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Flow Stability Analysis of Two-Phase Cooling Systems

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Abstract:
Effective and efficient removal of heat is of critical importance in a wide range of applications, from human comfort to high power electronic devices and high efficiency photovoltaic arrays. This paper considers the stability and control of a cooling system operating in the two-phase regime. Two-phase cooling uses boiling to transfer heat from the source to the working fluid. Compared with the usual single-phase operation which relies on the temperature differential for heat transfer, boiling has superior efficiency by taking advantage of the latent heat of vaporization. However, it is known that two-phase operation is susceptible to various types of instability. We consider a cooling cycle consisting of two heat exchangers, an evaporator to extract heat from the source and a condenser to dissipate the heat to the ambient environment, and a pump and valves to regulate the flow. The heat exchangers are modeled by one-dimensional partial differential equations based on mass balance, momentum balance and energy balance. By using a novel Lyapunov function and impose the condition that the thermal subsystem time constant (in terms of enthalpy) is much slower than the fluid subsystem (in terms of pressure and mass flow rate), we obtain explicit stability condition in terms of the pressure demand curves (pressure drop as a function of the mass flow rate) of the heat exchangers. Using this Lypunov funciton, we derive passivity conditions for the fluid system which may in turn be used for flow stabilization. The general Lyapunov framework presented here may be extended to the stability analysis and control design of more complex thermodynamic systems.

Keywords: Flow stability, cooling system, two-phase flow, passivity

1. INTRODUCTION

The past several decades have witnessed phenomenal advance of microelectronics technology with ever increasing density of electronic components packed into ever shrinking volumes. In applications such as data centers and electric or hybrid vehicles, high density electronic packaging results in increased amount of heat, which is now the key limiting factor of the performance and life time of electronic components [Schmidt and Notohardjono, 2002, Sharma et al., 2005]. For other applications with high heat load, such as photovoltaics, light emitting diodes and laser diodes [Gordon et al., 2004, Arik and Weaver, 2004, Karni et al., 2008], device efficiency is tightly coupled to the operating temperature. If not adequately removed, this heat load would lead to higher operating temperature which at best would compromise performance and at worst cause system malfunction.

The high heat load in these various applications is straining the capability of conventional single phase cooling solutions which rely only on the temperature differential between the heat source and cooling fluid. The latent heat of vaporization in two-phase cooling offers the promising prospect of much more efficient heat removal [Wallace, 2005]. However, it is well known that two-phase cooling is susceptible to thermal-hydrodynamic instability (from nuclear reactor design and operation) [Boure et al., 1973, Ishii and Hibiki, 2005], from the static Ledinegg instability (unstable equilibrium in two-phase regime) to dynamic pressure drop oscillation [Kakac and Bon, 2008].

Most of the past approaches, including our own work, have been based on simple lumped parameter approximations of the heat exchangers [Zhang et al., 2010b]. The few control-oriented approaches start with one-dimensional (1-D) distributed parameter model based on mass, momentum, and energy balance, and then discretize to a low order model [He et al., 1997, Rasmussen, 2005, Rasmussen et al., 2005]. Due to the complexity and nonlinearity of the model, linearization is typically applied with little insight into the physial nature of the system (e.g., preservation of the underlying conservation laws).

In this paper, we start from the same 1-D model for the heat exchangers, but propose a novel energy-motivated Lyapunov function to analyze the stability of the system. We show that the system stability is dependent on the slope of the "demand curve," which relates the channel pressure drop (at each position) to the mass flow rate. Furthermore, if the valve and pump pressures can be controlled, they form passive pairs with the heat exchanger inlet/outlet flow rates and may be used to stabilize the system using strictly passive feedback. Our approach is very general, but does rely on the assmption that the dynamics of the thermal subsystem (enthalpy) is much slower than that of the flow system (mass flow rate and pressure). This work is a major extension of our previous work [Zhang et al., 2010a] which only modeled the evaporator dynamics as a lumped parameter system, and the flow compressibiltiy was modeled by a tank. This paper focuses mainly on the flow stability and stabilization. The flow system is coupled to the thermal system through the coefficient of heat transfer. In our previous work [Zhang et al., 2010b], we have shown that extremum seeking type of technique may be used to find the mass flow rate corresponding to the most efficient heat transfer. This may be easily extended to the full cycle case as considered here.

