Passivity and Power Based Control of a Robot with Parallel Architecture *

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Abstract: This paper presents the design and implementation of two Passivity-based controllers (PBC) to the problem of trajectory tracking of a robot manipulator with a parallel architecture, one called IDA-PBC (Interconnection and Damping Assignment) and the other called Power-Shaping. For the first controller, the model described as a Port-controlled Hamiltonian (PCH), which provides the system be described in a appropriate form for applying balancing of energy. For the second controller, it is used a model described by Brayton-Moser equations, which describes the system for implementing the balancing of power. Through these two methods, at the end, it is obtained the same controller known as PD with gravity compensation, which is implemented in the manipulator in study.

Keywords: Parallel Robot, Passivity-based Control, Port-Controlled Hamiltonian Model and Power-shaping Control.

1. INTRODUCTION

Parallel mechanisms have been increasingly used in industry, e.g., in machine tools, high-speed pick-and-place robots, flight simulators, medical robots, among others. In relation to serial robots, parallel manipulators enable a higher loading, higher speeds and, in general, with a higher repeatability (Merlet [2000]). To achieve high speeds and accelerations for pick-and-place applications and greater precision in performing these tasks, efficient controllers, often based on accurate modeling, are necessary. However, it is generally difficult to obtain accurate models of the parallel robot dynamics, due to a high number of links and free joints (not actuated) and nonlinearities (such as friction).

Trajectory tracking control of robot manipulators is the most common task in robotic applications. Among the most common problems of robotic systems control is to perform tasks with accuracy tracking reference and in a relatively short time. Control design problems have traditionally been approached from a viewpoint of signal processing, i.e. the plant to be controlled and the controller are seen as signal processing devices that transform certain input signals into outputs. The control objectives are expressed in order to keep the error signals small and reduce the effect of certain disturbances, despite the presence of some unmodeled dynamics (Ortega et al. [2001]).

According to (Ortega et al. [2001]), this approach proves to be appropriate for linear time invariant systems, mainly because disturbances and unmodeled dynamics are being discriminated, via filtering, using frequency-domain considerations. However, the analysis and design of controllers for nonlinear systems is best suited when the plant to be controlled and the controller are viewed as devices that transform energy, breaking them down into simpler systems whose energies, after their interconnections, are added together to determine the behavior of the system as a whole. The control problem can then be redrafted in order to find the system dynamics and standard interconnection, such that the total energy function takes the desired shape. This approach, called energy-shaping, is the essence of passivity-based controllers (PBC), a controller design technique well known in mechanical systems.

When it comes to fully actuated mechanical system, which characterize the robotic system to be studied in this paper, the use of PBC methodology provides a natural procedure to model the system only from the potential energy, resulting in controllers with a clear physical interpretation (Ortega et al. [2002a]). However, unfortunately, modeling the total energy of the system destroys its physical structure. This means that the closed system no longer represents a Lagrangian system and the storage function of the map is not a passive energy function with a physical meaning. Thus, the IDA-PBC (Interconnection and Damping Assignment) method was developed, which aims at modeling the behavior of the energy of the closed loop system through the manipulation of interconnection and damping arrays (Ortega et al. [2002b]).

On the other hand, there is a new approach well suited to these types of systems, which takes as an example the concepts applied in IDA-PBC methodology, but shapes and designs controllers based on the modeling of power instead of energy. This
method is called power-shaping. According to (Ortega et al. [2001]), its main advantages are the possibility of inserting dynamics in the designed controllers to overcome the obstacle of the pervasive dissipation existence. However, this characteristic will not be considered in this article in order to achieve the same controller through both design methods. Recently, the power-shaping methodology has been applied to different fields as physical (García-Canseco et al. [2010]) and chemical systems (Favache and Dochain [2009]).

This article aims at drawing parallels between these two design methods of control of the system described in section 2, IDA-PBC (section 5) and Power-shaping Control (section 6), through demonstrating the equivalence of the controllers obtained under some design conditions. It is made by comparing the models (sections 3 and 4) and the controllers (section 7), followed by applications of these controllers on a robot manipulator with parallel architecture (section 8). Finally, the section 9 presents the final considerations about the methods applied in the study case.

2. THE PARALLEL ROBOT MANIPULATOR

The system used to study the application of the control techniques is a parallel kinematic machine (PKM) with two degrees of freedom (2-DOF) designed and built by Fatronik France (Baradat et al. [2008]), whose physical prototype is shown in Fig. 1.

