Min-Max Model Predictive Power Control Strategy for CDMA Cellular Networks

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Abstract: This paper applies an open-loop min-max MPC strategy to the uplink radio power control problem for a CDMA cellular network assessed over time-varying channels. The new power control algorithm is shown to robustly compensate for channel fading uncertainty as well as to reduce the effect of interference. This is achieved by implementing a computationally efficient min-max MPC mechanism based on a simple model of the tracking error that is estimated at each sampling time from the local SINR. The numerical efficiency of the resulting design is benchmarked against a number of existing strategies via a representative selection of simulation scenarios.

1. INTRODUCTION

In general, wireless power control systems are typically subject to external disturbances due to radio channel fading and multiple access interference (MAI) that cause a significant degradation in communication quality, hence network performance, Gunnarsson & Gustafsson [2003]. Moreover, when mobility is introduced, the problem becomes more difficult to solve. The problem is characterised by time varying attenuation in the received signal strength, thereby causing significant perturbations in the received signal-to-noise-plus-interference ratio (SINR). In this paper, the focus is on the uplink power control problem within a typical code-division multiple-access (CDMA) system, emphasising the utility of a model predictive control (MPC) strategy in compensating for the aforementioned uncertainties, so that an acceptable SINR level is achieved for all users, hence maximising the global system capacity. Here, outage probability, Kandukuri & Boyd [2002], is taken into account as a supplementary quality of service (QoS) type performance metric. Thus, power control can play a crucial role not only in satisfying the target SINR, but also in maintaining a level of SINR that is just above a prescribed threshold so that the base station can stay connected with the mobile user without unnecessary interference being generated across the network.

Transmit power control schemes can be classed as either centralised or decentralised. In the centralised scheme, a controller requires complete information of all the channel gains and interferences in the system to calculate appropriate power levels for all users. Conversely, a decentralised controller utilises only locally available information such as the measured SINR as a feedback signal. For a comprehensive survey please consult Koskie & Gajic [2006]. A decentralised approach is adopted in this work. Early work focussed on a fixed-step size power control algorithm as a solution, Ariyavisitakul & Chang [1993]. Some decentralised power control mechanisms have also been successfully used in cellular networks, see e.g., Foschini & Miljanic [1993], Yates [1995], Koskie & Gajic [2006].

Recent studies of the power control problem have focused on a satisfactory assessment of the influence of time-varying radio channel gain, interference, or noise on performance. In Jagannathan et al. [2006], an estimation based decentralised power control that assumes slow channel variation was presented. Based on this assumption, the channel gain has been accurately estimated, and power control has been implemented. In Lee et al. [2006], a robust power controller has been designed that uses a $\mathcal{H}_\infty$ filtering algorithm to minimise the worst-case effects of interference and noise regardless of the information that is being transmitted along the channel.

In Chisci et al. [2008], a queue-based power control scheme cast as an optimal control problem was considered. The algorithm features include fast on-line implementation, good performance and has exhibited significant power saving. However, a certain level of robust performance with respect to any or all of the aforementioned uncertainties has not been properly characterised thus far. In practical systems, the link information is not readily available. To address this limitation, a robust MPC is taken into account in this paper that applies a readily implementable open-loop min-max MPC algorithm, Muñoz de la Peña et al. [2007]. Such an approach explicitly takes into account the constrained SINR tracking error dynamics, and treats such uncertainties as bounded additive disturbances. In the decentralised framework that is formulated, the complex network dynamics of power control are represented by a simple state-space model of the SINR tracking error of each user, thus making the on-line optimisation problem tractable. In fact, given that in CDMA systems the power

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update rate tends to be very low, implementing the proposed min-max MPC by an explicit computation of the quadratic program (QP) that naturally arises, Muñoz de la Peña et al. [2007], is also shown to be quite feasible. As a result, this facilitates every mobile user to operate robustly at an acceptable SINR, while jointly satisfying a useful QoS requirement. The resulting controller is numerically evaluated via a radio network simulation model developed in Alavi & Hayes [2007] to demonstrate the feasibility and efficiency of this particular approach.