2. SYSTEM ARCHITECTURE

A common architecture for cooling system in buildings, vehicles, and high power electronic systems is a two-loop configuration as shown in Figure 1 [Lee and Mudawar, 2009, Webb et al., 2007].In the primary loop, a subcooled flow enters the heat exchanger (evaporator) with the heat source. Compared to saturated boiling, subcooled boiling elevates the critical heat flux, an important consideration for high heat flux electronics cooling. A secondary vapor compression cycle dissipates heat to the ambient by using a fluid-to-fluid heat exchanger (condenser). With the two-loop structure, distributed multiple heat loads could be handled with simple and small pump loops, all of which are coupled to a centralized chiller (secondary vapor compression cycle). The coupled vapor compression cycle enables the two-loop cooling system to remove heat from the cold side to the hot side – a feature that is desirable in high heat flux cooling in harsh environments and not achieved by the single loop pumped cooling systems [Chang et al., 2006].The inclusion of the vapor compression cycle in the two-loop system also elevates the temperature difference between the refrigerant and the ambient cooling media, resulting in increased system cooling capacity. The compressibility of the working fluid operating in the two-phase regime in the evaporator can lead to pressure-drop flow oscillations, which will significantly deteriorate the cooling performance and potentially cause burn out of the electronic components. The corresponding pressure-enthalpy diagram for the primary loop is shown in Figure 2.

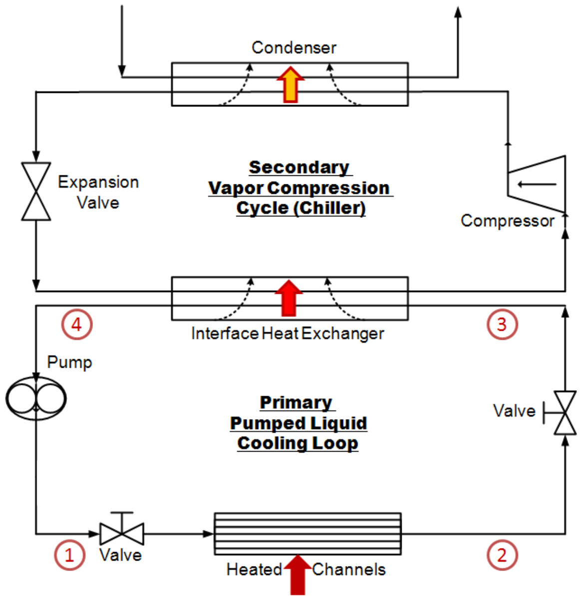

Fig. 1. Schematics of two-loop electronics cooling system

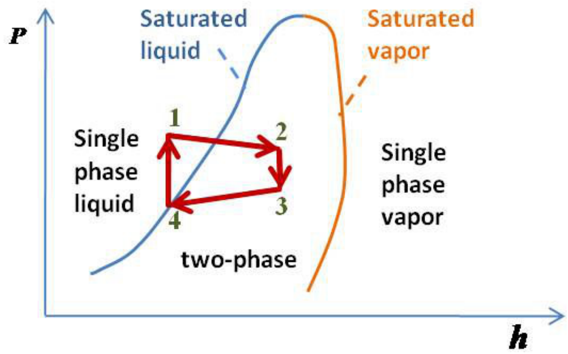

Fig. 2. Pressure-Enthalpy (P-h) diagram of a two-phase cooling loop driven by a pump

3. DYNAMIC MODEL

In this paper, we will focus on the primary loop only which shown in a simplified schematics as in Fig. 3. We consider the 1-D mass, momentum, and energy balance model for the heat exchangers as in [He et al., 1997, Rasmussen, 2005, Rasmussen et al., 2005]. For the evaporator, we have

$$\frac{\partial \rho_e}{\partial t} = -\frac{1}{A_e}\frac{\partial \dot{m}_e}{\partial z} \tag{1a}$$

$$\frac{\partial \dot{m}_e}{\partial t} = -A_e\frac{\partial P_e}{\partial z} - A_e\frac{\partial P_e^D}{\partial z} \tag{1b}$$

$$\frac{\partial(\rho_e h_e - P_e)}{\partial t} = -\frac{1}{A_e}\frac{\partial \dot{m}_e h_e}{\partial z} + \frac{1}{A_e}q_e' \tag{1c}$$