As can be seen in Fig. 1, the prototype robot has four arms, but only two of them are operated. Because of this, the other two arms are responsible only for adding a restriction of movement of the manipulator along a Cartesian axis (Y), plus all other possible restrictions on the rotation of the platform. This means that the platform moves only in two dimensions, i.e. along the axes X and Z. These are the reasons why the robot is characterized as 2-DOF. Thus, the system model takes into account only the axes X and Z, and describes the system in 2D. The simplified schematic of the robot is shown in Fig. 2.

The mechanism’s geometric relation provides equations that represent the effective position of the platform, defined by the coordinates \((x, z)\) of the point \(E\), shown in Fig. 2, depending on the angles \(q_1\) and \(q_2\), which are the actuation angles of the robot. \(L_1\) and \(L_2\) are the lengths of the arms (active components actuated by motors in the joints \(P_1\) and \(P_2\), specifically), \(l_1\) and \(l_2\) are the lengths of the forearms (passive components connected to the arms 1 and 2 by joints \(A_1\) and \(A_2\), respectively), and \(h\) is the length of the platform (passive component connected to the forearms 1 and 2 by joints \(B_1\) and \(B_2\), specifically).

By the mechanical construction of the robot, the point \(E(x, z)\) is the intersection of two circles. One of them, on the right, with the center at \((x_{11}, z_{11}) = (x_{A1} - h/2, z_{A1})\) and \(L_1\) radius, and another to the left, with the center at \((x_{02}, z_{02}) = (x_{A2} + h/2, z_{A2})\) and \(L_2\) radius. Equating the two circles around the mechanism, it is possible to obtain the kinematics and Jacobian relations that involve the joint space coordinates \((q_1, q_2)\) and the Cartesian space coordinates \((x, z)\).

The aim of this article is not to show the way of obtaining the classical kinematics and dynamic models of the manipulator. They are necessary only to validate others models that will be shown throughout the document. However, all necessary steps to determine these classical models are shown in Neves [2009].

3. PCH MODEL

According to (Ortega et al. [2001]), in order to characterize a class of stabilizable systems with energy balancing PBC and simplify the solution of PDEs, we need to incorporate more structure in the system dynamics, in particular, making explicit the damping terms and the dependence of the energy function. Therefore, the Port-controlled Hamiltonian (PCH) models, which cover a large class of nonlinear physical systems, will be used. They have been considered an alternative to the classical models and their standard form is expressed as:

\[
\Sigma : \begin{cases} 
    \dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\
    y = g^T(x) \frac{\partial H}{\partial x}
\end{cases}, \quad (1)
\]

where \(H(x) \in \mathbb{R}\) is the energy function, \(J(x) = -J^T(x) \in \mathbb{R}^{2n \times 2n}\) represents the interconnection structure, \(R(x) \in \mathbb{R}^{2n \times 2n}\) \((R(x) = R^T(x) \geq 0)\) is the matrix of dissipation, \(u, y \in \mathbb{R}^n\) are respectively the input and output of the system, \(g(x)\) is the function that couples the control to the system dynamics. Note that, since the system is fully actuated, \(u\) has the dimension of \(q\), i.e., each measured joint is actuated. For mechanical systems, the PCH model is written in terms of generalized coordinates \(x = [q\ p]^T\), which are composed by the generalized coordinates of position \(q = [q_1, \ldots, q_n]^T\) and the generalized coordinates of momentum \(p = [p_1, \ldots, p_n]^T\), where the relationship between them can be expressed as:

\[
\dot{q} = M^{-1}(q)p, \quad (2)
\]

where \(M(q) \in \mathbb{R}^{n \times n}\) \((M(q) = M^T(q) > 0)\) is the generalized mass matrix or matrix of inertia of the manipulator. The system \(\Sigma\), represented in (1), is passive from \(u\) to \(y\), because it satisfies the equation of energy balance:

\[
H[x(t)] - H[x(0)] = \int_0^t u^T(y(s))dt - d(t), \quad (3)
\]

where \(d(t)\) is a non-negative function that represents the phenomenon which describes the energy dissipation of the system. For the robotic manipulator in question, the energy function (Hamiltonian) chosen for the system is represented as:

