Optimization of a Failure – prone Manufacturing System with Regular Preventive Maintenance: an IPA Approach

Edwin D. Gomez U.*, Sophie Hennequin**, Nidhal Rezg***

* Henri Tudor Public Research Center, Service Science and Innovation dept., 29 avenue John F. Kennedy, 1855 Luxembourg - Kirchberg, Luxembourg (e-mail: edwin.gomez@tudor.lu)
** ENIM, INRIA/CosTeam-LGIPM, 1 route d’Ars Laquenexy, 57078 Metz Cedex 3, France (e-mail: hennequin@enim.fr) corresponding author
*** UPVM, INRIA/ CosTeam -LGIPM, 1 route d’Ars Laquenexy, 57078 Metz Cedex 3, France (e-mail: rezg@univ-metz.fr)

Abstract: This paper addresses the optimization of a single-stage single-product manufacturing system with a constant demand and a systematic preventive maintenance policy. The manufacturing system is modeled by a continuous-flow model. The machine is subject to time-dependent failures and its production speed is given by a hedging point policy. Times to failure and times to repair are random variables with exponential distribution. We propose a simulation-based optimization method for determining the optimal buffer level in order to minimize the long run average cost including inventory holding cost, backordering cost and corrective and preventive maintenance costs. The optimization algorithm is based on the Infinitesimal Perturbation Analysis (IPA) technique for estimation of gradients along the simulation. The unbiasedness of the IPA estimators is established. Numerical results based on the simulation algorithm are proposed to determine the optimal buffer level and to derive the optimal value of the period of the systematic preventive maintenance.

Keywords: Manufacturing system, corrective and systematic preventive maintenance, continuous-flow model, optimization, infinitesimal perturbation analysis.

1. INTRODUCTION

To be competitive in the actual industrial context, where the concurrence is increasing and demands are more and more complicated, companies must be as performed as possible and must be capable of adapting to market variations.

To contend this problematic, strategies have been developed to obtain more performing manufacturing systems. An approach consists to integrate the maintenance in the production strategy, i.e. create a compromise between maintenance and production so that the system becomes more efficient. The goal of the integrated maintenance is to define an integrated optimal plan of production and maintenance which allows minimizing the costs linked to the production and the maintenance. In this context, the operational research is more and more used to solve this problem, and the stochastic optimization algorithms represent an interesting and necessary tool to get an efficient solution. In the set of techniques used, we find the Infinitesimal Perturbations Analysis (IPA) which is particularly powerful to solve several complex problems. IPA was developed by Ho and Cao (1991), for the efficient computation of n-dimensional gradient vector of performance measure, ∇f(θ), of a discrete event dynamic system with respect to its parameter vector θ, such as buffer size, inflow rate, service rate, etc. (see also Glasserman, 1990). IPA technique has been used successfully in several works, like (Yu & Cassandras 2004; Mourani, et al., 2006, Yu & Cassandras 2006; Melamed, et al., 2007; Turki, et al., 2009; Cassandras, et al., 2010).

The goal of this paper is to apply IPA in a single-stage failure-prone manufacturing system, integrating production and maintenance, throughout a continuous-flow model. This type of model has been largely studied. The first work that used continuous-flow model to study the production control of a failure-prone manufacturing system was (Kimemia & Gershwin 1983), which shows that the optimal production policy has a special structure called hedging point policy, in which a non negative production surplus should be maintained to avoid the non satisfaction of the demand when failures occurs. This policy corresponds to the fact that there is a buffer level h∗ which minimizes the cost and allows satisfying the demand.

This paper deals with the control problem of a stochastic manufacturing system consisting of one machine producing one part type. The stochastic nature of the system is due to the fact that the machine is subject to random breakdowns and repairs. For this reason, a preventive maintenance policy is considered. It has been widely demonstrated that implementing preventive maintenance strategies for several failure-prone manufacturing systems can extend the equipments life and reduce operating costs (Barlow and Proschan, 1965). To guarantee the continuous supply of the subsequent production unit during a corrective or preventive
maintenance, many authors have studied the case of building buffer stocks. The study realized by Van der Duyn Schouten and Vanneste (1995) for a system composed of two machines and one buffer between them, proposes a preventive maintenance policy based on the age of the machines and the preventive maintenance of constant durations. Mourani, et al., (2008) studied a single-stage failure-prone manufacturing system with constant demand and transportation delay from the machine to the inventory, without preventive maintenance policy. The objective of this work is to determine the optimal buffer level, in a hedging point policy, using IPA.

