Simple PID tuning rules with guaranteed $M_s$ robustness achievement

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Abstract: The design of the closed-loop control system must take into account the system performance to load-disturbance and set-point changes and its robustness to variation of the controlled process characteristics, preserving the well-known trade-off among all these variables. This paper face with this combined servo/regulation performance and robustness problem. The proposed method is formulated as an optimization problem for combined performance (not for independent operation modes), including also the robustness property as a constraint. The accomplishment of the claimed robustness is checked and then, the PID controller gives a good performance with also a precise and specific robustness degree. The proposed robust based PID control design is tested against other tuning methods.

Keywords: PID control, Process control, robustness, performance

1. INTRODUCTION

Since their introduction in 1940 (Babb, 1990; Bennett, 2000) commercial Proportional - Integrative - Derivative (PID) controllers have been with no doubt the most extensive option that can be found on industrial control applications (Åström and Hägglund, 2001). Their success is mainly due to its simple structure and to the physical meaning of the corresponding three parameters (therefore making manual tuning possible). This fact makes PID control easier to understand by the control engineers than other more advanced control techniques. In addition, the PID controller provides satisfactory performance in a wide range of practical situations.

With regard to the design and tuning of PID controllers, there are many methods that can be found in the literature over the last sixty years. Special attention is made of the IFAC workshop PID’00 - Past, Present and Future of PID Control, held in Terrassa, Spain, in April 2000, where a glimpse of the state-of-the-art on PID control was provided. Moreover, because of the widespread use of PID controllers, it is interesting to have simple but efficient methods for tuning the controller.

In fact, since the initial work of Ziegler and Nichols (1942), an intensive research has been done, developing autotuning methods to determine the PID controller parameters (Skogestad, 2003; Åström and Hägglund, 2004; Kristiansson and Lennartson, 2006). It can be seen that most of them are concerned with feedback controllers which are tuned either with a view to the rejection of disturbances (Cohen and Coon, 1953; López et al., 1967) or for a well-damped fast response to a step change in the controller set-point (Rovira et al., 1969; Martín et al., 1975; Rivera et al., 1986). Moreover, in some cases the methods considered only the system performance (Ho et al., 1999), or its robustness (Åström and Hägglund, 1984; Ho et al., 1995; Fung et al., 1998). However, the most interesting cases are the ones that combine performance and robustness, because they face with all system’s requirements Ho et al. (1999); Ingimundarson et al. (n.d.); Yaniv and Nagurka (2004); Vilanova (2008).

O’Dwyer (2003) presents a complete collection of tuning rules for PID controllers, which show their abundance. Taking into account that in industrial process control applications, it is required a good load-disturbance rejection (usually known as regulatory-control), as well as, a good transient response to set-point changes (known as servo-control operation), the controller design should consider both possibilities of operation.

Despite the above, the servo and regulation demands cannot be simultaneously satisfied with a One-Degree-of-Freedom (1-DoF) controller, because the resulting dynamic for each operation mode is different and it is possible to choose just one for an optimal solution.

Considering the previous statement, the studies have focused only in fulfilling one of the two requirements, providing tuning methods that are optimal to servo-control or to regulation-control. However, it is well known that if we optimize the closed-loop transfer function for a step-response specification, the performance with respect to load-disturbance attenuation can be very poor and vice-versa (Arrieta and Vilanova, 2010). Therefore, it is desirable to get a compromise design, between servo/regulation, by using 1-DoF controller.

The proposed method considers 1-DoF PID controllers as...
an alternative when explicit Two-Degree-of-Freedom (2-DoF) PID controllers are not available. Therefore, it could be stated that the proposed tuning can be used when both operation modes may happen and it could be seen as an implicit 2-DoF approach (because the design takes into account both objectives, servo and regulation modes) (Arrieta et al., 2010).

Moreover, it is important that every control system provides a certain degree of robustness, in order to preserve the closed-loop dynamics, to possible variations in the process. Therefore, the robustness issue should be included within the multiple trade-offs presented in the control design and it must be solved on a balanced way.

With respect to the robustness issue, during the last years, there has been a perspective change of how to include the robustness considerations. In this sense, there is variation from the classical Gain and Phase Margin measures to a single and more general quantification of robustness, such as the Maximum of the Sensitivity function magnitude.