For the consenser, we have

$$\frac{\partial \rho_c}{\partial t} = -\frac{1}{A_c}\frac{\partial \dot{m}_c}{\partial z} \tag{2a}$$

$$\frac{\partial \dot{m}_c}{\partial t} = -A_c\frac{\partial P_c}{\partial z} - A_c\frac{\partial P_c^D}{\partial z} \tag{2b}$$

$$\frac{\partial(\rho_c h_c - P_c)}{\partial t} = -\frac{1}{A_c}\frac{\partial \dot{m}_c h_c}{\partial z} - \frac{1}{A_c}q_c' \tag{2c}$$

where $(\rho_e, \dot{m}_e, h_e, P_e)$ and $(\rho_c, \dot{m}_c, h_c, P_c)$ are the density, mass flow rate, enthalpy, and pressure for the evaporator and condenser, respectively. The heat exchangers are assumed to have uniform cross-sectional areas, A_e and A_c, and lengths, L_e and L_c, respectively. Under uniform heating/cooling conditions, the rate of heat input into the evaporator is denoted by $q_e = q_e' L_e$ and the rate of heat extraction from the condenser is denoted by $q_c = q_c' L_c$. For simplicity in this paper, we ignore the evaporator wall temperature and condenser cooling water dynamics. They may be easily added and will be addressed in our future work. The channel demand pressure drops are denoted by $\frac{\partial P_e^D}{\partial z}$ and $\frac{\partial P_c^D}{\partial z}$ and they are functions of the mass flow rate, $\dot{m}_e$ and $\dot{m}_c$, respectively.

Note that in the two-phase regime, pressure, density, and enthalpy are related (i.e., only two independent variables). For a two-phase mixture with given P and h, the vapor quality is

$$x(h, P) = \frac{h - h_{s\ell}(P)}{h_{sv}(P) - h_{s\ell}(P)} \tag{3}$$

where $h_{s\ell}(P)$ and $h_{sv}(P)$ are the saturated liquid enthapy and saturated vapor enthapy corresponding to the pressure P. The density may be calculated as

$$\rho = \left[\rho_{s\ell}^{-1}(P) + x(h, P)\left(\rho_{sv}^{-1}(P) - \rho_{s\ell}^{-1}(P)\right)\right]^{-1}. \tag{4}$$

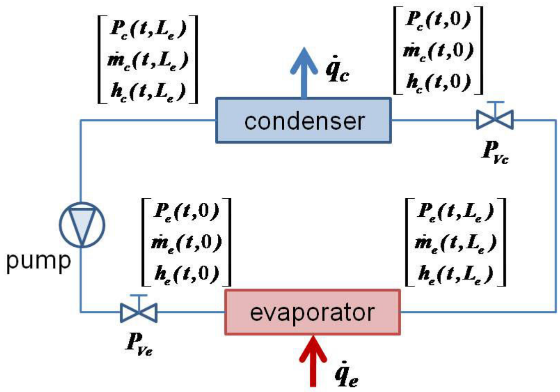

Fig. 3. Schematics of a pump heat transfer loop

Pump and valve normally have much faster dynamics than evaporator and condenser, so their characteristics can be assumed to be static in the pump loop. The boundary conditions for evaporator and condenser are given by

$$\dot{m}_e(t, L_e) = \dot{m}_c(t, 0) \tag{5a}$$
$$h_e(t, L_e) = h_c(t, 0) \tag{5b}$$
$$P_e(t, L_e) - \Delta P_V^c = P_c(t, 0) \tag{5c}$$
$$\dot{m}_e(t, 0) = \dot{m}_c(t, L_c) \tag{5d}$$
$$h_e(t, 0) = h_c(t, L_c) + \frac{\dot{W}_m}{\dot{m}_c(t, 0)} \tag{5e}$$
$$P_c(t, L_c) + \Delta P_m - \Delta P_V^e = P_e(t, 0) \tag{5f}$$

where ΔP_m is the pump differential pressure given by the pump curve (function of the mass flow rate and pump speed, which is controllable), ΔP_V^c and ΔP_V^e are valve pressure drops (may be controlled by adjusting the valve openings) and $\dot{W}_m$ is the pumping power given by

$$\dot{W}_m(t) = \frac{\Delta P_m \dot{m}_c(t, L_c)}{\eta \rho_c(t, L_c)} \tag{6}$$

where η is the pump efficiency.