In this paper, we consider a preventive maintenance policy based on a type-bloc strategy (Barlow & Proschan 1996), which consists to realize a preventive maintenance at predetermined time periods, independently of the use of the machine and the state of wear. In the literature, we find another kind of replacement strategy, which is the type-age strategy (Charlot, et al., 2007). This preventive maintenance policy consists to realize a preventive maintenance action after an “age” or a specific operation-period of the machine. Furthermore, with respect to the nature of failures, we consider the case when failures are time-dependent, i.e., machine deteriorates over time, even if it doesn’t work. Another kind of failures exists; it is the operations-dependent failures (Buzacott & Hanifin, 1978).

We propose an algorithm allowing get an optimal solution, taking into account the buffer level, the preventive maintenance period, and the corrective maintenance, guaranteeing a minimal cost. Several works, like (Chelbi & Ait-Kadi 2004; Gharbi, et al., 2007), have treated integrated maintenance problems, but all these works realize a surface study to optimize the system. We propose a study based on IPA technique to realize the optimization.

The rest of the paper is organized as follows. Section 2 introduces the studied system throughout a continuous-flow model, where we present the machine failure nature, the hedging point control policy, the dynamic of the system and the cost function, including inventory and maintenance cost. Section 3 presents the infinitesimal perturbations analysis applied to our manufacturing system. We study the system sample paths and then we derive the sample gradient with respect to the hedging point policy. The sample gradient is proved to be unbiased. Section 4 uses this sample gradient in an optimization algorithm based on the bisection method, to plot the curve of the cost function with respect to the hedging point, and determine the optimal buffer level which controls the hedging point policy. Other simulation is realized, varying the preventive maintenance period to get a completely optimal configuration of the system. Finally, we conclude in Section 5.

2. STUDIED SYSTEM

This paper considers a continuous flow model of a single-stage single-product manufacturing system (see Fig. 1). Material flows continuously from outside the system to the machine, denoted by M, where it joins the finished product buffer, denoted by B. We assume that the demand arrives at buffer size, which are both used to determine when to realize a preventive maintenance. A two-machine system with fixed-capacity buffer between them was studied by Meller, et al., (1996), which has considered constant machines failure rates, repair times distributed exponentially and this buffer B with a constant demand rate D. The machine is subject to time-dependent failures (Buzacott & Hanifin, 1978). The failure-repair epochs of the machines form a stochastic process that does not depend on the production control policy of the system. Times to failure (TBF) and times to repair (TRT) of the machine M are independent and identically distributed (i.i.d.) random variables.

![Fig. 1. Manufacturing system](image)

The downstream buffer capacity is infinite.

The maintenance strategy under consideration is the systematic preventive maintenance policy (Barlow & Proschan 1996). It consists of submitting the machine to a preventive maintenance action as soon as the time reaches a certain period Tm or a failure, whichever comes first.

We assume that the machine is never starved. The machine M is either up or down. The state of the machine at time t, denoted α(t), is given by:

\[
α(t) = \begin{cases} 1 & \text{if } M \text{ is operating} \\ 0 & \text{if } M \text{ requires a maintenance action} \end{cases}
\]

When the machine is producing, the production speed of M, denoted by u(t), could take any value between 0 and U, where U is the maximal production speed (the machine capacity). When a maintenance action is required, the machine could not produce and a corrective or preventive maintenance must be realized. According to that, we define another boolean variable, denoted β(t), which represents the kind of maintenance realized at time t. Then, if α(t) = 0, 3 β(t) such as:

\[
β(t) = \begin{cases} 1 & \text{if the maintenance is preventive} \\ 0 & \text{if the maintenance is corrective} \end{cases}
\]