Taking also into account the importance of the explicit inclusion of robustness into the design, the aim is to look for an optimal tuning for a combined servo/regulation index, that also guarantees a robustness value, specified as a desirable Maximum Sensitivity requirement.

The previous cited methods study the performance and robustness jointly in the control design. However, no one treats specifically the performance/robustness trade-off problem, nor consider in the formulation the servo/regulation trade-off or the interacting between all of these variables.

In addition, this work raises a point not found in the literature, such as the fulfillment of the robustness specification. Many of the existing tuning rules that include robustness constraints never check the accomplishment of such constraint. Therefore, the robustness of the resulting control system is not known. On this respect, we attempt to generate tuning rules (in fact simple tuning rules) that fulfill such constraint, providing at the same time, the maximum combined servo/regulation performance. Therefore, it can be stated the main contribution presented in this paper.

The paper is organized as follows. Section 2 introduces the control system configuration, as well as the general framework. The optimization problem setup is stated in Section 3, where it is defined the combined servo/regulation performance index and the robustness constraint. The proposed PID tuning with guaranteed robustness achievement is in Section 4 and some examples are provided in Section 5. The paper ends in Section 6 with some conclusions.

2. GENERAL FRAMEWORK

2.1 Control system configuration

We consider the feedback control system shown in Fig. 1, where \( P(s) \) is the controlled process, \( C(s) \) is the controller, \( r(s) \) is the set-point, \( u(s) \) is the controller output signal, \( d(s) \) is the load-disturbance and \( y(s) \) is the system output. The output of the ISA-PID controller Åström and Hågglund (2006) is given by

\[
u(s) = K_p \left( 1 + \frac{1}{T_i s} \right) e(s) - K_p \left( \frac{T_d s}{1 + (T_d/N)s} \right) y(s)
\] (1)

Figure 1. Closed-loop control system.

where \( e(s) = r(s) - y(s) \) is the control error, \( K_p \) is the controller static gain, \( T_i \) the integral time constant, \( T_d \) the derivative time constant and the derivative filter constant \( N \) is taken \( N = 10 \) as it is usual practice in industrial controllers.

Also, the process \( P(s) \) is assumed to be modelled by a First-Order-Plus-Dead-Time (FOPDT) transfer function of the form

\[P(s) = \frac{K}{1 + Ts} e^{-Ls}
\] (2)

where \( K \) is the process gain, \( T \) is the time constant and \( L \) is the dead-time. This model is commonly used in process control because is simple and describes the dynamics of many industrial processes approximately (Åström and Hågglund, 2006).

The availability of FOPDT models in the process industry is a well known fact. The generation of such model just needs for a very simple step-test experiment to be applied to the process. From this point of view, to maintain the need for plant experimentation to a minimum is a key point when considering industrial application of a technique.

2.2 Performance

One way to evaluate the performance of control systems is by calculating a cost function based on the error, i.e. the difference between the desired value (set-point) and the actual value of the controlled variable (system’s output). Of course, as larger and longer in time is the error, the system’s performance will be worse.

In this sense, a common reference for the evaluation of the controller performance, is a functional based on the integral of the error like: Integral-Square-Error (ISE), or Integral-Absolute-Error (IAE).

Some approaches had used the ISE criterion, because its definition allows an analytical calculation for the index (Zhuang and Atherton, 1993). However, nowadays can be found in the literature that IAE is the most useful ans suitable index to quantify the performance of the system (Chen and Seborg, 2002; Skogestad, 2003; Åström and Hägglund, 2006; Kristiansson and Lennartson, 2006; Tan et al., 2006). It can be used explicitly in the design stage or just as an evaluation measure.

The formulation of the criterion is stated as

\[ IAE \doteq \int_0^\infty |e(t)| \, dt \approx \int_0^\infty |r(t) - y(t)| \, dt \] (3)

where the index can be measure for changes in the set-point or in the load-disturbance.

2.3 Robustness

Robustness is an important attribute for control systems, because the design procedures are usually based on the
use of low-order linear models identified at the closed-loop operation point. Due to the non-linearity found in most of the industrial process, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system.