2. PROBLEM STATEMENT

In order to implement a MPC, we need a reduced complexity model of the CDMA power control process. In this section, we present a linear state-space tracking error model that we will use to design a controller to robustly maintain the SINR around a target value, while preserving a satisfactory QoS for all mobiles.

Throughout the paper, the logarithmic (e.g., dB) value of a variable \( x \) is denoted by \( \bar{x} \), namely, \( \bar{x} = 10 \log_{10} x \).

2.1 Radio Link Model

We consider a single-cell CDMA uplink of \( n \) mobiles, labeled \( 1, \ldots, n \) and the base station. Assuming that the user \( i \) is transmitting using the power level \( p_i > 0 \), the corresponding connected base station will experience a received signal power given by \( s_i(k) = g_i(k)p_i(k) \). The term \( g_i \), which is positive, represents the time-varying path gain from the \( i \)-th user to the base station. Correspondingly, this gain can be modeled as the product of different time-varying random quantities,

\[
g_i(k) = g_{p,i}(k)g_{s,i}(k)g_{f,i}(k),
\]

where \( g_{p,i} \), \( g_{s,i} \), and \( g_{f,i} \) represent path loss, log-normal shadowing, and Rayleigh fading, which limit the network performance during the transmission and can be determined according to

\[
g_{p,i}(k) = \frac{A_p}{r_{p,i}(k)},
\]

\[
g_{s,i}(k) = 10^{0.1 \xi_{s,i}(k)},
\]

\[
g_{f,i}(k) = X^2.
\]

In the path loss model (2), \( r_{p,i} \) is the distance from the \( i \)-th mobile to the base station, \( \alpha \) the typical path loss exponent ranging from 2 (free-space propagation) to 5 (dense urban area), and \( A_p \) is a constant that depends on the antenna characteristics and the average channel attenuation. Note that \( A_p = 1 \) has been adopted in this work. For shadow fading, empirical studies have shown that \( g_{s,i} \) follows a log-normal distribution, Rappaport [2002], hence implying \( g_{s,i} \) is Gaussian. A simple and realistic model of \( g_{s,i} \) can be given by (3) that incorporates the mobile velocity \( v \) and the decorrelation distance \( X_c \), where \( \xi_{s,i} \) is a Gaussian random variable with zero mean and variance \( \sigma^2_{\xi_{s,i}} = 1 - a^2_{s,i} + \sigma^2_{\xi_{s,i}} \). The term \( \sigma_{s,i} \) denotes the log-standard deviation, and the coefficient \( a_{s,i} \) is given by \( a_{s,i} = \exp(-vT_s/X_c) \), where \( T_s \) denotes the sampling period, Rappaport [2002].

In a Rayleigh fading environment, the received signal envelope due to channel fading typically has a Rayleigh distribution, Rappaport [2002]. It is typical to model this

\[\gamma_i(k) = s_i(k)/I_i(k),\]

as the product of the received signal power \( s_i(k) \) and the interference plus noise power \( I_i(k) \). The received signal power \( s_i(k) \) is given by

\[s_i(k) = p_i(k)g_i(k),\]

while the interference \( I_i(k) \) is the sum of the interference from all other users, the noise power, and the noise power on the uplink, wherein it is corrupted by time-varying uncertain interference, noise, and channel gain.

At sampling instant \( k \) at the base station, the measured SINR is compared to the target SINR \( \bar{\gamma}_i \) for each user. This measurement can be written as

\[\gamma_i(k) = s_i(k)/I_i(k),\]

and the SINR tracking error \( \bar{e}_i(k) \) can be expressed as

\[\bar{e}_i(k) = \gamma_i(k) - \bar{\gamma}_i(k).\]

For signal reception to occur, the SINR must exceed a given threshold \( \gamma^0 \), reflecting a certain QoS requirement, i.e., the outage probability.

2.2 Radio Power Control Model for Prediction

A closed-loop SINR-based radio power control system in the logarithmic domain is depicted in Fig. 1. Note that in a decentralised power control framework, only local SINR measured at the base station (i.e., minimal feedback information) is required.