The control policy, given by a hedging point policy (Kimemia & Gershwin 1983), can be formally described as follows: when the machine M is up, (i) it is blocked if its downstream buffer reaches its hedging point value, the buffer is said full and its production speed is slowed down to the constant value of the demand; (ii) it produces at its maximum production rate in any other case; (iii) when the machine is down or a preventive maintenance is made, the production speed is equal to 0. The control policy is formally defined as follows:

\[
u(t; x, α) = \begin{cases} U \text{ si } x(t) < h \text{ et } α(t) = 1 \\ D \text{ si } x(t) = h \text{ et } α(t) = 1 \\ 0 \text{ si } x(t) > h \text{ ou } α(t) = 0 \end{cases}
\]

10423
$U$ is the maximal production rate and $h$ is the value of the hedging point which represents the maximal buffer level.

**Assumption (1).** $U > D$.

**Remark 1:** This assumption allows us to be sure to satisfy the customer’s demand $D$.

**Assumption (2).** $h > 0$.

**Remark 2:** This assumption allows us to exclude meaningless cases.

The continuous flow model is a piecewise linear system and its dynamics change at the occurrence of different discrete events including the failure of the machine $M$, the repair of $M$, buffer full (i.e. $x(t)$ reaches $h$), buffer empty (i.e. $x(t)$ becomes 0).

From equations (1), (2) and (3), it is clear that the production speed of the machine does not change between two events. The discrete event dynamics of the system are given by the following equation:

$$\frac{dx(t)}{dt} = \begin{cases} U - D & \text{si } x(t) < h \text{ et } a(t) = 1 \\ 0 & \text{si } x(t) = h \text{ et } a(t) = 1 \\ -D & \text{si } x(t) > h \text{ ou } a(t) = 0 \end{cases}$$

which can be rewritten as:

$$x(t; h, \xi) = x(0) + \int_0^T (u(t) - D) \, dt$$

The inventory cost is given by:

$$g(x(t)) = \begin{cases} c^+x(t) & \text{si } x(t) \geq 0 \\ -c^-x(t) & \text{si } x(t) < 0 \end{cases}$$

where $c^+$, $c^-$ denote the unit inventory holding and backlog costs, respectively, with $c^+ > 0$, $c^- > 0$. These costs are known and constant. Generally the inventory cost is convex and nonnegative in function of $x$.

The maintenance cost based on a type-bloc strategy is given by (Chelbi & Ait-Kaidi 2004):

$$C_M(T_p) = \frac{c_c \cdot N(T_p) + c_p}{T_p}$$

$T_p$ is the preventive maintenance period, $c_c$ and $c_p$ denote the cost of one corrective action and one preventive action, respectively, and $N(T_p)$ is the mean number of failures at interval $[0, T_p]$.

Nevertheless, such as a simulation based algorithm will be defined, we prefer to define the following equation for the maintenance cost:

$$C_M(t) = c_c \cdot N_c(t) + c_p \cdot N_p(t)$$

where $N_c(t)$ and $N_p(t)$ denote the number of corrective actions and preventive actions, respectively.

Furthermore, in this paper the following hypothesis are considered for the maintenance:

- The costs of preventive and corrective maintenance are known and constant.
- Each maintenance action leaves the machine as good as new.
- Failures are detected instantaneously.
- All of the resources needed to perform the maintenance actions are available at the right time and place.

The objective of this paper is to determine the value of the hedging point $h$ which minimizes the expected total average cost (inventory plus maintenance). In this study, we are going to estimate the long-run average cost, which in a finite horizon, is given by:

$$L(t; h, \xi) = \lim_{T \to \infty} \frac{1}{T} \int_0^T g(x(t; h, \xi)) \, dt + C_M(T)$$

**Assumption (3).** $C_M(T)$ is not a function of $h$, i.e. $T_p$ is $h$-independent.

By assumption (3), we consider that $C_M(T)$ depends only on the preventive maintenance period $T_p$, and $T_p$ is independent of the hedging point $h$.

The expected long-run average cost at interval $[0, \infty)$ is then given by:

$$\min_J(h) = E[L(t; h, \xi)]$$

**Assumption (4).** $J(h)$ is continuously differentiable in $h$.

The proof for this last assumption is similar to the proof given in (Akella & Kumar 1986).