As an indication of the system robustness (relative stability) the Sensitivity Function peak value will be used. The control system Maximum Sensitivity is defined as

\[ M_s = \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + C(j\omega)P(j\omega)|} \]

(4)

and recommended values for \( M_s \) are typically within the range 1.4 - 2.0 Åström and Hägglund (2006). The use of the maximum sensitivity as a robustness measure, has the advantage that lower bounds to the Gain, \( A_m \), and Phase, \( \phi_m \), margins (Åström and Hägglund, 2006) can be assured according to

\[ A_m > \frac{M_s}{M_s - 1} ; \phi_m > 2 \sin^{-1} \left( \frac{1}{2M_s} \right) \]

Therefore, to assure \( M_s = 2.0 \) provides what is commonly considered minimum robustness requirement (that translates to \( A_m > 2 \) and \( \phi_m > 29^\circ \), for \( M_s = 1.4 \) we have \( A_m > 3.5 \) and \( \phi_m > 41^\circ \)).

In many cases, robustness is specified as a target value of \( M_s \), however the accomplishment of the resulting value is never checked.

3. OPTIMIZATION PROBLEM SETUP

From the above definitions for performance and robustness specifications, there appears the need to formulate a joint criteria that faces with the trade-off between the performance for servo and regulation operation and also that takes into account the accomplishment of a robustness level.

3.1 Servo/Regulation trade-off

As it is known, there is a trade-off behavior between the dynamics for servo and regulation control operation modes. It is not enough just to consider the tuning mode, it is also necessary to include the system operation in the controller’s design.

Using some of the exposed ideas, we can say that \( J^*_r \) represents the criteria (3) taking into account the operation mode \( x \), for a tuning mode \( z \). From this, we can post the following definitions:

- \( J^*_r \) is the value of performance index for the set-point tuning operating in servo-control mode.
- \( J^*_d \) is the value of performance index for the load-disturbance tuning operating in servo-control mode.
- \( J^*_o \) is the value of performance index for the load-disturbance tuning operating in regulatory-control mode.

Obviously \( J^*_r \) is the optimal value for servo-control operation, \( J^*_o \), and \( J^*_d \) is the optimal one for regulation, \( J^*_o \). An intermediate tuning between servo and regulation operation modes should have higher values than the optimal ones, when the tuning and operation modes are the same, but the indexes would be lower when the modes are different. So, for each operation mode we have the following relationships,

\[ J^*_r \leq J^*_r \leq J^*_r \leq J^*_r \]
\[ J^*_d \leq J^*_d \leq J^*_d \leq J^*_d \]

(5)

where \( J^*_r \) and \( J^*_d \) are the performance values of the intermediate tuning for servo and regulation control operation, respectively.

The previous ideas can be represented graphically, the results are shown in Fig. 2, where the performance indexes are plotted in the \( J_r - J_d \) plane. It can be seen that the point \((J^*_r, J^*_d)\) is the “ideal” one because it represents the minimum performance values taking both possible operation modes, servo and regulation, into account. However, this point is unreachable due the differences in the dynamics for each one of the objectives of the control operation modes. Therefore our efforts must go towards getting the minimum resulting distance, meaning the best balance between the operation modes.

On this way, a cost objective function is formulated in order to get closer, as much as possible, the resulting point \((J^*_r, J^*_d)\), to the “ideal” one, \((J^*_r, J^*_d)\). Therefore,

\[ J_r = \sqrt{(J^*_r - J^*_r)^2 + (J^*_d - J^*_d)^2} \]

(6)

where \( J^*_r \) and \( J^*_d \) are the optimal values for servo and regulation control respectively, and \( J^*_r \) and \( J^*_d \) are the performance indexes for the intermediate tuning considering both operation modes. In Fig. 2, index (5) is represented by the arrow between the “ideal” point and the corresponding to the intermediate tuning.

From the above analysis, the optimization problem setup considers the model’s normalized dead-times, \( \tau \), in the range 0.1 \( < \tau \leq 2.0 \), to obtain the PID controller optimum parameters such that

\[ \tau_0 := [K_p, T_i, T_d] = \arg \left[ \min_\tau J_r \right] \]

(6)

where \( \tau \) is the PID controller parameters vector. Here, optimization is done using genetic algorithms technique (Mitchell, 1998).