A key attribute of a heat exchanger is its demand pressure characteristics, P_e^D or P_c^D, as illustrated in Figure 4. It is a function of the mass flow rate and the heat load, and can vary with the channel location z. When the flow rate is high, the exit flow is sub-cooled liquid. (corresponding to the right side of the curve). As the mass flow rate is reduced while other conditions are unchanged, flow boiling will commence. Further reduction in the flow rate will gradually cause vigorous boiling. Since frictional and acceleration pressure drops tend to increase as the void fraction (or flow quality, i.e., the portion of vapor in the two-phase mixture) increases, the pressure-drop will reach a minimum before starting to increase as flow rate continues to decrease. This point is sometimes termed the onset of flow instability (OFI). This region of negative slope in the demand curve, corresponding to the two-phase flow, is the key source of flow instability (and hence operational challenge) of two-phase heat transfer systems [Kakac and Bon, 2008].

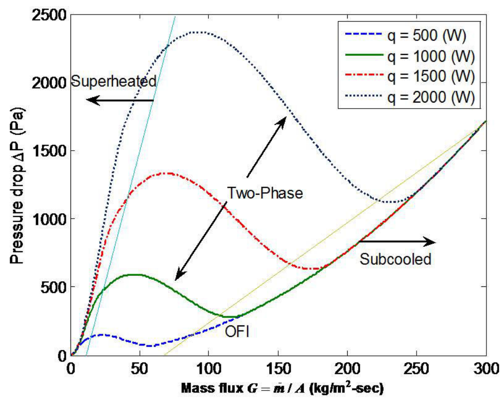

Fig. 4. Typical demand pressure drop curve as a function of mass flux, $G = \dot{m}/A$, and the heat load q

The wall temperature of the evaporator may be modeled by a lumped equation

$$C_{pw} M_w \dot{T}_w = -\alpha_e(\dot{m}_e, q_e) S_e (T_w - T_e(P_e, h_e)) + q_e \tag{7}$$

where T_w is the evaporator wall temperature, $C_{pw} M_w$ is the wall thermal inertia, α_e is the coefficient of heat transfer (dependent on the mass flow rate $\dot{m}_e$ and the heat load q_e), S_e is the evaporator surface area, and T_e is the *averaged* evaporator fluid temperature.

The condenser heat extraction may be modeled by the lumped approximation

$$q_c = \alpha_c S_c (T_c - T_a) \tag{8}$$

where $T_c(P_c, h_c)$ is the averaged condensing fluid temperature, T_a is the chiller cooling water temperature, S_c is the condenser surface area, and α_c is the condensing heat transfer coefficient (assumed to be constant).

4. STABILITY ANALYSIS

Our focus in this paper is primarily on the flow stability. Thermal stability will be briefly discussed, but will be left as future work. Consider the steady state condition for a given pump pressure differential ΔP_m, valve pressure drops, ΔP_V^c and ΔP_V^e, evaporator heat load q_e, condenser heat extraction q_c. Set the right hand side of the flow dynamics (1)–(2) to zero, we have:

$$\dot{m}_e^* = \dot{m}_c^* = \dot{m}^* \tag{9a}$$
$$P_e^*(z) + P_e^D(\dot{m}^*, z)^* = c_e \tag{9b}$$
$$P_c^*(z) + P_c^D(\dot{m}^*, z)^* = c_c \tag{9c}$$
$$h_e^*(z) = h_e^*(0) + \frac{q_e'}{\dot{m}^*} z \tag{9d}$$
$$h_c^*(z) = h_c^*(0) - \frac{q_c'}{\dot{m}^*} z \tag{9e}$$

where $\dot{m}^*$, c_e, c_c, $h_e^*(0)$, $h_c^*(0)$ are constants to be determined. Impose the pressure boundary conditions, we obtain:

$$c_e - P_e^D(\dot{m}^*, L_e) - \Delta P_V^c = c_c - P_c^D(\dot{m}^*, 0) \quad (10a)$$
$$c_e - P_e^D(\dot{m}^*, 0) = c_c - P_c^D(\dot{m}^*, L_c) + \Delta P_m - \Delta P_V^e \quad (10b)$$