**Assumption (5).** We suppose that $J(h)$ is convex in $h$.

This assumption is necessary to apply the infinitesimal perturbation analysis. Considering that the random variables are given by exponential laws, it could be easily demonstrate that this assumption is verified (see Mourani et al. (2008)).

### 3. INFINITESIMAL PERTURBATIONS ANALYSIS

We turn our intention in this section to the perturbation analysis study. We will study the sample paths of the buffer level. We observe and analyze two sample paths, one is the nominal sample path and the other is the perturbed sample path (see Figure 2). We assumed that the optimal buffer level is increased by a perturbation, denoted by $\Delta h$. In this paper, we consider $\Delta h > 0$ (we can consider $\Delta h < 0$ in a similar way) and we evaluate the resulting changes in the cost function using geometric arguments.

In Figure 2, the red line describes the nominal sample path $x(t; h, \xi)$ of the system, under a hedging point (maximal buffer level) $h$; the blue line corresponds to the perturbed sample path $x(t; h + \Delta h, \xi)$, which is constructed from the nominal sample path by perturbing the hedging point $h$ by $\Delta h$. Thus, the hedging point for the perturbed sample path become $h + \Delta h$. 

---

**References**

- Akella & Kumar (1986)
- Mourani et al. (2008)
Assumption (5). The two sample paths have the same random variable distribution for the times between failures (TBF), times to repairs (TTR) and also the same parameter values.

Assumption (6). Indeed, in this paper, to simplify the theoretical study, we assume that we have no material in the buffer at \( t=0 \).

From Figure 2, we can compare the two sample paths analytically, as follows.

For simplicity, let \( x(t; h, \xi) = (t) \) and \( x(t; h + \Delta h, \xi) = x^{\Delta h}(t) \).

**Theorem (1).** \( x(t) \leq x^{\Delta h}(t) \leq x(t) + \Delta h, \forall t \in [0, T] \).

**Proof.** \( \forall t \in [0, T] \), the nominal sample path is given by:

\[
x(t) = x(0) + \int_0^t (u(t) - D) dt
\]

which amounts to:

\[
x(t) = x(0) + \int_0^{t_1} (u(t) - D) dt + \int_{t_1}^T (u(t) - D) dt
\]

where \( t_1 \) corresponds to a buffer full for the nominal sample path (\( x(t) \) reaches its hedging point).

On the other hand, \( \forall t \in [0, T] \), the perturbed sample path is given by:

\[
x^{\Delta h}(t) = x(0) + \int_0^{t_1} (u(t) - D) dt + \int_{t_1}^T (u(t) - D) dt + \Delta h.
\]

Then, \( \forall t < t_1 \), \( x^{\Delta h}(t) = x(t) = x(0) + \int_0^t (u(t) - D) dt \), and \( \forall t \geq t_1 \), \( x^{\Delta h}(t) = x(t) + \Delta h = x(t_1) + \int_{t_1}^T (u(t) - D) dt + \Delta h \).

Then, \( x(t) \leq x^{\Delta h}(t) \leq x(t) + \Delta h, \forall t \in [0, T] \).

**Corollary (1).** \( 0 \leq x^{\Delta h}(t) - x(t) \leq \Delta h, \forall t \in [0, T] \)

**Proof.** From theorem (1),

\[
x^{\Delta h}(t) - x(t) = 0, \forall t < t_1 \quad \text{and} \quad x^{\Delta h}(t) - x(t) = \Delta h, \forall t \geq t_1.
\]

Then, \( 0 \leq \Delta x(t) \leq \Delta h, \forall t \in [0, T] \).

From Figure 2, we observe that the necessary times, in order that both nominal and sample path reach their respective hedging point, can be expressed by:

\[
t_1 = \frac{h - x(0)}{U - D}
\]

and

\[
t_1^{\Delta h} = \frac{(h + \Delta h) - x(0)}{U - D}
\]

Thereafter, we present the average total cost function expression on an infinite horizon \( T \).

