Disturbance Rejection in Linear Discrete Multivariable Systems: Inverse Model Approach

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Abstract: The problem of unknown and unmeasurable disturbance rejection in linear discrete multivariable control systems is considered, at that problem decomposition into disturbance estimation and compensation is suggested. It has been shown, that control structure with disturbance observer and disturbance compensator used for unmeasurable disturbance estimation and compensation may be treated as an Inverse Model Control approach. Moreover, the connection between inverse model design problem and unknown input observer theory has been established in order to give a practical way to inverse model parameterization and design. The properties of closed-loop feedback/feedforward combined control system with inverse model-based controller have been also investigated for the purpose of attainable disturbance rejection accuracy assessment.

Keywords: Invariance, compensators, disturbance rejection, inverse dynamic control, multivariable feedback control, observers, output regulation.

1. INTRODUCTION

The problem of unknown and immeasurable disturbance rejection in multivariable systems along with reference signal tracking (output regulation problem) is one of the most important in control theory. So far as uncertainties of the controlled object may be treated as a parametric disturbance of nominal plant model, the disturbance rejection closely connected with general problems of control systems invariance and robustness (Paraskevopoulos P.N., F. N. Koumboulis, K. G. Tzierakis, 1993, Guang-Ren Duan, Ying-Xin Yan, Dong Honglin, 2006).

There are two main approaches to such a problem. First, namely disturbance attenuation methods, use the available a priori information about disturbances in statistical or uncertain (set-membership) form. At that the design solution is obtained in a class of classical feedback control structures and is formalized as an optimization problem with the averaged or guaranteed cost function, the requirements of controller internal stability are used as a supplementary restriction. The cost functions in the form of a norm of closed-loop transfer function are widely used and a solution may be obtained using $H_2$ or $H_\infty$ optimal control methods (Francis B.A. 1987, Zhou, K., Doyle, J. C., Glover, K., 1996). It is known, that the systems, which are optimal with respect to a class of disturbances, usually doesn't ensure the high accuracy for all disturbances realizations. The most difficult case is the situation where the spectrums of reference signal and disturbances are essentially intersected. Moreover, in most practical applications the typical situation is characterized by the lack of a priori information, which is quite enough for disturbance modeling.

Another approach is based on the utilization of current information about disturbances obtained by the direct or indirect measurements. Such an approach realized in innovative control structures known as "two-degree-of freedom controllers" (Wolovich W.A. 1995). is realized in combined feedback/feedforward control systems. The appropriate design methods using the different types of plant's and disturbances models in control loop, known as Internal Model Control (IMC) method, are very popular in robust process control problems (Morari M. and Zafirov E., 1989). In (Tsypkin Ya. Z., Holmberg U., 1995) was shown, that IMC approach ensures selective invariance properties of closed-loop system, i.e. rejection of a certain class of disturbance. So far as the idea of selective invariance was initially developed for SISO systems with scalar disturbance, its development and generalization for multivariable systems are interest (Lyubchik L.M., 1995). Recently in such a way a number of model-based control methods have been developed for disturbance rejection in multivariable systems taking into account the requirements of accuracy, dynamic performance, stability and robustness.

The perspective modification of the approach mentioned above in the inverse model control (IMC) method (Chang, J., Yu, C. 1991, Lyubchik L.M., 1995, Chia-Shang Liu, Huei Peng, 2002). The IMC approach includes disturbance estimation, output controlled object reaction prediction and disturbances effect compensation. The appropriate control structure consists of the disturbance observer (DO) and feedback/feedforward controller with disturbance compensator (DC). Such an approach ensures not only the closed-loop system stabilization, but also high accuracy reference signal tracking and immeasurable arbitrary disturbance rejection.

In this paper for multivariable disturbance rejection problem a unified approach to DO and DC design problem is proposed based on the inverse models (IM) of the controlled object channels. At that, the IM are used both for disturbance
estimation (indirect measurement) and for prediction and compensation in order to ensure selective invariance properties of closed-loop system.

The basic of IMC approach is the IM state space representation (structural synthesis) and IM parametric design in order to ensure the desired dynamic properties. If the invertibility conditions take place (Silverman L.M., 1969, Seraji H., 1989), the structure inversion algorithm may be applied, in this case the structure and parameters of inverse models are strictly determined by the parameters of controlled object channels, for example, for nonminimum-phase object the inverse models will be unstable. So the IM design method must include the suitable parameterization of its equations, whereupon free tuning parameters are selected from the simultaneous conditions of stability and desired dynamic properties. The proposed approach for IM structural synthesis based on unknown-input observer (UIO) theory (Hou M., 1999), the structure inversion algorithm may be based on unknown-input observer (UIO) theory (Hou M., 1999), and then the observer equation combined with the unknown input signal estimate may be treated as the designed inverse model. Furthermore, DO and DC parametric synthesis come to relevant modal control problems. In such a way IM based design procedure for DO and AC may be suggested.

2. PROBLEM STATEMENT

Consider an output control problem for multivariable control object described by the linear state-space nominal model with signal and parametric disturbance:

\[
\begin{align*}
    x_{k+1} &= (A + \delta A_k)x_k + B_1u_k + D_1f_k, \\
    y_k^1 &= C_1x_k, \\
    y_k^2 &= C_2x_k,
\end{align*}
\]

where \( x_k \in \mathbb{R}^n \) - state vector at instant \( k \), \( u_k \in \mathbb{R}^m \) - control vector, \( y_k^1 \in \mathbb{R}^q_1 \) - output controlled variables, \( y_k^2 \in \mathbb{R}^q_2 \) - vector of measured variables, \( f_k \in \mathbb{R}_d \) - input disturbance, \( \delta A_k \) - parametric disturbance.