The aim of minimizing (6) is to achieve a balanced performance for both operation modes of the control system.
3.2 Robustness constraint criterion

The cost functional (5) proposed before, even though face with the trade-off problem between the operation modes of the system, just takes into account characteristics of performance. However, there is a need to include a certain robustness for the control-loop. In that sense, we want to use (4) as a robustness measure. So, the optimization problem (6) is subject to a constraint of the form

\[ |M_s - M_s^d| = 0 \]  

(7)

where \( M_s \) and \( M_s^d \) are the Maximum Sensitivity and the desired Maximum Sensitivity functions respectively. This constraint tries to guarantee the selected robustness value for the control system.

4. SERVO/REGULATION PID TUNINGS WITH ROBUSTNESS CONSIDERATION

From the previous formulation, we look for a tuning rule that faces with the trade-off problem between the performance for servo and regulation modes and providing, at the same time, a certain degree of robustness (if necessary).

As it has been stated, we solve the optimization problem (6) subject to constraint (7). In that sense, a broad classification can be established, using specific values for \( M_s \), within the suggested range between 1.4 – 2.0. This will allow a qualitative specification for the control system robustness. So, the rating is described here as

- Low robustness level - \( M_s = 2.0 \)
- Medium-low robustness level - \( M_s = 1.8 \)
- Medium-high robustness level - \( M_s = 1.6 \)
- High robustness level - \( M_s = 1.4 \)

According to this principle, the above mentioned four values for \( M_s \) are used here as desirable robustness, \( M_s^d \) into the robustness constraint (7), for the problem optimization (6). Additionally, an unconstrained optimization is done, that can be seen as the \( M_s^d \) free case.

In order to provide results for autotuning methodology, the optimal sets for the PID parameters with the corresponding desired robustness, are approximated in equations for each controller’s parameter. This fitting procedure looks for simple expressions that allow for an homogenized set, to preserve the simplicity and completeness of the approach.

Therefore, the resulting controller parameters will be, expressed just in terms of the FOPDT process model parameters (2) as

\[ K_p K = a_1 \tau^{b_1} + c_1 \]
\[ T_i = a_2 \tau^{b_2} + c_2 \]
\[ T_d = a_3 \tau^{b_3} + c_3 \]  

(8)

where the constants \( a_1, b_1 \) and \( c_1 \) are given in Table 1, according to the desired robustness level for the control system.

It is important to note that, although there may be other tuning equations that provide a good fit, the choice of the proposals (8) represents an option to retain the simplicity that can be seen as the \( M_s^d \) free case.

In the literature, there are many control designs that include robustness in the formulation stage and even more, in some cases the consideration is regarded as a parameter design directly. However, none of these methods check the accomplishment of the claimed robustness and this should be an aspect that deserves much attention.

The deviation of the resulting value of \( M_s \) with respect to the specified target has a direct influence (as a trade-off) in the performance of the system (Vilanova et al., 2010). In order to guarantee the selected robustness, the constraint stated in (7) forces the optimization problem to fulfill the fixed value \( M_s^d \) and for this, the minimum of the performance index \( J_{rd} \) is achieved.

Here, the resulting robustness, applying the proposed methodology, is compared to the desired one, in order to check the accomplishment of the claimed robustness. Fig. 3 shows that the robust tuning has a very good accuracy for the \( M_s \) values for all the range of processes, therefore assuring that performance is the best one that can be achieved for that robustness value. From the very well known performance-robustness trade-off, the increase of the system’s robustness from the \( M_s^d \)-free case (no robustness constraint), is reflected in a deterioration of the system’s performance, and vice-versa. Similar to Fig. 3, where it can be seen the robustness increasing, in Fig. 4 it is shown how the performance index \( J_{rd} \) varies, for each one of the proposed robustness levels. If we use the information of Fig. 3 and Fig. 4, and the unconstrained case as the starting point, it is possible to see that for

![Figure 3. Accomplishment for each claimed robustness level.](image)
Figure 4. Combined index $J_{rd}$ for each robustness level tuning.