For the nominal sample path:

\[
J_T(h) = E[L_T(t; h, \xi)]
\]

\[
L_T(h) = \frac{1}{T} \int_0^T g(x(t; h, \xi)) dt + C_M(T)
\]

For the perturbed sample path:

\[
J_T(h + \Delta h) = E[L_T(t; h + \Delta h, \xi)]
\]

\[
L_T(h + \Delta h) = \frac{1}{T} \int_0^T g(x(t; h + \Delta h, \xi)) dt + C_M(T)
\]

For an infinitesimal perturbation, the cost function \( L_T(h + \Delta h) \) is given as follows:

\[
L_T(h + \Delta h) = L_T(h) + \frac{1}{T} \int_0^T [c^* \Delta h \cdot T_1(h) - c^* \Delta h \cdot T_2(h)] + C_M(T)
\]

where \( 1(x(t) \geq 0) = 1 \) if \( x(t) \geq 0 \), and null otherwise. A similar definition is used for \( 1(x(t) < 0) \), so,

\[
L_T(h + \Delta h) = L_T(h) + \frac{1}{T} \int_0^T [c^* \Delta h \cdot T_1(h) - c^* \Delta h \cdot T_2(h)] + C_M(T)
\]

where \( T_1(h) \) is the total time when \( x(t) \geq 0 \) and \( T_2(h) \) is the total time when \( x(t) < 0 \).

Finally, we take the derivative of the cost function with respect to the buffer level, to get the IPA estimator, as follows:

\[
\frac{\partial L_T(h)}{\partial \Delta h} = \frac{c^* \cdot T_1(h) - c^* \cdot T_2(h)}{T}
\]

In this paper, the expected value of the sample path derivative obtained by simulation is used instead of the derivative of the expected cost. Therefore, we need to establish the unbiasedness of the gradient estimator (see theorem 2).

**Theorem (2).**

\[
E \frac{\partial L_T(h)}{\partial \Delta h} = E \frac{\partial L_T(h)}{\partial h}
\]

i.e.

\[
\frac{\partial J_T(h)}{\partial h} = \frac{\partial L_T(h)}{\partial h}
\]

**Proof.** To prove this theorem, we must verify the two following conditions (Wardi & Melamed 2001):

\[
\forall h \in H, \quad \text{where} \ H \in \mathbb{R} \quad \text{is an opened interval, the derivative} \ \frac{\partial L_T(h)}{\partial h} \quad \text{exists w.p.1.}
\]

The random function \( L_T(h) \) has a Lipschitz constant with one first finite moment.
To prove the validity of these both conditions, we can use the dominated convergence theorem, which consist in our case, to prove that:

$$\lim_{\Delta h \to 0} \frac{L_T(h + \Delta h) - L_T(h)}{\Delta h} = \frac{\partial L_T(h)}{\partial h}$$

Then, by replacing for our values:

$$\lim_{\Delta h \to 0} \frac{\Delta h [T_1(h) \cdot c^+ + T_2(h) \cdot c^-] - [L_T(h) \cdot T]}{\Delta h}$$

$$= \lim_{\Delta h \to 0} \frac{T_1(h) \cdot c^+ - T_2(h) \cdot c^-}{T} = \frac{\partial L_T(h)}{\partial h}$$

In the following, a simulation based optimization, using the dichotomy method, is given to illustrate our results.

### 4. NUMERICAL RESULTS

To generate the sample paths, we consider exponential random variables for times between failures and times to repair. The parameters are defined in table 1.

#### Table 1. Simulation parameters

<table>
<thead>
<tr>
<th>MTBF (hours)</th>
<th>MTTR (hours)</th>
<th>MTTPM (hours)</th>
<th>Tp (hours)</th>
<th>U (units/hour)</th>
<th>D (units/hour)</th>
<th>c+ (€/unit/hour)</th>
<th>c- (€/unit/hour)</th>
<th>cp (€/action)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>20</td>
<td>10</td>
<td>50</td>
<td>15</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

**MTBF** is the mean time between failures, **MTTR** is the mean time to repair, **Tp** is the preventive maintenance period, **U** is the maximal production speed, **D** is the demand, **c+** is the holding unitary cost, **c-** is the backlogging unitary cost, **cp** is the corrective action cost and **cp** is the preventive action cost.

The simulation was run 20 times, for 11 millions of preventive actions. The simulation results are presented in table 2.