From (6)-(8), the next-step SINR tracking error can be derived as the following error dynamic equation,

\[\bar{e}_i(k + 1) = \bar{e}_i(k) - \bar{u}_i(k) + \bar{w}_i(k),\]

where \( \bar{w}_i \) denotes the uncertain interference, fading, and noise. This allows the complex network dynamics of power control to be simplified, and then represented by using a state-space model applied for each mobile.
\[ \dot{x}_i(k+1) = x_i(k) - u_i(k) + w_i(k) \]  
\[ e_i(k) = x_i(k), \]
with \( x_i \) representing the state of the SINR tracking error. The model is clearly decentralised and has less complexity than the centralised model, whose dimension depends on the number of users in the cell. In the next section, a decentralised robust MPC controller is presented based on the model of the SINR tracking error for a given agent. For simplicity of the notation, the subscript \( i \) will now be suppressed.

**Remark 1:** All computations in MPC are based on a prediction model, and the cost function to be minimised is typically quadratic in the state and in the control input. From (10), the dimension of state and input is small (say, each equal to 1), thus largely reducing the complexity of the optimisation problem.

### 3. MIN-MAX MPC BASED POWER CONTROL

In order to deal with the additive uncertainty taken into account in the derived SINR tracking error model (10), we propose to use min-max MPC, Witsenhausen [1968]. In this paradigm, the objective function is minimised for the worst possible realisation of the uncertainty. In particular, we consider open-loop min-max MPC with state feedback control law and quadratic cost function, Muñoz de la Peña et al. [2007]. In what follows, we revise the formulation of this class of controllers presented here. In this paper, it was shown that this class of controllers can be implemented by solving a QP problem.

System (10) belongs to the following class of discrete linear time-invariant system with bounded uncertainty,

\[ \hat{x}(k+1) = A \hat{x}(k) + B \hat{u}(k) + D \hat{w}(k), \]

where \( \hat{x}(k) \in \mathcal{R}^{n_x} \) is the state, \( \hat{u}(k) \in \mathcal{R}^{n_u} \) the control input, \( \hat{w}(k) \in \mathcal{R}^{n_w} \) the bounded uncertainty, and hence \( \hat{w}(k) \in W, \) where \( W \) is a closed polyhedron that contains the origin.

In order to introduce the effect of feedback in the predictions, we assume that the control input is defined as follows,

\[ \hat{u}(k) = -K \hat{x}(k) + \hat{v}(k), \]

where \( K \) is a linear gain. This implies that the control moves \( \hat{u}(k) \) for updating the transmit power, are corrected by \( \hat{v}(k) \) that will be computed by the MPC controller.

The constrained min-max predictive power control problem considered here is

\[ J^*(\hat{x}) = \min_{\hat{v}} \max_{\hat{w} \in W_{N_w}} V(\hat{x}, \hat{v}, \hat{w}) \]

s.t. \( \hat{x}(k+j) \in X, \forall \hat{w} \in W_{N_w}, j = 0, \ldots, N_p \)
\[ \hat{u}(k+j) \in U, \forall \hat{w} \in W_{N_w}, j = 0, \ldots, N_p - 1, \]

where \( N_p \) is the prediction horizon, \( \hat{x}(k) = \hat{x} \) is the initial state, \( \hat{x}(k+j|k) = \hat{x}(k+j) \) the predicted state and control input, respectively, \( \hat{v} = [\hat{v}(k|k)^T, \ldots, \hat{v}(k+N_p-1|k)^T]^T \) is the sequence of correction control inputs, \( \hat{w} = [\hat{w}(k|k)^T, \ldots, \hat{w}(k+N_p-1|k)^T]^T \) represents a possible sequence of input disturbances to the system, \( W_{N_w} \subseteq \mathcal{R}^{N_w \times n_w} \) denotes the set of possible disturbance sequences of length \( N_w, W_{N_w} = W \times W \times \cdots \times W, \) where \( \times \) denotes the cartesian product, \( X \) and \( U \) are polyhedra defined by the state and input constraints respectively and \( V(\hat{x}, \hat{v}, \hat{w}) \) is the objective function defined as

\[ V(\hat{x}, \hat{v}, \hat{w}) = \sum_{j=0}^{N_p-1} [\hat{x}(k+j|k)^T Q \hat{x}(k+j|k) + \hat{u}(k+j|k)^T R \hat{u}(k+j|k)] + \hat{x}(k+N_p|k)^T P \hat{x}(k+N_p|k), \]

with \( Q \geq 0, P \geq 0, \) and \( R > 0. \)

### 3.1 MIN-MAX MPC Computation: a QP Formulation

Min-max optimisation problems in general exhibit a very high computational complexity. In Muñoz de la Peña et al. [2007], it was shown that the optimisation problem (13) can be reformulated into a QP problem. We review in what follows the equivalent QP formulation.