\[
H(q, \dot{q}) = T(q, \dot{q}) + V(q) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + V(q), \quad (4)
\]
where $T(q, \dot{q}) \in \mathbb{R}$ is the kinetic energy, $V(q) \in \mathbb{R}$ is the potential energy. The energy function of the system can also be written as

$$H(q, p) = \frac{1}{2} p^T M^{-1}(q) p + V(q). \quad (5)$$

Thus, the PCH model of the manipulator gets the following form:

$$\begin{align*}
\dot{q} &= \left[ \begin{array}{c}
0 \\
0 \\
\dot{V}_q H \\
\end{array} \right] + \left[ \begin{array}{c}
\dot{q} \\
\end{array} \right] + \left[ \begin{array}{c}
\dot{q} \\
\end{array} \right] = g^T(x) \nabla H,
\end{align*} \quad (6)$$

where $u \in \mathbb{R}^n$ is the vector of external inputs (torques applied on the motors), $\nabla_q H = \frac{\partial}{\partial q}$, $\nabla p H = \frac{\partial}{\partial p}$, $G$ is the input matrix, and $D \in \mathbb{R}^{nxn}$ is a damping matrix. For our application, the system is fully actuated. Then, $G = I_n$ and $D = 0$. Notice that the system is affine in control, because $g(x) = [0 \quad L_n]^T$. Thus, the output vector is precisely the generalized velocities ($y = \dot{q}$).

In order to demonstrate the equivalence of the PCH and dynamic models, the PCH model will be rearranged so that the dynamic models can be obtained. Thus, starting from the PCH system model definition (6) and making some multiplications, we get:

$$\begin{align*}
q &= \nabla_q H - M^{-1}(q) p \\
p &= -\nabla p H - R(q) \nabla_q H + u.
\end{align*} \quad (7)$$

The introduction of the derivative of $p$ ($p = M(q) \dot{q} + M(q) \dot{q}$) in the second line of (7) and its manipulation, yield:

$$M(q) \ddot{q} + M(q) \dot{q} + \frac{\partial T(q, \dot{q})}{\partial q} + \frac{\partial V(q)}{\partial q} + R(q) \dot{q} = u. \quad (8)$$

Using the relationships

- $M(q) \ddot{q} = C(q, \dot{q})$, the Coriolis vector;
- $\frac{\partial V(q)}{\partial q} = G(q)$, the gravity vector;
- $R(q) \dot{q} = F_{fric}(\dot{q})$, the friction force vector;

it can be obtained the classical dynamic model of this parallel robot manipulator (equation (9)), just as shown in Neves [2009] and Merlet [2000].

$$M(q) \ddot{q} + C(q, \dot{q}) + G(q) + F_{fric}(\dot{q}) = u. \quad (9)$$

4. BM MODEL

The introduction of models written in the form of Port-controlled Hamiltonian allowed the description of a large number of nonlinear physical systems using a different view, which considers systems (plants, controllers, etc) as energy transformers. Its popularity comes from its wide applicability with respect to the analysis and control systems. However, as mentioned in [Ortega et al. [2001]], energy-balancing control is stymied by the existence of pervasive dissipation.

Several control methodologies have been developed to overcome this dissipation obstacle. One of these methodologies, called power-shaping, was originally introduced in [Ortega et al. [2003]] and later extended to general nonlinear systems in [García-Cansaco et al. [2010]]. The method relies on the solution of a PDE, which allows us to write the original dynamics in terms of the Brayton-Moser equations (Brayton and Moser [1964]), and yields storage functions which have units of power.

In general, the BM models have the following form:

$$Q(x) = \frac{\partial R(x)}{\partial x} + B(x)u, \quad (10)$$

where $x \in \mathbb{R}^{2n}$ is the states vector of the system, $Q(x) \in \mathbb{R}^{2nx2n}$ a full rank symmetric matrix, $B(x) \in \mathbb{R}^{2nxn}$ the input matrix and $P(x) \in \mathbb{R}$ a mixed potential function, which has the unity of power, e.g., watt in SI. It has been shown in [Jeltsema and Scherpen [2007]] that the BM model of a mechanical system described by equation (10) can be obtained from the PCH model making:

$$\begin{align*}
\dot{x} &= [q \quad p]^T; \\
Q(q, p) &= \frac{\partial V}{\partial q} + \frac{1}{2} \frac{\partial p^T M^{-1}(q)p}{\partial q} + \frac{\partial p^T M^{-1}(q)}{\partial q} p - M^{-1}(q) \dot{p}; \\
P(q, p) &= \frac{\partial V}{\partial q} + \frac{1}{2} \frac{\partial p^T M^{-1}(q)p}{\partial q} + \frac{\partial p^T M^{-1}(q)}{\partial q} p.
\end{align*} \quad (11)$$

