Methods of Trajectory Tracking for Flexible Joint Space Manipulators

Steve Ulrich and Jurek Z. Sasiadek

Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, K1S5B6, Canada
(Tel: 613-520-2600 ext. 1833; e-mail: sulrich@connect.carleton.ca).

Abstract: This paper presents a comparative study of four control strategies for a flexible joint space
manipulator. The considered control strategies consist in the classical Slotine and Li algorithm, a simple
proportional derivative controller, a singular perturbation-based controller and a nonlinear backstepping
controller. All control methodologies are assessed in numerical simulations for endpoint positioning while
tracking a 12.6 × 12.6 m square trajectory by a two-link flexible joint space robot, thus providing a
common framework to compare the respective performance of the considered control schemes. Simulation
results indicate that controlling both nonlinearities and joint flexibility effects improve the closed-loop
behavior of the space robot where the control of nonlinearities is of greater importance in the sense that it
provides the most significant improvements. While each control scheme tracks the trajectory with a
different degree of accuracy, the best performance for this particular application is obtained with the
nonlinear backstepping control strategy.

1. INTRODUCTION

The control of flexible joint space robotic manipulators represents a very challenging problem, mainly because the
number of degrees of freedom of the system is twice as the number of control actions. Although it is well known that
space robots equipped with planetary gears exhibit joint vibration effects (Kahraman and Vijayakar, 2001), the
flexible joint problem is primarily caused by the use of harmonic drives, i.e. a type of gear mechanism that is
increasingly popular for use in terrestrial and space robotic applications due to its low backlash, low weight,
compactness, high torque capability, wide operating temperature range and good repeatability. However, with
harmonic drives, the joint flexibility problem is significant. Considering the joint flexibility in the analysis and design of
control systems is therefore essential, especially when accurate endpoint positioning must be achieved. In some
cases, joint flexibility can lead to instability when neglected in the control design, as explained by Sweet and Good

Over the past decades, several solutions have been proposed to address the flexible joint control problem, among them
proportional derivative (PD) techniques. Tomei (1991) proposes a simple PD regulator for flexible joint robots, similar to that used for rigid robots. Simulation results for a regulation problem about a reference position are provided.

An extension of the PD regulator for robot manipulator considering joint flexibility, actuators dynamics as well as friction is presented by Lozano et al. (1999). More recently, a PD controller with on-line gravity compensation was proposed for regulation tasks by De Luca, Siciliano and Zollo (2005). Besides PD controllers, singular perturbation-based (SPB) strategies have been investigated by several researchers, such as Spong (1989, 1995), Ghorbel, Hung and Spong (1989), Chang and Daniel (1992), and Huang, Ge and Lee (2004). Using the principle under which a flexible manipulator acts on a two-scale behavior, SPB controllers consist of a slow control action designed on the basis of a rigid robot model and a fast control action designed to damp the elastic oscillations at the joints. Ott, Albu-Schaffer and Hirzinger (2002) experimentally verified the so-called Slotine and Li (SLI) control law designed by Slotine and Li (1988) for rigid joint manipulators in combination with a fast joint torque controller. The experiments have been conducted with a two joint light-weight robot at the German Aerospace Center (DLR). Recently, a model reference adaptive control (MRAC) approach proposed by Ulrich and Sasiadek (2010a) for rigid joint space manipulators was successfully extended to flexible joint space robots using the SPB approach (Ulrich and Sasiadek, 2010b). The inherent nonlinearities associated with flexible joint robots also naturally led to the development of nonlinear control schemes, namely by De Luca, Isidori and Nico1 (1985) and Khorasani (1990). Inspired from Kokotovic (1991), nonlinear backstepping approaches have been proposed by Nicosia and Tomei (1993), Brogliato, Ortega and Lozano (1995) and Oh and Lee (1999). An adaptive backstepping method using tuning functions has also been developed by Macnab, D’Eleuterio and Meng (2004). A detailed survey on flexible joint control techniques is provided by Ulrich and Sasiadek (2009).