Taking into account (11) and (12), as the predictions \( \hat{x}(k+j|k) \) and \( \hat{u}(k+j|k) \) depend linearly on \( \hat{v}, \hat{w}, \) the constraints of (13) can be written as \( F \hat{x} + G \hat{v} \leq m + M \hat{w}, \) \( \forall \hat{w} \in W_{N_w} \). It follows that the robust constraints of the problem (13) are equivalent to \( F \hat{x} + G \hat{v} \leq d, \) where \( d \) is a vector such that its \( i \)-th entry satisfies \( d_i = m_i + \max_{\hat{w} \in W_{N_w}} (M_i \hat{w}), \) and \( m_i \) and \( M_i \) are the \( i \)-th element and row of \( m \) and \( M \), respectively.

In addition, taking into account (14), matrices \( H_x, H_v, \) and \( H_\omega \) can be found, Camacho & Bordons [2004], in such a way that the cost function \( V(\hat{x}, \hat{v}, \hat{w}) \) can be evaluated as the following quadratic functional,

\[ V(\hat{x}, \hat{v}, \hat{w}) = \| H_x \hat{x} + H_v \hat{v} + H_\omega \hat{w} \|_2^2. \]

As the function \( V(\hat{x}, \hat{v}, \hat{w}) \) is convex in \( \hat{w} \), the maximum can be obtained by evaluating the cost function at the set of vertices of the polyhedron \( W_{N_w} \), denoted by \( V(W_{N_w}) \). As a result, the problem (13) can be rewritten as

\[ J^*(\hat{x}) = \min_{\hat{v}} \max_{\hat{w} \in W_{N_w}} V(\hat{x}, \hat{v}, \hat{w}) \]

s.t. \( F \hat{x} + G \hat{v} \leq d \)

\[ \hat{v} \geq V(\hat{x}, \hat{v}, \hat{w}) - V(\hat{x}, \hat{v}, 0), \]

\[ \forall \hat{w} \in V(W_{N_w}), \]

with

\[ V(\hat{x}, \hat{v}, \hat{w}) - V(\hat{x}, \hat{v}, 0) = \hat{w}^T H_x^T H_x \hat{w} + 2 \hat{w}^T H_x^T (H_x \hat{x} + H_v \hat{v}). \]

In order to obtain an equivalent problem to (17) in which the functional does not depend on the state vector, the following variable change is introduced,

\[ \bar{\hat{v}} \triangleq \hat{v} + [H_x^T H_x]^{-1} H_x^T H_x \hat{x}. \]
As a result, the min-max problem (13) is now equivalent to
\[
J^*(\tilde{x}) = \tilde{x}^T Y \tilde{x} + \min_{\tilde{z}, \gamma} \frac{1}{2} \tilde{z}^T H \tilde{z} + \gamma
\]  
(20)
\[
s.t. \quad G_m \tilde{z} + g_m \gamma \leq W_m + S_m \tilde{x}
\]  
(21)
\[
G_c \tilde{z} \leq W_c + S_c \tilde{x}
\]  
(22)
The constraints (21) described by matrices \(G_m, g_m, W_m\), and \(S_m\) correspond to the maximisation of the functional, while (22) defined by \(G_c, W_c, \) and \(S_c\) represent the robust constraints of the problem. All these matrices can be obtained from the system model and the cost function.

Therefore, the application of the min-max MPC to the radio power control system can be written as:

**Algorithm:**
1. At sample \(k\), solve the optimisation problem (20)-(22).
2. Obtain \(\tilde{v}\) from the variable change defined in (19).
3. Then, set \(\tilde{v}(k) = \tilde{v}(0)\), and apply \(\tilde{u}(k) = -K \tilde{x}(k) + \tilde{v}(k)\) for the power control update command.
4. Get new SINR tracking error measurement, and repeat the optimisation at sample \(k + 1\).