5. IDA-PBC CONTROL

5.1 Concepts Associated with the IDA-PBC Method

Consider the nonlinear system as

$$\dot{x} = f(x) + g(x)u. \quad (15)$$

Assume the existence of $g^+(x)$, $J_d(x) = -J_d^T(x)$, $R_d(x) = R_d^T(x)$ $\geq 0$ and the function $H_d : \mathbb{R}^{2n} \to \mathbb{R}$ which satisfy the PDE

$$g^+(x)f(x) = g^+(x)[J_d(x) - R_d(x)] \nabla H_d, \quad (16)$$

where $g^+(x)$ is a full rank annihilator of $g(x)$, i.e., $g^+(x)g(x) = 0$ and $H_d(x)$ is such that

$$x^* = \arg \min H_d(x), \quad (17)$$

with $x^* \in \mathbb{R}^{2n}$ an equilibrium point to be stabilized. Then, the closed loop system (15), where

$$u(x) = [g^+(x)g(x)]^{-1} g^+(x)[J_d(x) - R_d(x)] \nabla H_d - f(x), \quad (18)$$

takes the PCH form

$$\dot{x} = [J_d(x) - R_d(x)] \nabla H_d(x), \quad (19)$$

with $x^*$ a stable equilibrium (locally). It will be asymptotically stable if, in addition, $x^*$ is an isolated minimum of $H_d(x)$ and the largest invariant set inside the closed loop dynamics (19) contained in

$$\{x \in \mathbb{R}^{2n} \mid \nabla H_d(x)^T R_d(x) \nabla H_d = 0\} \quad (20)$$

equal to $x^*$. An estimation of the respective domain of attraction is given by the highest level of a limited set $\{x \in \mathbb{R}^{2n} \mid H_d(x) \leq c\}$.

5.2 Implementation of IDA-PBC Method

The method used in this work does not consider the friction forces applied at the joints of the system. Thus, the model used for the controller design will be defined in (6), but without the presence of $R_d$ matrix, which represents the friction applied to the robotic system. Therefore, the system takes the form:

$$\begin{align*}
\dot{q} &= \left[ \begin{array}{c}
0 \\
L_n \\
\dot{V}_q H \\
\end{array} \right] + \left[ \begin{array}{c}
\dot{q} \\
\end{array} \right] + \left[ \begin{array}{c}
\dot{q} \\
\end{array} \right] = g^T(x) \nabla H, \\
y &= g^T(x) \nabla H \quad (\dot{g}(x)).
\end{align*} \quad (21)$$

As shown previously, the total energy of this system is given by (5). Similarly, we can naturally define the desired energy function as:

$$H_d(q, p) = \frac{1}{2} p^T M_d^{-1}(q)p + V_d(q), \quad (22)$$
where the index \( d \) indicates that this is the reference. Since the energy function of the system was determined, the matrix of interconnection will be also determined:

\[
J_d(q, p) = \begin{bmatrix}
0 & M^{-1}(q)M_d(q)
\end{bmatrix}.
\]

(23)

Taking the control action given by (18), and in order to make clear the role of each term of the equation, we can divide the control action into two terms: the energy-shaping term and damping term \( a_d(x) = a_d(x) + u_d(x) \). In this case we have that (Ortega and Canseco [2004]):

\[
\begin{align*}
& u_d(x) = \left[ g^T(x)\nabla f(x) \right]^{-1} g^T(x) \left[ J_d(x) \nabla H_d - f(x) \right], \\
& u_d(x) = -K Q_d(x) \nabla H_d.
\end{align*}
\]

(24)

We can now make some assumptions to simplify the control law and get a simpler law, but not necessarily less efficient. The following simplifications are made:

- It will only be done the energy-shaping of the potential energy, which naturally implies that \( M_d(q) = M(q) \).
- The tuning parameter \( J_d(q, p) \) will not be used (\( J_d(q, p) = 0 \)).

The above simplifications lead to \( J_d = J \), i.e., the desired interconnection matrix is not modified and is equal to the interconnection matrix of the system.