Besides the MRAC strategies developed by Ulrich and Sasiadek (2010a, 2010b), the tracking performance of all aforementioned joint-based control schemes have been assessed (either numerically or experimentally) while tracking smooth trajectories, such as sine or cosine waveforms of low-frequencies. In addition, those controllers have all been tested on manipulators having relatively short links. The problem with such robots is that joint elasticity effects are not as noticeable as they would be on robots with long links where joint vibrations may have greater impacts on the endpoint positioning.
Within this context, the objective of this study is to compare the performance of four of the most widely used flexible joint controllers in a 12.6 × 12.6 m square trajectory tracking scenario. More specifically, the Slotine and Li (SLI) algorithm for rigid joint robots developed by Slotine and Li (1998), the PD controller proposed by Tomei (1991), the SPB controller designed by Spong (1989) and the nonlinear backstepping control law introduced by Brogliato, Ortega and Lozano (1995) are considered. For the sake of brevity, the theoretical aspects of the original controllers are not recalled and only the modified versions suitable for space applications – where the dynamics is not influenced by gravitational forces – are presented. The interested readers are referred to the above cited papers for more details. The choice of these controllers enables the clear dissociation of the effects of nonlinearities from the joint elasticity effects. Indeed, once the rigid-based SLI controller is implemented, it will be possible to assess how the flexible ones improve the closed-loop behavior. Similarly, it will be possible to see the effects of using nonlinear controllers instead of the linear PD controller. Moreover, these controllers will allow the comparison of the results obtained from a very simple way of compensating for the joint flexibility effects (SPB controller) to a more complex controller (backstepping controller).

Within this context, the developments and results presented in this paper represent an extension to space manipulators of previous numerical and experimental comparative studies on flexible joint control schemes presented by Brogliato, Ortega and Lozano (1995), Brogliato et al. (1998) and Brogliato and Rey (1998).

2. DYNAMICS MODELING

The two-link flexible robot shown in Fig. 1 has planar motion and vibration modes. As with previous studies on the control of flexible space manipulators (Green and Sasiadek, 2005) (Cao and de Silva, 2006), an inertially-stabilized platform assumption is adopted in this work. The gravitational potential energy is omitted for space robot applications.

The well-established dynamic model of a flexible joint robot proposed by Spong (1987) is given by
\[
\begin{align*}
\mathbf{M}_r(q) \ddot{q} + \mathbf{C}_r(q, \dot{q}) \dot{q} - \mathbf{k}(q_m - q) &= 0 \\
\mathbf{J}_m \dot{q}_m + \mathbf{k}(q_m - q) &= \mathbf{\tau}
\end{align*}
\]
where \(\mathbf{M}_r(q)\) is the inertia matrix, \(\mathbf{C}_r(q, \dot{q})\) is related to Coriolis and centrifugal efforts, \(\mathbf{k}\) is the constant diagonal spring stiffness matrix, \(\dot{q}\) is the link angle vector, \(q_m\) is the motor shaft angle vector, \(\mathbf{J}_m\) is the actuator inertia matrix and \(\mathbf{\tau}\) is the control input vector. In such a system model, the link dynamics equation (1) and the actuator dynamics equation (2) are only coupled by the elastic torque term \(q - q_m\).

3. CONTROL METHODOLOGIES

In this section, the four flexible joint control schemes considered in this comparative study are presented.

3.1 Slotine and Li Control

The SLI control law for rigid joint robots consists of a PD feedback term and a full dynamics feedforward compensation term. An advantage of this control scheme is that it is computationally simple. In particular, it requires neither feedback of joint accelerations nor inversion of the manipulator inertia matrix. For space applications, giving a twice differentiable and bounded link reference trajectory \(q_r\) and omitting the gravitational effects, the resulting SLI control law is given by
\[
\mathbf{\tau}_r = \mathbf{M}_r(q) \dot{q}_r + \mathbf{C}_r(q, \dot{q}_r) \dot{q}_r - \mathbf{K}_s \mathbf{s}
\]
where \(\mathbf{K}_s\) is a diagonal matrix of positive gains and where
\[
\begin{align*}
\dot{q}_r &= \dot{q}_r + \lambda (\dot{q}_r - \dot{q}) \\
\dot{q}_r &= \dot{q}_r + \lambda (\dot{q}_r - \dot{q}) \\
\mathbf{s} &= -(q_r - \dot{q}_r) - \lambda (\dot{q}_r - \dot{q}) = q_r - \dot{q}_r
\end{align*}
\]
In (4) to (6), \(\lambda\) is a diagonal positive constant gain matrix. Note that the control law given by (3) does not contain a \(\mathbf{K}_s\) term since the position error \(q_r - \dot{q}_r\) is already included in \(\dot{q}_r\).