### 4. SIMULATIONS

The min-max MPC based power control algorithm described in the previous section has been numerically studied by using the CDMA network simulation model developed in Alavi & Hayes [2007]. This provides a benchmark simulation platform for the evolution of this class of network simulation. The simulation parameters to form the network can be summarised as follows. In this scenario, a single-cell uplink CDMA system has been simulated with 10 mobile users. The hexagonal cell with radius of 1 km is controlled by one base station centred at the origin. The mobile users are randomly distributed over this area according to a Gaussian distribution. The mobility of each user is arbitrarily chosen at the beginning of the simulation to create several test situations. The simulations have been performed based on the operating frequency of 900 MHz and the bandwidth \(B_w\) of each channel assumed to be 1.23 MHz. The data bit rate \(R_d\) is set at 9.6 kbps. For simplicity, perfect SINR measurement is supposed and the target SINR \(\gamma_t\) is set to be -10 dB for all users. Moreover, the standard deviation of noise power at the base station has been selected as a random value between 0.5 and 1. An urban area setting has been taken into account for the operational environment. The parameters settings of the radio link propagation employed in the simulation model that follows in such a circumstance is: path loss exponent \(\alpha = 4\), log-standard deviation of shadowing \(\sigma_{s,i} = 2.7\), and the Rayleigh distribution parameter has been set to 0.5.

#### 4.1 Uncertainty Bounds

To apply the min-max MPC, the uncertainty bounds have been set according to the error computed using the predicted state which came from the prediction model (9) with \(\tilde{w}_c(k) = 0\). The errors computed using this model are shown in Fig. 2. The uncertainty bounds have been set to -2.5 and 2.5, since 95.13% of the errors are within these bounds.

#### 4.2 Benchmark Comparison

To benchmark the advantages of the proposed algorithm, the min-max MPC is compared with other existing control laws.

**Power control with fixed-step size** In this technique, Ariyavisitakul & Chang [1993], the base station transmits the control decision to the mobile to either up or down its power by a typical fixed-step size of 1 dB, i.e., if the measured SINR is above the target, then the update command is to decrease power by -1 dB, otherwise the control action taken is to increase power by 1 dB.

**Power control with robust \(\mathcal{H}_\infty\)** In Lee et al. [2006], a robust \(\mathcal{H}_\infty\) power controller with fixed state feedback gain \(K_s\) was proposed to minimise the worst-case effects of interference and noise on SINR variance by solving an \(\mathcal{H}_\infty\) optimal tracking problem, subject to some linear matrix inequality (LMI) constraints. Here, \(R_1\) and \(R_2\) are respectively positive weighting factors for tracking and disturbance rejection, and \(\gamma\) is a prescribed value. With this method, a resulting optimal \(\mathcal{H}_\infty\) gain \((K_s)\) of 1 has been obtained for the case studied.

**Power control with nominal MPC** To test the robustness of the proposed strategy, a nominal MPC has also been considered in this benchmark comparison. The settings of the controller parameters have been reasonably well tuned: \(N_p = 3\), \(Q = P = 2\), and \(R = 1\).

**Power control with min-max MPC** The following tuning and weighting factors have been used during the application of the proposed min-max MPC scheme: \(N_p = 2\), \(Q = P = R = 1\), and \(K = -0.618\). The min-max MPC optimisation problem (20)-(22) to be solved at each sample \(k\) is subject to the state constraint \(\|\tilde{x}(k)\|_\infty \leq 4\), corresponding to the SINR threshold value, and the input constraint \(|\tilde{u}(k)| \leq 10\). Note that these constraints have also been set for the nominal MPC.