For the case of this mechanical system, with \( y = \dot{q} \), we have \( g(x) = \begin{bmatrix} 0 & L \end{bmatrix}^T \). Therefore, the part of the control related to the energy-shaping is:

\[
\begin{align*}
& a_d(x) = \nabla V - \nabla V_d, \\
& a_d(x) = \left[ 0 - K \right] \nabla H_d = -K \dot{q}.
\end{align*}
\]

(26)

Solving the equation (25), we obtain:

\[
\begin{align*}
& a_d(x) = \nabla V - \nabla V_d. \\
& a_d(x) = \nabla V - K \dot{q}.
\end{align*}
\]

(27)

Considering a reference of position. Since we know what is the desired potential energy of the system, and that we defined the desired energy function, we can go further and rewrite equation (26):

\[
\begin{align*}
& a_d(x) = \nabla V - K \dot{q}.
\end{align*}
\]

(28)

In the same way, the control action responsible for the damping can be calculated:

\[
\begin{align*}
& u_d(x) = \left[ 0 - K \right] \nabla H_d = -K \dot{q}.
\end{align*}
\]

(29)

Finally, the whole control action is:

\[
\begin{align*}
& a(x) = \nabla V - K \dot{q} - K \dot{q}.
\end{align*}
\]

(30)

The controller of the equation (30) is the well known PD (proportional with derivative action) with gravity compensation (Wang and Goldsmith [2008], Ortega et al. [2001], Takegaki and Arimoto [1981]), widely used in mechanical control systems.

6. POWER-SHAPING CONTROL

The purpose of this section is to establish a control signal to stabilize the system and to adjust the mixed potential function \( P(x) \) into a function with the desired shape \( P_d(x) \). Thus, it uses the same idea as in IDA-PBC control, which also adjusts the system into the desired shape. In this case, the shaping does not happen in the total energy function (Hamiltonian) of the system, but in the mixed potential function \( P(x) \), which also results in shaping the \( Q(x) \) matrix. These matrix changes of the system lead to the desired shape

\[
Q_d(x) = \frac{\partial P_d(x)}{\partial x},
\]

(31)

where the \( d \) index refers to “desired” matrices.

The method used in this article is based on the method described in (García-Cansco et al. [2010]), which takes into account the definition of the nonlinear system in state-space form as

\[
\begin{align*}
& x = f(x) + g(x)u, \\
& y = h(x).
\end{align*}
\]

(32)

(33)

With the system described in this way, we can get the driver through the proposition 1 described below.

Proposition 1. Consider the general nonlinear system (32). Given an equilibrium point \( x^* \in X^* \subset \mathbb{R}^n \), where \( X^* := \{ \dot{x} = 0 \} \), and \( g^*(x) \) is a full-rank left annihilator of \( g(x) \). Assume

A.1 There exists a nonsingular matrix \( Q : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n} \) that

(i) solves the partial differential equation

\[
\nabla (Q(x)f(x)) = [\nabla (Q(x)f(x))]^T,
\]

(34)

(ii) and verifies \( Q(x) + Q^T(x) \leq 0 \) in neighborhood of \( x^* \).

A.2. There exists a scalar function \( P_d : \mathbb{R}^n \rightarrow \mathbb{R} \), positive definite in a neighborhood of \( x^* \), that verifies

(iii) \( g^*(x)Q^{-1}(x)\nabla P_d(x) = 0 \),

(iv) \( \nabla P_d(x) = 0 \), \( \nabla^2 P_d(x^*) > 0 \), with \( P_d(x) = P(x) + P_d(x) \), and \( P(x) \) satisfies \( \nabla P(x) = Q(x)f(x) \).

Then, the control law

\[
u = [g^*(x) \nabla Q(x)]^{-1} \nabla (Q(x)f(x)) \nabla P_d(x)
\]

(35)

ensures \( x^* \) is a (locally) stable equilibrium with Lyapunov function \( P_d(x) \). Assume, in addition,

A.3 \( x^* \) is an isolated minimum of \( P_d(x) \) and the largest invariant set contained in the set

\[
\{ x \in \mathbb{R}^n \mid \nabla^2 P_d(x) > 0 \} = \{ x \mid x^* \}
\]

(36)

equals \( \{ x^* \} \).

Then, \( x^* \) is an asymptotically stable equilibrium and an estimate of its domain of attraction is given by the largest bounded level set \( \{ x \in \mathbb{R}^n \mid P_d(x) \leq c \} \).

