (for each wind turbine)

$$C_P(\beta, \lambda) \approx p_{00} + p_{10}\beta + p_{01}\lambda + p_{20}\beta^2 + p_{11}\lambda \beta$$

$$+ p_{02}\lambda^2 + p_{30}\lambda^3 + p_{21}\beta^2\lambda + p_{12}\beta\lambda^2 + p_{03}\lambda^3$$

$$C_T(\beta, \lambda) \approx k_{00} + k_{10}\beta + k_{01}\lambda + k_{20}\beta^2 + k_{11}\lambda$$

$$+ k_{02}\lambda^2 + k_{30}\lambda^3 + k_{21}\beta^2\lambda + k_{12}\beta\lambda^2 + k_{03}\lambda^3,$$

where $\lambda = (\Omega R/V)$, and the pitch angle $\beta$ is considered constant in low wind speed. Linear approximations for $C_T(\ldots)$ and $C_P(\ldots)$ are as follows:

$$C_T(V, \Omega) = \gamma_0 + \gamma_1 V + \gamma_2 \Omega,$$

$$C_T(V, \Omega) = c_{\alpha_0} + \alpha_1 V + \alpha_2 \Omega,$$

where $\gamma_i$ and $\alpha_i$ are linearization factors.

Therefore, based on the definition (17), the cost function for the wind farm will be expressed in terms of $(x, u)$. The optimal control problem is to find a control vector $u(t)$ and an associated state vector function $x(t)$, defined on the fixed time interval $[t_0, t_1]$ that will minimize

$$J(u) = \int_{t_0}^{t_1} [\gamma_0 + \alpha_2 + \gamma_1 x - \gamma_1 x u^{-2} - \gamma_0 u^{-2} + \gamma_2 u] dt, (21)$$

where $x = (x_1, x_2, ..., x_n)$, and $u = (u_1, u_2, ..., u_m)$, and $u^{(j)}$ stands for every element of $u$ to the power. Moreover, the initial conditions are

$$x(t_0) = x_0, \quad x(t_1) \text{ is free, \quad } t \in [t_0, t_1]$$

$$u \in [u_{min}, u_{max}], (22)$$
subject to the differential equations (3).

The problem is to find among all admissible control functions \( u(t) \) which bring \( x(t) \) from the initial point to a point satisfying the given terminal condition, one which makes the integral in (21) as large as possible.

The Maximum Principle transfers the problem of finding a \( u(t) \) which maximizes the integral in (21) subject to the given constraints, to the problem of maximizing the Hamiltonian function.

\[
H(x^*, u, t) = P_0 f(x^*, u, t) + P f_0(x^*, u, t), \quad P_0 = 1 \text{ or } 0.
\]

Where \( f \) is the integrand in (22), and \( f_0 \) is the equation for \( \dot{x} \). For current problem the Hamiltonian is:

\[
H(x^*, u, t) = f(x, u) + P \begin{pmatrix} u^T A_1 \\ u^T A_2 \\ \vdots \\ u^T A_n \end{pmatrix} x + PB_t u,
\]

where

\[
f(x_j, u_i) = \gamma_0 + \alpha_2 + \gamma_1 x_j - \gamma_1 x_j u_i - 2 - \gamma_0 u_i^2 + \gamma_2 u_i.
\]

Then there exists an absolutely continuous function \( P(t) \) such that for all \( t \in [t_0, t_1] \), \( u^*(t) \) maximizes \( H(x^*(t), u, p(t), t) \). Except at the points of discontinuities of \( u^*(t) \):

\[
\dot{P}(t) = -\frac{\partial H(x^*, u, t)}{\partial x},
\]

then:

\[
\dot{P}(t) = \frac{\partial f(x, u)}{\partial x} + P \begin{pmatrix} u^T A_1 \\ u^T A_2 \\ \vdots \\ u^T A_n \end{pmatrix} x + B_t u.
\]

For \( u^*(t) \) to minimize the Hamiltonian it is necessary (but not sufficient) that Kirk [2004]:

\[
\frac{\partial H(x^*, u, t)}{\partial u} = 0,
\]

then:

\[
\frac{\partial f(x, u)}{\partial u} + \frac{\partial}{\partial u} P \begin{pmatrix} x^T A_1 \\ x^T A_2 \\ \vdots \\ x^T A_n \end{pmatrix} u + B_t u = 0,
\]

therefore,

\[
\frac{\partial f(x, u)}{\partial u} + P \begin{pmatrix} x^T A_1 \\ x^T A_2 \\ \vdots \\ x^T A_n \end{pmatrix} + B_t = 0.
\]

The problem is now a two point boundary value problem; hence, difficult to solve analytically.

4.1 Existence of solution in low wind speed

In this section we will prove that the proposed optimal problem has a solution.

Proposition 1. There exist a solution to the general optimal control problem (21)-(22).

Proof. We will use the Filipov-Cesari theorem based on Seierstad and Sydsæter [1993]. The theorem defines the set \( N(x, U, t) \) in \( \mathbb{R}^{n+1} \), for each \( (x, t) \), by

\[
N(x, U, t) = \{ f(x, u, t) + \eta, f_1(x, u, t), \ldots, f_n(x, u, t) \} : \eta \leq 0, \quad u \in U.
\]

It says that for every standard optimal control problem, there exist an optimal pair \( (x^*(t), u^*(t)) \) (where \( u^*(t) \) is measurable), if:

- There exists an admissible pair \( (x(t), u(t)) \).
- \( N(x, u, t) \) is convex for each \( (x, t) \).
- This condition can be dropped if all the \( f_i \) functions are linear in \( x \); i.e.

\[
f_i(x, u, t) = \sum_{i=1}^{n} l_{ij}(x_j + g_i(u, t), t),
\]

where \( l_{ij}(t) \) and \( g_i(u, t) \) are continuous functions; see Neustadt [1963].
- \( U \) is closed and bounded.
- There exists a number \( b \) such that \( \|x(t)\| \leq b \) for all \( t \in [t_0, t_1] \) and all \( (x(t), u(t)) \).