There are three constants but only two equations. The conservation of mass is needed for the unique solution of these constants:

$$M = A_e \int_0^{L_e} \rho_c \, dz + A_c \int_0^{L_c} \rho_c \, dz \quad (11)$$

where M is the total refrigerant charge in the system.

The enthalpy boundary conditions are

$$h_e^*(0) + \frac{q_e'}{\dot{m}^*} L_e = h_c^*(0) \quad (12a)$$
$$h_e^*(0) = h_c^*(0) - \frac{q_c'}{\dot{m}^*} L_c + \frac{\dot{W}_m}{\dot{m}^*}. \quad (12b)$$

The two equations are clearly linearly dependent, with the sum of the equations simply the energy balance:

$$q_e' \cdot L_e = q_c' \cdot L_c - \dot{W}_m. \quad (13)$$

Similar to the conservation of mass, conservation of energy is now needed to uniquely determine the remaining constant:

$$U = \int_0^{L_e} A_e \left[\rho(h_e^*, P_e^*) h_e^*(z) - P_e^*\right] dz + \int_0^{L_e} A_c \left[\rho(h_c^*, P_c^*) h_c^*(z) - P_c^*\right] dz \quad (14)$$

where U is the total internal energy of the working fluid. Notice that in (11) and (14), both pump and valve fluid holdups are ignored.

For the flow stability, we first consider the following scalar function (which will be used in the Lyapunov analysis):

$$V_1 = \frac{1}{2A_e} \int_0^{L_e} \tilde{\dot{m}}_e^2 \, dz + \frac{1}{2A_c} \int_0^{L_c} \tilde{\dot{m}}_c^2 \, dz, \quad (15)$$

where

$$\tilde{\dot{m}}_e = \dot{m}_e - \dot{m}^*, \quad \tilde{\dot{m}}_c = \dot{m}_c - \dot{m}^*.$$

The derivative of V_1 along the system trajectory is

$$\begin{aligned}\dot{V}_1 &= A_e^{-1} \int_0^{L_e} \tilde{\dot{m}}_e \frac{\partial \tilde{\dot{m}}_e}{\partial t} \, dz + A_c^{-1} \int_0^{L_c} \tilde{\dot{m}}_c \frac{\partial \tilde{\dot{m}}_c}{\partial t} \, dz \\ &= - \int_0^{L_e} \tilde{\dot{m}}_e \frac{\partial \tilde{P}_e}{\partial z} \, dz - \int_0^{L_c} \tilde{\dot{m}}_c \frac{\partial \tilde{P}_c}{\partial z} \, dz \\ &\quad - \int_0^{L_e} \tilde{\dot{m}}_e \frac{\partial \tilde{P}_e^D}{\partial z} \, dz - \int_0^{L_c} \tilde{\dot{m}}_c \frac{\partial \tilde{P}_c^D}{\partial z} \, dz.\end{aligned}$$

Using the equilibrium conditions (9b) and (9c) and applying integration by parts, we have

$$\begin{aligned}\dot{V}_1 &= -\tilde{\dot{m}}_e \tilde{P}_e \Big|_0^{L_e} - \tilde{\dot{m}}_c \tilde{P}_c \Big|_0^{L_c} \\ &\quad + \int_0^{L_e} (P_e - P_e^*) \frac{\partial \dot{m}_e}{\partial z} \, dz + \int_0^{L_c} (P_c - P_c^*) \frac{\partial \dot{m}_c}{\partial z} \, dz \\ &\quad - \int_0^{L_e} \tilde{\dot{m}}_e \frac{\partial \tilde{P}_e^D}{\partial z} \, dz - \int_0^{L_c} \tilde{\dot{m}}_c \frac{\partial \tilde{P}_c^D}{\partial z} \, dz\end{aligned}$$

where $\tilde{P}_i = P_i - P_i^*$, $i = e, c$.