7. LINKING IDA-PBC CONTROL AND POWER-SHAPING CONTROL

Let us compare the different ways used to describe the system model, (10) and (32). By multiplying (10) by \( Q^{-1}(x) \), we notice that \( f(x) = Q^{-1}(x) \dot{p} \) and \( g(x) = Q^{-1}(x)B(x) \). Therefore, replacing \( Q(x)g(x) \) by \( B(x) \) in the control law (35), one gets

\[
u = \left[ B^T(x)B(x) \right]^{-1} B^T(x) \nabla P_d(x)
\]

(37)

To be able to obtain the \( Q_d(x) \) and \( P_d(x) \) matrices, and still put the expression of the control action in a form that can be compared with the classic controls, we must perform some manipulations of the system. Thus, subtracting (10) from (31), we get

\[
\dot{Q}(x) = [Q_d(x) - \dot{Q}(x)] = \frac{\partial P_d(x)}{\partial x} - B(x)u.
\]

(38)

By (38), we notice that to obtain the expression (37), it is necessary that \( \dot{Q}(x) = 0 \). This can be done with the following choice of \( Q_d(x) \) and \( P_d(x) \).
$$Q_d(q, p) = \begin{bmatrix}
\frac{\partial^2 V_d}{\partial q^2} + \frac{1}{2}\frac{\partial^2 (p^T M^{-1}(q)p)}{\partial q^2} - \partial \left(\frac{\partial(p^T M^{-1}(q)p)}{\partial q}\right)
\frac{\partial (p^T M^{-1}(q))}{\partial q}
\frac{\partial (M^{-1}(q)p)}{\partial q}
\end{bmatrix}.$$ (39)

and

$$P_d(q, p) = \begin{bmatrix}
\frac{\partial^2 V_d}{\partial q^2} + \frac{1}{2}\frac{\partial^2 (p^T M^{-1}(q)p)}{\partial q^2} - \partial \left(\frac{\partial(p^T M^{-1}(q)p)}{\partial q}\right)
\frac{\partial (p^T M^{-1}(q))}{\partial q}
\frac{\partial (M^{-1}(q)p)}{\partial q}
\end{bmatrix}.$$ (40)

To show the equivalence of the control obtained by this method with that obtained via IDA-PBC, we must expand (38) as:

$$\begin{bmatrix}
\frac{\partial^2 V_d}{\partial q^2} - \frac{\partial^2 V}{\partial q^2}
0
0
\end{bmatrix} = \begin{bmatrix}
\frac{\partial^2 P_a(x)}{\partial q} - \frac{\partial^2 P_d(x)}{\partial q}
0
0
\end{bmatrix} - M^{-1}(q)u. \quad (41)$$

Using equations (41), (40), (14), and taking into account the assumption A.2 (iv), we obtain:

$$M^{-1}(q)a = \frac{\partial P_a(x)}{\partial p} - \frac{\partial P_d(x)}{\partial p} = \left(\frac{\partial V}{\partial q} - \frac{\partial V_d}{\partial q} - M^{-1}(q)K_v\right)M^{-1}(q). \quad (42)$$

Whereas the purpose of control is the same as the IDA-PBC, i.e., perform an energy-shaping of the potential energy plus a damping assignment, we must use the desired mixed potential function as $V_d = \frac{1}{2}(q - q_d)^T K_v(q - q_d)$, where $q_d$ is the desired angular position. And finally, knowing that $M(q)$ and $M^{-1}(q)$ are symmetric matrices, $q = M^{-1}(q)p$ and $\ddot{q} = (q - q_d)$, we get

$$u = \nabla \dot{V} - K_v \ddot{q} - K_q q. \quad (43)$$

Note that the controller obtained is exactly the one obtained by the IDA-PBC method, i.e., a PD controller (proportional with derivative action) with compensation of gravity. It is important to highlight that, as demonstrated by (García-Cansco et al. [2010]) in proposition 1, we can prove the asymptotic stability of the closed loop system using the controller designed.

8. SIMULATIONS AND RESULTS

This section is responsible for presenting the simulation performed, along with the subsequent results. Since it was not possible to access the physical plant of the real parallel robot, it was performed a simulation of the PD controller with compensation of gravity in Simulink environment using the plant as the nonlinear dynamic model, as shown in equation (9). For this simulation, it was used a pick-and-place trajectory in three stages with their respective times: moving up in the Z axis ($T_{z_1}$), travel in the X axis ($T_x$), moving down in the Z axis ($T_{z_2}$). Additionally, it is desired that the robot remains motionless for $T_{pp}$ at the points of picking and placing. As described in section 2, the Y axis movement was discarded, making the robot’s workspace only within the XZ plane.