3.2 Proportional Derivative Control

Taking benefit of the passivity properties and of the particular structure of the dynamic equations, the simple PD controller proposed by Tomei (1991) can globally stabilize about a reference position robots having elastic joints. The PD controller consists of a linear position and velocity feedback, plus a term to compensate the gravitational forces. Omitting the gravitational forces, the PD control law can be rewritten as
\[
\mathbf{\tau} = \mathbf{K}_p(q_r - q_m) - \mathbf{K}_d \dot{q}_m
\]
where $K_p$ and $K_d$ are the constant proportional and derivative control gains matrix, respectively. As seen in (7), the PD control law can be implemented by using only position and velocity measurements where the proportional gain matrix multiplies the error between $q$ and $q_m$. With the original controller designed by Tomei (1991), the desired link angle vector, $q_m$, is related to the desired motor shaft angle vector, $q_{mc}$, by subtracting a term of the form $k^{-1}g$ to $q_{mc}$, where $g$ is the gravity vector. Therefore, for space applications, the term $k^{-1}g$ is null and and $q_{mc}$ is directly equal to $q_m$. From a robustness point of view, the stability of the original PD controller is therefore affected by uncertainties in joint stiffness coefficients (since they appear explicitly in the control law). Since the space-based PD control law given by (7) is now independent of the joint stiffness matrix, its stability is therefore not dependent on an accurate knowledge of the stiffness matrix, although a given set of control gains (tuned for a given system with a specific joint stiffness matrix) may not necessarily be robust to large variations or modeling errors in the joint stiffness matrix.

3.3 Singular Perturbation-Based Control

Under the assumption of weak joint elasticity, the SLI control law which achieves robust tracking of the reference trajectory for perfectly rigid robots was modified by Spong (1989) to control flexible joint robots using the singular perturbation-based (SPB) methodology. Specifically, the SPB approach consists in a fast control term designed to damp the elastic vibrations at the joints which is added to a slow control term designed on the basis of a rigid robot model. As explained by Kokotovic, Khalil and O'Reilly (1999) a SPB (or composite) control law is of the form

$$
\tau = \tau_s + \tau_f
$$

where $\tau_s$ is designed on the basis of a rigid robot model and $\tau_f$ is designed to damp the elastic oscillations at the joints. The resulting SPB controller applicable to flexible joint space robots consists in the SLI scheme given by (3) using link variables $q$, $\dot{q}$ plus a fast control term consisting in a linear correction term of the form $K_c^f(q - q_m)$, as follows

$$
\tau = \tau_s + K_c^f(q - q_m)
$$

where $K_c^f$ is a constant diagonal gain matrix.

3.4 Nonlinear Backstepping Control

Brogliato, Ortega and Lozano (1995) present three nonlinear design techniques for global stabilization of flexible joint robots, among them the so-called nonlinear backstepping control methodology. The backstepping control scheme combines the cascade decomposition property of the model and the integrator augmentation stabilization of Kokotovic and Sussmann (1989). The resulting nonlinear backstepping controller represents a more general case the one proposed by Nicosia and Tomei (1993) and is given by

$$
\tau = J_m[q_{mc} - 2(q_{mc} - q_m) - 2(q_m - q)]
$$

where

$$
q_{mc} = k^{-1}\tau_s + q
$$

Hence, for space robots, $\tau_s$ and $\tau_f$ (which are introduced by the use of $q_{mc}$ and $\dot{q}_{mc}$, respectively) are given by

$$
\tau_s = M(q)\ddot{q} + C(q, \dot{q})\dot{q} - K_s \dot{q}
$$

and

$$
\tau_f = M(q)\dddot{q} + 2M(q)\dot{q} + C(q, \dot{q})\ddot{q} + C(q, \dot{q})\dot{q} - K_f \dot{q}
$$

In practice, no acceleration or jerk measurements are required to implement this controller. Indeed, the second derivative of the link angular position vector is obtained analytically by inverting the link dynamics equation (1), and the third derivative is obtained by time-differentiating the second derivative expression. As shown in (10)-(13), the resulting backstepping controller has a complex structure involving physical parameters of the system in a highly nonlinear way.

4. SIMULATION RESULTS

In this section, the results obtained by implementing the control schemes reviewed in this study (SLI, PD, SPB, and backstepping) are analyzed. As discussed earlier, this allows a clear dissociation between the effects of the nonlinearities and the effects of the flexibility (once the SLI rigid-based controller is implemented, one can see how the flexible ones improve the closed-loop behavior). Moreover, this enables the comparison of the results obtained from a very simple way of compensating for the flexible effects (SPB scheme) to a more complex controller (backstepping). For each controller, a reference (or commanded) 12.6 m \times 12.6 m square trajectory has been tested, in a counter-clockwise direction starting at the lower-right-hand corner. Unlike the work of Brogliato et al. (1998) in which the desired trajectories have been chosen to be smooth (sine-type trajectories), a large square trajectory represents greater control challenges. Indeed, the corner of the square consists in abrupt changes in directions. Hence, it is required that the endpoint reaches each corner and then redirects itself along an orthogonal direction with minimum overshoot and settling time. Moreover, the commanded square trajectory is fast enough to render the nonlinearities and flexibility effects significant (60 seconds for the complete square trajectory).