Both the nominal MPC and the min-max MPC quadratic optimisation problems have been solved using Matlab’s *quadprog*.
4.3 Performance Assessment

In order to compare the performance of the proposed min-max MPC control strategy with the other controllers, several simulation test scenarios that include variable mobile speeds for each user in the range 20-100 km/h have been considered. For each scenario, the simulation has been iteratively performed for 10 runs with a duration of 200 samples in each run. Note that for consistency the type of motion for each user was randomly determined at the beginning of each simulation and not changed when an experiment is conducted for each controller. During the simulation, dynamic data of all users were stored, and at the end the mean measured values were produced according to the following two performance criteria:

- Outage probability, Kandukuri & Boyd [2002]:
  \[ P_o = \text{Prob}(\gamma_i < \gamma^\text{th}), \]
  where \( \gamma^\text{th} \) denotes the SINR threshold, a minimum value required for signal reception to occur. The SINR threshold is given by \( \gamma^\text{th} = \left( \frac{E_b}{N_0} \right) \), where \( E_b \) denotes the energy dissipated per information bit and \( N_0 \) the total interference and noise power spectral density. In order to achieve an acceptable bit error rate (BER) of \(< 10^{-3}\), the target \( \frac{E_b}{N_0} = 7\text{dB} \) is required for every user, which results in \( \gamma^\text{th} = -14\text{dB} \).

- Standard deviation of the SINR tracking error:
  \[ \sigma_{\bar{c}} = \left( \frac{1}{N} \sum_{k=1}^{N} (\gamma^i - \bar{\gamma}_i(k))^2 \right)^{\frac{1}{2}}, \]
  where \( N \) is the total number of samples.

4.4 Results Analysis

Impact of channel uncertainty (one user case) To investigate the system performance in the presence of channel fading, only one user is considered. The results shown in Fig. 3 and 4 illustrate the evolution of the achieved SINR level and the transmission power in each iteration for a user (traveling at 100 km/h) using the different controller designs. It is readily observed that in this highly uncertain environment the min-max MPC offers a significant improvement in respect of maintaining the SINR above a threshold value, i.e., the number of measurements that violate the SINR floor constraint is much lower in the min-max MPC case than for other competing strategies). The min-max MPC approach also exhibits the lowest average transmission power in these experiments.

Effect of increasing number of users In this scenario, the effect of 10 users utilising the network simultaneously is considered. Figure 5 and 6 depict the evolution of SINR and transmission power for a randomly observed user traveling at 100 km/h in such a scenario. As expected, the min-max MPC approach provides the best robust tracking performance with the smallest variations in the received SINR, thus resulting in the least number of samples in which the threshold value has been crossed. Furthermore, Fig. 6 suggests that the effect of induced MAI causes the overall power consumption to be much better in the min-max MPC case, in particular with respect to the robust controller design. This suggests that the use of a min-max MPC approach avoids an unnecessarily high user (or link) cost, and will thereby result in a (much) improved battery lifetime.

Effect on \( P_o \) and \( \sigma_{\bar{c}} \) In this example, the effect of variable mobile velocity ranging from 20-100 km/h on both the outage probability and tracking performance is addressed. As mobility increases, the fluctuation in channel gain becomes more rapid. The results are aggregated for the performance criteria (23) and (24) in Fig. 7 and 8, respectively. It is clear from Fig. 7 that transmit power control using the min-max MPC performs best with a significant enhancement in a certain level of robustness to outage events with respect to the other methods over the set of mobile velocities considered. Figure 8 demonstrates the fact that the min-max MPC approach exhibits smaller standard deviation with respect to SINR tracking error than the other schemes over the test set.

5. CONCLUDING REMARKS

In this paper, the problem of robust model predictive power control within a wireless CDMA cellular network in the presence of uncertain channel fading and interference has been considered. The problem has been formulated as an open-loop min-max MPC problem in a decentralised fashion. The efficacy of the proposed robust power control law has been illustrated by various numerical test scenarios carried out on a CDMA network simulation model. The results, achieved in terms of the robust SINR tracking,
Fig. 5. Evolution of the SINR for a randomly selected mobile user traveling at 100 km/h. (Scenario 2)

Fig. 6. Evolution of the transmission power for a randomly selected mobile user traveling at 100 km/h. (Scenario 2)

Fig. 7. Outage probability with varying mobile velocity for 10 mobile users

red reduction in transmit power, reduced occurrence of outage events, and improvements in the observed standard deviation of the tracking error, indicate that the min-max MPC exhibits superior performance when compared to other approaches.

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