In each selected level, the robustness is improved achieving smaller values for $M_s$, but at the same time having larger values (i.e. worse) for the performance index $J_{rd}$.

It is also important to note that, the relation between the loss of performance and the robustness increase (for each level of $M_s$) is nonlinear, neither for the $\tau$ range. For example, in Fig. 4 the difference between the performance for cases $M_s^d = 1.8$ and $M_s^d = 1.6$, is much smaller than the one for $M_s^d = 1.6$ and $M_s^d = 1.4$, despite that the levels are equally separated.

In general terms, it is possible to say that the robustness requirements are fulfilled, facing at the same time, to the performance servo/regulation trade-off problem.

5. COMPARATIVE EXAMPLE

This section presents an example in order to evaluate the characteristics of the proposed tuning rule. The analysis is not only for a specific process, but for the whole set of plants provided in their range of validity, $\tau \in [0,1,2.0]$.

In order to show the global advantages that the proposal can provide.

The robust tuning rules that can be found in the literature consider different specifications for $M_s$. They range from the considered minimum robustness; $M_s = 2.0$; to a high robustness; $M_s = 1.4$.

Here, we compare the tuning proposed in Section 4 with the following methods:

- AMIGO method (Åström and Hägglund, 2004) provides tuning with a design specification of $M_s = 1.4$.
- Kappa-Tau ($\kappa - \tau$) method (Åström and Hägglund, 1995) provides tuning with a design specification of $M_s = 1.4$ and $M_s = 2.0$.
- Tavakoli method (Tavakoli et al., 2005) provides tuning with a design specification of $M_s = 2.0$.

Fig. 5 shows the achieved $M_s$ values for 1.4 and 2.0 cases, for the compared tuning rules. With this information and the one in Fig. 3, it seems that the proposed tuning is the option that provides the best accuracy for the selected robustness.

With the aim to establish a more precise and quantitative measure of the claimed robustness accomplishment for the whole range of models, the next index is stated

$$I_{M_s} = \int_{\tau_o}^{\tau_f} |M_s^d(\tau) - M_s^d(\tau)|d\tau$$  \hspace{1cm} (9)

where $M_s^d$ and $M_s^d$ are the desired and resulting $M_s$ values, respectively. As $I_{M_s}$ is smaller, the accuracy is better. In Table 2 there are the values (9) for the analyzed tuning rules. Now, from the plots in Figs. 3 and 5, and the measured values (9) in Table 2, it is possible to say that the proposed PID tuning (using the levels classification), is the one that provides the best accomplishment between the desirable and the achieved robustness.

Once the robustness accomplishment has been verified, it is important to see the resulting performance for the compared tuning rules. Fig. 6 shows the combined servo/regularization performance index (5). For the $M_s^d = 2.0$ it is obvious that the proposed tuning is the one that provides the best robustness accomplishment and at the same time, the best achievable performance. For $M_s^d = 1.4$ case, because AMIGO method does not fulfill the robustness requirements, having a somewhat lower robustness, it has values slightly lower for $J_{rd}$ index compared to the proposed tuning. However, the proposal is more accurate for the claimed robustness with also good performance (see Fig. 6). This fact strongly confirms the importance of the relation between robustness and performance variations.

6. CONCLUSIONS

In process control, it is very important to guarantee some degree of robustness, in order to preserve the closed-loop performance servo/regulation.
dynamics, to possible variations in the control system. Also, at the same time, it must be provided the best achievable performance for servo and regulation operation. All of the above specifications, lead to different trade-offs, between performance and robustness or between servo and regulation modes, that must be solved on a balanced way. Here, we looked for a PID controller tuning rule that faces to the general problem. This tuning is optimal, as much as possible, to a proposed performance index that takes into account both system operation modes, including also a certain degree of robustness, specified as a desirable Maximum Sensitivity value.

Autotuning formulae have been presented for two approaches. First, robustness is established using a qualitative levels classification and then, the idea is extended to an issue that offers a generic expression, to allow the specification in terms of any value of robustness in the range $M_\varepsilon \in [1.4,2.0]$. Moreover, taking into account the performance/robustness trade-off, the accuracy of the claimed robustness is a point that has been verified, achieving flat curves for the resulting values. In short, both approaches are the main contributions presented in this paper.

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