Note that the selection of feedback gains for the proposed controllers is far from evident. Indeed, even in the linear case, it is in general not possible to place the poles arbitrarily. This
Fig. 2. PD trajectory tracking results

Fig. 3. SLI trajectory tracking results

Fig. 4. SPB trajectory tracking results
is due to the structure of the input, which does not reduce to a pole-placement controller even if the plant would be linear. This is confirmed by the work of Brogliato et al. (1998) in which the authors were not able to tune the gains of their different backstepping controllers to yield stable closed-loop systems. For these reasons, the controllers’ parameters have been tuned (even for the linear case) in numerical simulations to obtain optimal tracking results. Concerning the SLI algorithm, one must replace $q$ with $q_m$, that is, use the motor variables instead of the link variables, since the system is assumed rigid. The SLI control gains are selected as $\Lambda = I$ and $K_x = 2I$, the PD control gains as $K_p = 3025I$ and $K_d = 1000I$, the SPB control gains as $\Lambda = I$, $K_p = 2I$ and $K_d = 5I$ and the backstepping control gains as $\Lambda = I$ and $K_d = 2I$. The parameters of the two-link flexible joint robot are: $I_1 = I_2 = 4.5$ mN·m, $m_1 = m_2 = 1.5075$ kg, $J_{m1} = J_{m2} = 1$ kg·m$^2$ and $k_1 = k_2 = 500$ N·m/rad.

The commanded and actual control trajectories, and elastic joint vibrations, $q - q_m$, for the PD, SLI, SPB and backstepping control strategies are depicted in Figs. 2 to 5, respectively. Although the linear PD algorithm provides a stable closed-loop behavior, the resulting tracking errors are relatively large: $+0.70/-0.75$ m along the x axis and $+0.45/-0.70$ m along the y axis. As shown in Fig. 2, this is caused as a result of the sustained elastic vibrations at the joints. As shown in Fig. 3 the SLI scheme provides better tracking results, illustrating the importance of controlling the nonlinearities. However, the improvements obtained with the SLI controller over the PD algorithm come at the cost of higher control torques (not shown in figures), which are required to damp the elastic joint vibrations in a few seconds. The results obtained with the SPB scheme show that controlling the flexibility effects can further improve the tracking results, as illustrated in Fig. 4 where the tracking error is $+0.15/-0.17$ m along the x axis and $+0.13/-0.16$ m along the y axis with minimal overshoots occurring at the direction switches, compared to $+0.23/-0.29$ m along the x axis and $+0.27/-0.16$ m along the y axis for the SLI strategy. As with the SLI scheme, the SPB strategy requires high torques in order to damp the elastic vibrations at the joints. Finally, the results shown in Fig. 5 obtained with a more complex strategy to control the joint flexibilities; the nonlinear backstepping control method, exhibit rapid settling to a zero-tracking error in steady-state, with no overshoots at each corner of the trajectory.

These results clearly demonstrate that both nonlinearities and joint flexibility effects control improve the closed-loop behavior where the control of nonlinearities is of greater importance in the sense that it provides the most significant improvements. Also, the simulation results obtained suggest that a nonlinear model-based method to control the flexibility effects provides the best tracking results. For these reasons that are supported by simulation results, the most promising technique for this specific application, from a trajectory tracking point of view, would be the backstepping control method. However, this controller may not be the best from a viewpoint of practical use, since the high torque levels which are required to perfectly track the trajectory could not be implemented by typical joint motors ($4.0 \times 10^7$ N·m for joint 1 and $9.3 \times 10^7$ N·m for joint 2, initially). Note that this study did not aim at classifying several design techniques from a general point of view, as it is clear that the results depend on the nature of the process to be controlled, and that they might significantly vary if applied to another plant.

5. CONCLUSION

In this paper, four different flexible joint control schemes have been numerically compared in a $12.6 \times 12.6$ square trajectory tracking scenario. The considered control techniques are: the so-called Slotine and Li control scheme, a PD controller, a singular perturbation-based (SPB) algorithm and a backstepping controller. Initially, imprecise trajectory tracking results are obtained using the PD controller and then greatly improved results are achieved using the nonlinear Slotine and Li scheme designed as if the system was perfectly rigid. Further improvements have been obtained with the
SPB method which considers the joint flexibilities. Finally, the nonlinear backstepping strategy further improved the tracking performance resulting in an almost perfect square trajectory with no overshoots at each corner. The main conclusion is that although a simple PD controller may provide stable behavior, it is worth compensating for the nonlinearities and taking the joint flexibilities into account in the control design. Amongst these four controllers, the best tracking performances have been obtained with the nonlinear backstepping methodology. However, one problem with this nonlinear control law is that it yields impractical control torques.

REFERENCES