We next consider the following scalar function:

$$V_2 = A_e \int_0^{L_e} \int_{\rho_0}^{\rho_e} (P_e - P_e^*) d\xi dz + A_c \int_0^{L_c} \int_{\rho_0}^{\rho_c} (P_c - P_c^*) d\xi dz. \quad (16)$$

where ρ_0 is an arbitrary constant reference density. Note that P_i is a function of h_i and ρ_i, $i = e, c$. For the stability analysis, we need to make the following assumption:

Assumption: For the stability analysis, we assume $\frac{\partial P_e}{\partial h_e} \dot{h}_e$ and $\frac{\partial P_c}{\partial h_c} \dot{h}_c$ are sufficiently small so can be neglected.

Since the thermal system is in general much slower in term of its dynamical response than the flow system, this assumption is reasonable. However, more rigorous justification (e.g., through singular perturbation) is necessary but is left as future work. A consequence of this assumption is that the system becomes one-way decoupled in the sense that the thermal system does not affect the flow stability but the flow dynamics does affect the behavior of the thermal system (through the coefficient of heat transfer).

Now differentiate V_2 along the solution and impose the assumption, we have

$$\dot{V}_2 = - \int_0^{L_e} (P_e - P_e^*) \frac{\partial \dot{m}_e}{\partial z} \, dz - \int_0^{L_c} (P_c - P_c^*) \frac{\partial \dot{m}_c}{\partial z} \, dz. \quad (17)$$

Let $V = V_1 + V_2$. Then in $\dot{V}$, the $(P_i - P_i^*) \frac{\partial \dot{m}_i}{\partial z}$, $i = e, c$, terms cancel, and we have

$$\begin{aligned}\dot{V} &= \tilde{\dot{m}}_e(t, 0) \tilde{P}_e(t, 0) - \tilde{\dot{m}}_e(t, L_e) \tilde{P}_e(t, L_e) \\ &\quad + \tilde{\dot{m}}_c(t, 0) \tilde{P}_c(t, 0) - \tilde{\dot{m}}_c(t, L_c) \tilde{P}_c(t, L_c) \\ &\quad - \int_0^{L_e} \tilde{\dot{m}}_e \frac{\partial \tilde{P}_e^D}{\partial z} dz - \int_0^{L_c} \tilde{\dot{m}}_c \frac{\partial \tilde{P}_c^D}{\partial z} dz.\end{aligned}$$

Substituting in the boundary conditions, we have

$$\begin{aligned}\dot{V} &= \tilde{\dot{m}}_e(t, 0)(\Delta \tilde{P}_m - \Delta \tilde{P}_V^e) - \tilde{\dot{m}}_c(t, 0) \Delta \tilde{P}_V^c \\ &\quad - \int_0^{L_e} \tilde{\dot{m}}_e \frac{\partial \tilde{P}_e^D}{\partial z} dz - \int_0^{L_c} \tilde{\dot{m}}_c \frac{\partial \tilde{P}_c^D}{\partial z} dz.\end{aligned} \quad (18)$$

We can make several observations based on this expression.

- If the demand curves for the heat exchangers about their equilibria are positive in the L_2 sense:
$$\left\langle \tilde{\dot{m}}_e, \frac{\partial \tilde{P}_e^D}{\partial z} \right\rangle_{L_2[0, L_e]} \geq 0 \quad (19a)$$
$$\left\langle \tilde{\dot{m}}_c, \frac{\partial \tilde{P}_c^D}{\partial z} \right\rangle_{L_2[0, L_c]} \geq 0 \quad (19b)$$
then the system is passive between $(\Delta \tilde{P}_m - \Delta \tilde{P}_V^e, -\Delta \tilde{P}_V^c)$ and $(\tilde{\dot{m}}_e, \tilde{\dot{m}}_c)$.
- If the pressure drop in the evaporator and condenser, $\Delta P_e^D(\dot{m}_e)$ and $\Delta P_c^D(\dot{m}_c)$, have positive slopes at $\dot{m}^*$ for all z, then the flow system is locally asymptotically stable in the open loop (i.e., $\Delta \tilde{P}_V^e = \Delta \tilde{P}_V^c = \Delta \tilde{P}_m = 0$). This is the case for single-phase operation.
- The pump can enhance the condenser stability. For example, $\Delta \tilde{P}_m$ may be chosen as a strictly passive map from $\tilde{\dot{m}}_e(t, 0)$. However, it cannot directly enhance the condenser stability. Fortunately, condenser characteristics tends to be stable (due to the nature of