We can note that for our numerical application, the optimal buffer level $h^* = 771.50 \pm 7.14$ units. That is, $h^* = 771.50$ units, with a percentage error of 0.93%.

### Table 2. Simulation results

<table>
<thead>
<tr>
<th>Optimal buffer (h)</th>
<th>Minimal cost</th>
<th>Optimal buffer (h)</th>
<th>Minimal cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>769.3241</td>
<td>987.0280</td>
<td>769.3148</td>
<td>991.6139</td>
</tr>
<tr>
<td>770.1462</td>
<td>993.1721</td>
<td>776.4967</td>
<td>997.3301</td>
</tr>
<tr>
<td>773.5926</td>
<td>998.0458</td>
<td>768.3774</td>
<td>998.4997</td>
</tr>
<tr>
<td>769.1005</td>
<td>987.5054</td>
<td>773.3394</td>
<td>992.9843</td>
</tr>
<tr>
<td>778.3931</td>
<td>1007.5148</td>
<td>771.2167</td>
<td>984.6541</td>
</tr>
<tr>
<td>772.7277</td>
<td>992.9838</td>
<td>765.7588</td>
<td>983.1076</td>
</tr>
<tr>
<td>764.3540</td>
<td>978.5697</td>
<td>771.4851</td>
<td>993.3886</td>
</tr>
<tr>
<td>770.1584</td>
<td>992.3897</td>
<td>773.4800</td>
<td>993.4551</td>
</tr>
<tr>
<td>776.6920</td>
<td>997.2340</td>
<td>776.5576</td>
<td>996.5344</td>
</tr>
<tr>
<td>773.8050</td>
<td>991.6664</td>
<td>765.6532</td>
<td>985.6583</td>
</tr>
</tbody>
</table>

The cost function curve, corresponding to our simulation, is given in figure 3. This curve shows the convexity of our experience. Then, the minimal point of the curve, which denotes the buffer level which minimizes the long run average cost, can easily be estimated. But, it is clear that the system is not completely optimal, because we considered a static preventive maintenance period, which can also be optimized.

![Cost function curve](image1)

**Fig. 3. Cost function curve**

Another interesting graph corresponds to figure 4, which gives the curve of the optimal buffer level with respect to the preventive maintenance period. We note that there is a point which is the minimal of minimum, and it corresponds to a configuration of $h = 73.41$ units and $Tp = 276.7093$ hours. We also observe in this curve that for $Tp > 1000$ hours, the value of $h^*$ is quasi the same.

![Optimal buffer level with respect to the systematic preventive maintenance period](image2)

**Fig. 4. Optimal buffer level with respect to the systematic preventive maintenance period**

Now, in figure 5, we show through a surface graph, the existing relation between the long-run average cost and the two decision variables, $Tp$ and $h$. 

![Surface graph](image3)
Fig. 5. Surface graph

5. CONCLUSIONS

In this paper, we have studied a single-stage single-product failure-prone manufacturing system with constant demand. Machine is subject to time-dependent failures, so a systematic preventive maintenance policy is implemented. A hedging point policy is considered to control the production. By using IPA technique, sample gradient estimator of the cost function with respect to the hedging point is derived. A simulation based on a bisection-method algorithm is run to evaluate the sample path gradient estimator and to determine the optimal hedging point. Another simulation, for different values of preventive maintenance period, is run to determine the optimal configuration (hedging point and preventive maintenance period) of the system in terms of cost. Different graphs show the optimality characteristics of the system.

For future works, our idea is to apply directly infinitesimal perturbation analysis method on the preventive maintenance period. This will allow us to know with precision and with short computation times, the preventive maintenance period and the buffer level which minimize the long-run average cost, under an integrated maintenance policy. We also wish to see the impact of the results of the infinitesimal perturbation analysis when the systematic preventive maintenance period and the buffer level are linked. The application will become much more complex because the estimator will be biased. We also plan to apply this methodology to optimize more complex manufacturing systems, containing several machines, random demands, taking into account other resources, type-age replacement strategies, several types of distribution to represent the times between failures, times to repair and times to preventive maintenance. We will also study the availability as an objective function to maximize.

REFERENCES