Based on the definition \( N(x, u, t) \) for this problem is:

\[
N(x, u, t) = \{ (\gamma_0 + \alpha_2 + \gamma_1 x - \gamma_1 x u - 2 - \gamma_0 u^2 + \gamma_2 u) + \eta, f_1(x, u, t), \ldots, f_n(x, u, t) \} : \eta \leq 0, \quad u \in U.
\]

\( N(x, U, t) \) is not convex but since all the \( f_i \) functions are linear in \( x \), this condition can be dropped. \( u \) is the rotor speed and it can not be infinity, so it is bounded, \( u \in [u_{min}, u_{max}] \).

\( x(t) \) is the wind speed and it has been derived from the differential equations describing the whole farm, (2) and (4). For all inputs the differential equation has a solution, which is bounded. Because neither the wind speed nor the thrust coefficient can be unbounded. Hence,there exists a number \( b \) such that \( \|x(t)\| \leq b \) for all \( t \in [t_0, t_1] \) and all \( (x(t), u(t)) \).

All the above mentioned conditions for the problem are satisfied, so based on the theorem, we conclude that the problem has an optimal solution.

4.2 Finding the solution

Finding an analytic solution for the above mentioned two point boundary value problem, is not easy. Therefore, to make the problem simpler the differential equation will be linearized around the optimal \( C_T(\ast) \). In other words the flow equations will be solved considering the steady flow, which the wind speed is constant. Assuming the flow is steady, the differential equation (4) does not include \( u \) as a variable. In this case the problem to be solved is:
\[
H(x^*, u) = f(x, u) + P \begin{bmatrix}
\hat{A}_1 \\
\hat{A}_2 \\
\vdots \\
\hat{A}_n
\end{bmatrix} x + P \hat{B}, \tag{34}
\]

where,
\[
f(x_j, u_i) = \gamma_0 + \alpha x_j - \gamma_1 x_j u_i - \gamma_2 u_i^2 - \gamma_0 u_i^2 + \gamma_2 u_i.
\tag{35}
\]

Then there exists a function \( P(t) \) such that for all \( t \in [t_0, t_1] \), \( u^*(t) \) maximizes \( H(x^*(t), u, p(t), t) \):
\[
\dot{P}(t) = -\frac{\partial H(x^*, u)}{\partial x^*}, \tag{36}
\]
then:
\[
\dot{P}(t) = P \begin{bmatrix}
\hat{A}_1 \\
\hat{A}_2 \\
\vdots \\
\hat{A}_n
\end{bmatrix} + \gamma_1 (1 - u^{-2}) \tag{37}
\]
For \( u^*(t) \) to minimize the Hamiltonian Kirk [2004]:
\[
\frac{\partial H(x^*, u)}{\partial u} = 0, \tag{38}
\]
then:
\[
\frac{\partial H}{\partial u} = \frac{\partial f(x, u)}{\partial u} + \frac{\partial}{\partial u} P \begin{bmatrix}
x^T \hat{A}_1 \\
x^T \hat{A}_2 \\
\vdots \\
x^T \hat{A}_n
\end{bmatrix} + \dot{\hat{B}} = 0, \tag{39}
\]
so:
\[
\frac{\partial f(x, u)}{\partial u} = 0, \tag{40}
\]
Therefore:
\[
u(t) = \left(-\frac{-\gamma_1}{2(\gamma_1 x(t) + \gamma_0)}\right)^{-1}, \tag{41}\]

The problem is solved for a sample wind farm composed of five wind turbines in a row. The wind direction is considered to be parallel to the wind turbine row, to have the maximum interaction between wakes of the wind turbines.

Rotor speed is calculated from (41), since \( u = \Omega \), and is compared with the numerical solution of (30). Rotor speed reference signals will be sent from the farm controller to each wind turbine controller as a command. The wind turbine controller is responsible for tracking these reference signals. Reference rotor speeds are presented in Table 1.

A comparison between power production in this method and the conventional strategy in low wind speed has been shown in Fig. 1. The green graph is achieved after implementing the farm controller proposed in this paper, the blue graph is after implementing the reference signals calculated from numerical results, and the red graph is the conventional strategy, which each wind turbine produces as much energy as possible without load control. The total power produced with the controller is 6.77 MW and from the numerical results 7.24 MW, which are less than produced power from conventional strategy, 7.38 MW. The reason is that the farm controller has made a trade-off between load and power.

Tower vibration of first and second wind turbine in fore–aft direction has been shown in Fig. 2. The red graph presents the vibrations when there is no wind farm control, and the green graphs present the damped vibrations after using the wind farm controller. The load control on each individual wind turbine controller has not been implemented, and the results are due to increase the damping factor in the tower dynamics. Moreover, the graphs are computed from numerical results.
5. CONCLUSIONS AND FUTURE WORKS

5.1 Conclusions

A wind farm controller has been developed in this paper, which aims to extract the maximum power from the farm and reducing the structural loads. This paper focuses on optimization of both power and load simultaneously, and the approach is founded on the dynamical model of the flow in wind farms. The wind farm model delivered wind speed in the vicinity of each wind turbine, and the proposed controller considered the turbine performance in low wind speed. The control algorithm determined the reference signals of the rotor speed for each wind turbine controller.

5.2 Shortcomings

Developing the farm controller for the high wind speed scenario using a similar approach will be the future work.

REFERENCES