ΔP_c^D), therefore, only evaporator stability is usually of concern.

- The valves may be used to enhance stability. Any positive feedback of $\tilde{\dot{m}}_e$ to $\Delta\tilde{P}_V^e$ enhances the stability of the evaporator and a positive feedback of $\tilde{\dot{m}}_c$ to $\Delta\tilde{P}_V^c$ enhances the stability of the condenser.
- To operate in the two-phase regime, ΔP_e^D and ΔP_c^D may have negative slope with respect to flow rate. In that case, a combination of pump and evaporator inlet valve would be sufficient to stabilize the system.
- The implementation of the mass flow rate feedback to the pump and valve pressure differentials requires the steady state pressure values, corresponding to the desired mass flow rate, $\dot{m}^*$. These values depend on the model, which may be uncertain. With the Lyapunov analysis, we may replace the steady state pressure values with their adaptive estimates, which is just integral feedback of $\tilde{\dot{m}}_e$ and $\tilde{\dot{m}}_c$.

So far we have analyzed the stability of the flow subsystem. For the thermal subsystem, consider the total internal energy of the system same as (14):

$$V_T = A_e \int_0^{L_e} (\rho_e h_e - P_e)\, dz + A_c \int_0^{L_c} (\rho_c h_c - P_c)\, dz. \quad (20)$$

Its derivative along the solution is

$$\dot{V}_T = \int_0^{L_e} q_e' dz - \int_0^{L_c} q_c' dz + \dot{W}_m = q_e - q_c + \dot{W}_m = 0 \quad (21)$$

where the last equality follows from the energy balance (i.e., the equilibrium condition). This shows that the total internal energy is conserved.

For the evaporator wall temperature regulation, consider the temperature loop given by (7). This system is always stable, but the heat transfer coefficient $\alpha_e(\dot{m}_e)$ is highly dependent on $\dot{m}_e$ with a general shape as shown in Figure 5. This figure shows that the subcooled flow directly transitions to film boiling [Carey, 1992] under relatively low flow rate. Once the flow rate is increased, nucleate/convective boiling becomes dominant, and the heat transfer performance is significantly enhanced. When the flow rate is further increased, the imposed heat load would not be enough to boil the fluid. Therefore, the heat transfer coefficient for single-phase liquid will decrease again since the thermal conductivity decreases with increasing flow rate for a fixed heat load. However, in the low-medium flow rate range, experimental data are limited because nucleate flow boiling is very sensitive to operating conditions, therefore large uncertainties are associated with the resulting boiling heat transfer model. In general, the nucleate or convective boiling has superior heat transfer performance, but it is very challenging to characterize accurately.

Our strategy for the temperature control is to regulate the flow dynamics to achieve the lowest wall temperature possible. The evaporator wall temperature equilibrium condition under constant q_e is given by :

$$T_w^* = T_e(P_e^*, h_e^*) + \frac{q_e}{\alpha_e(\dot{m}^*, q_e) S_e}. \quad (22)$$

Similar to our previous work [Zhang et al., 2010b], we will manipulate $\dot{m}^*$ to maximize α_e (and therefore minimize T_w^*), and then using the flow stabilizing control described above to achieve the desired $\dot{m}^*$. Since α_e is not directly measurable, we estimate α_e by using the measured T_w to infer its time constant. We are also currently investigating directly minimizing T_w^*.