Two different trajectories were simulated, both with the same displacement, but with different speeds and accelerations. The times of the trajectory stages are shown in Table 1. The Fig. 3 shows the first desired and obtained pick-and-place trajectories on the XZ plane and the Fig. 4 shows the errors curves of X and Z axes separately for the same trajectory. It can also be seen in sections 5.2 and 6 that the controller design method of energy-shaping and power-shaping provide a class of controllers which

<table>
<thead>
<tr>
<th>$T_{z_1}$</th>
<th>$T_x$</th>
<th>$T_{z_2}$</th>
<th>$T_{pp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traj. 1</td>
<td>500ms</td>
<td>500ms</td>
<td>500ms</td>
</tr>
<tr>
<td>Traj. 2</td>
<td>500ms</td>
<td>100ms</td>
<td>50ms</td>
</tr>
</tbody>
</table>

Table 1. Trajectory times.

showed should be calibrated according to the final application. For this type of system, parallel manipulators performing pick-and-place tasks, the controlled acceleration of the robotic mechanism is a critical requirement in the control design. Thus, for the simulation results, the control system was first calibrated with the aid of a Simulink toolbox called Design Optimization, for the trajectory defined in this section. This tool tunes parameters in the controller to meet specified constraints in the closed-loop system. The constraints include bounds on signal amplitudes and matching of reference signals. The optimized values found for $K_p$ and $K_v$ were diagonal matrices with the elements $K_{p_1} = 6.288$, $K_{p_2} = 11.951$, $K_{v_1} = 12.3$ and $K_{v_2} = 10.4$.

Although the method presented a satisfactory result, once the speed for the pick-and-place trajectory was increased, it became harder to optimize the controller. Even though the same optimization method as described above was performed, the best control parameters to fit in a control action saturation of 500N.m found were $K_{p_1} = 30.000$, $K_{p_2} = 30.000$, $K_{v_1} = 304.5$ and $K_{v_2} = 406$. It can be shown in the Fig. 5 that the robot motion was degraded if compared to the slower trajectory. In this way, in order to improve performance, another control method should be implemented, involving an integral or adaptive action or even other control theories.

9. CONCLUSION

It was presented in this article, the application of one controller obtained through two different methods - one Passivity-based (PBC) and another power-based - to the problem of trajectory tracking of a manipulator robot with a parallel architecture. With the first method, called Interconnection and Damping Assignment (IDA-PBC), it is possible to modify the kinetic energy of mechanical systems by modeling its potential energy.
Figure 5. Obtained and desired trajectory in XZ plane with higher accelerations.

This class of controllers was used before by (Ortega and Spong [2000]), seeking to reach the global stability of an inverted pendulum with an inertial disk and a ball-bar system, but the methodology extends to a wider class of systems, nonlinear systems in general. With the second method, called Power-shaping, the Brayton-Moser equations are used to modify the mixed potential energy function of the system, which has the same unity as power.

Simulations of the controlled system were performed using the classical dynamic model of the robot, involving nonlinearities such as friction, to mimic, as much as possible, the real physical system. It was used one single trajectory of a pick-and-place manipulator robot operating with two different profiles of velocity: a slow one and a fast one. For each case it was made a calibration of the control system. In the slow operation, satisfying results were obtained. In the fast operation, the position control of the robot showed less precision, pointing the possibility of using other control methods (Shang et al. [2009]) or adding new dynamics in the controller to increase its performance, as friction compensation (Shang et al. [2008]) or integrative dynamics.

The control methodologies IDA-PBC and Power-shaping consists of an alternative way in the analysis and project of control systems. Instead of considering the signals involved in the process, the energy of systems is manipulated. The results showed in this paper indicate that the controllers designed with these methodologies are not always unique, i.e., they can be obtained through other synthesis methods. However, energy based systems do not represent a simple controller design technique, but a wider way of understanding systems. The role of energy and the interconnections between subsystems provide the basis for various control strategies. Thus, energy can serve as a common language to facilitate communication among scientists and engineers from different fields (Jeltsema and Scherpen [2009]).

REFERENCES