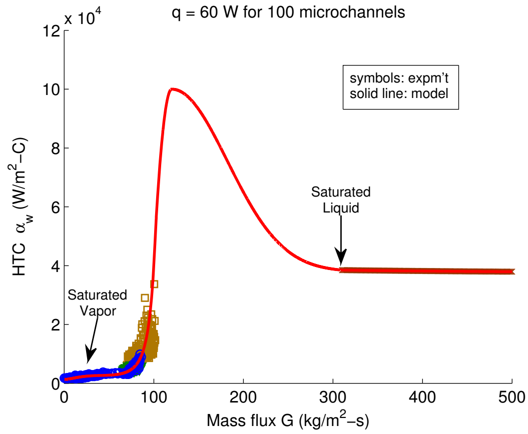

Fig. 5. Typical shape of the heat transfer coefficient, α_e, as a function of the mass flow rate $\dot{m}_e = GA$ at certain heat load [Zhang et al., 2010b].

5. SIMULATION

A pump loop cooling system using water as the working fluid is used as an illustrative example. We use the technique in [Astrom and Bell, 2000] to obtain the pressure drop demand curves for the evaporator and condenser in Fig. 6. Note that when the condenser inlet flow is of two phase, the condenser is always stable (the slope of the demand curve is always positive), but the evaporator becomes unstable in the two-phase regime (the slope of the demand curve is negative).

In this study, the same minichannel size (D=4 mm, L=0.8 m) is used for evaporator and condenser, the imposed heat load q_e=600 W, and the initial mass flux (mass flowrate over cross-sectional area), G=80 kg/m^2-s, as indicated by square symbol in Fig. 6. For simplicity, lump evaporating and condensing flow models are used during simulation.

In the open loop region, the two-phase flow instability is evident, where the valve is full open. At $t = 200s$, the valve controller is turned on, and the flow is stabilized as shown in Fig. 7.

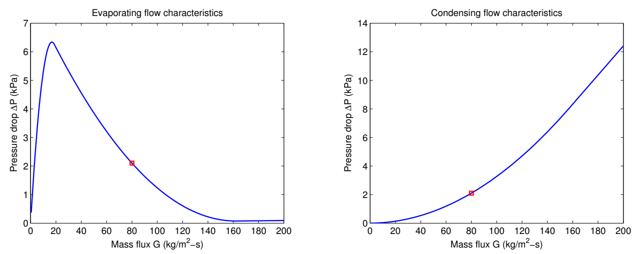

Fig. 6. Evaporator (left) and condenser (right) demand curves for the example pumped cooling loop (squares: steady states)

6. CONCLUSION

This paper considers the dynamics of a two-phase pump cooling loop. The 1-D distributed parameter model based

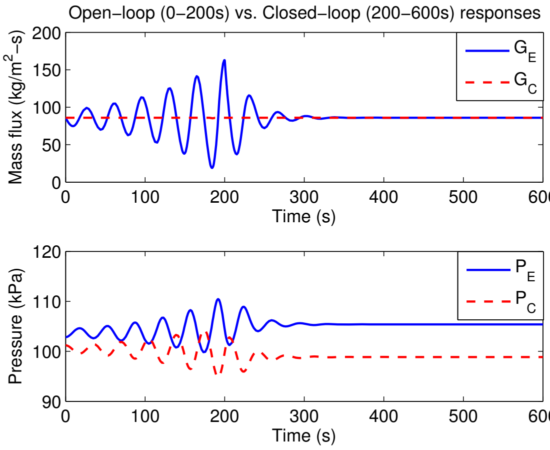

Fig. 7. Flow stabilization with active evaporator inlet valve control (constant pump speed and no condenser valve)

on mass, momentum, and energy balance is used. A novel Lyapunov analysis is performed to analyze the stability about the equilibrium and shows the effectiveness of pump and valves to enhance the stability of the open loop stable system or stabilize the open loop unstable system when it operates in the two-phase regime. A critical assumption in the analysis is that the thermal subsystem is much slower than the flow subsystem. We are currently extending the analysis to rigorously justify this assumption by using singular perturbation. The general framework presented in this paper may be extended to more complex architectures, including vapor compression cycle, inclusion of by-pass storage tanks, and multiple heat exchangers.

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