Let only \( p \leq n \) rows of object dynamic matrix \( A \) subject to parametric disturbance, which may be factorized as \( \delta A_k = D_2 \cdot \Delta \), where \( \Delta \) is composed from the object parameters deviation from the nominal values, and \((n \times p)\) matrix \( D_2 \) describes the parametric disturbance structure.

At that the controlled object model may be represented as:

\[
\begin{align*}
    x_{k+1} &= Ax_k + B_1u_k + B_2w_k, \\
    y_k^1 &= C_1x_k, \\
    y_k^2 &= C_2x_k,
\end{align*}
\]

where \( B_2 = (D_1, D_2) \), \( w_k = (f_k, \Delta x_k) \in \mathbb{R}^{m_2} \) - equivalent signal disturbance, \( m_2 = p + d \).

Hereby in multivariable object structure may be picked out the following four channels of signal transfer:

- control input – controlled output channel “C-C” with transfer function \( G_{11}(z) = C_1(zI_n - A)^{-1}B_1 \),
- disturbance input – measured output channel “D-M” with transfer function \( G_{22}(z) = C_2(zI_n - A)^{-1}B_2 \),
- control input – measured output channel “C-M” with transfer function \( G_{21}(z) = C_2(zI_n - A)^{-1}B_1 \),
- disturbance input – controlled output channel “D-C” with transfer function \( G_{12}(z) = C_1(zI_n - A)^{-1}B_2 \).

The output regulation problem (Seraji H., 1989) is to find the control sequence \( \{u_k\} \), depending from the measured output variables \( y_k^2 \), which ensure the reference signal \( y_k^1 \) tracking and disturbances \( w_k \) rejection. The requirement of closed-loop system stabilization along with the disturbance rejection leads to the disturbance rejection problem with stability. Moreover, as long as the state vector of the system can’t be measured directly, the formulation of the disturbances rejection problem by measurement feedback can be defined.

Following the proposed approach, the control structure for disturbance rejection and closed-loop system stabilization selected as a combination of DO and DC, both based on the designed IM of the respective controlled object channels.

From practical point of view it is desirable to decompose the problem into two steps, namely, the structural synthesis of the designed DO and DC renders the fixed and free parameters tuning methods in order to satisfy the design goals, such as pole-placement, performance optimization and so on.

3. DISTURBANCE OBSERVER DESIGN

3.1 Disturbance Inverse Model-Based Observer

By analogy with the dynamic observers theory, the discrete dynamic system

\[
\begin{align*}
    x_{k+1} &= A^tx_k + B_1^ty_k + B_2^ty_{k+d} + B_1^tu_k, \\
    w_k &= C^tx_k + D_1^t y_k + D_2^t y_{k+d},
\end{align*}
\]

with state vector \( \bar{x}_k \in \mathbb{R}^{n-q} \) will be referred to as asymptotic reduced-order disturbance observer, if dynamic system (3) is asymptotically stable and the following conditions take place: \( \|\bar{x}_k - Rx_k\| \to 0 \), \( \|w_k - w_k\| \to 0 \), \( k \to \infty \), where \( R \in \mathbb{R}^{n-q \times n} \) is the appropriate aggregate matrix, such that rank \( (C_2^t : R^t) = n \), rank \( R = n - q \).

It is obvious, that equation (3) match the equation of inverse system for channel “D-M” of object (2).

At that vector \( w_k \) may be treated as input disturbance estimation, obtained by the IM (3). In such a case, integer \( r \geq 1 \) determines the minimal input disturbance estimation delay, which coincide with channel “C-C” relative order, namely, minimal integer, such as matrix Markov parameter \( S_{22}(r) = C_2A^{-1}B_2 \neq 0 \).
The minimal state-space realization of IM may be obtained by means of reduced order UIO. For the purpose of simplicity, consider only the case of single relative order, when \( S_{22} = C_{22} B_{2} \neq 0 \). Treated \( w_{k} \) as an unknown input signal, the state vector estimate \( \hat{x}_{k} \in \mathbb{R}^{n} \) may be obtained by minimal-order UIO as follows:

\[
x_{k+1} = F x_{k} + G y_{k} + G_{0} u_{k}, \quad x_{k} = \hat{x}_{k} + P \hat{e}_{k},
\]

where \( \hat{x}_{k} \in \mathbb{R}^{n-q}; \) – estimation of aggregated state vector \( Rx_{k} \), \( \hat{x}_{k} \) – observer state vector.

Matrices \( P_{1} \in \mathbb{R}^{n_{1}q_{1}} \), \( Q_{1} \in \mathbb{R}^{n_{2}n_{2}} \) are uniquely determined by selected aggregate matrix \( R \) and defined as:

\[
(P \ Q) = \begin{pmatrix} C^{-1} & 0 \end{pmatrix} R, \quad CP = I_{q_{1}}, \quad RQ = I_{q_{2}}, \quad PC + QR = I_{n},
\]

(5)

Equations of observer (4) dynamic part may be transformed to the equivalent form

\[
\tilde{x}_{k+1} = F \tilde{x}_{k} + \left[ G - F \overline{H} \right] y_{k}^2 + H \tilde{x}_{k+1} + G_{0} u_{k}.
\]

(6)

At that state vector estimation error \( e_{k} = x_{k} - \tilde{x}_{k} \) and aggregated variables deviation \( \tilde{e}_{k} = \tilde{x}_{k} - \tilde{x}_{k} \) are connected by linear transformation \( e_{k} = Q \tilde{e}_{k} \), thus \( \tilde{e}_{k} \) will be given by following equation:

\[
\sigma_{k+1} = F \sigma_{k} + \left[ RA - FR - (G - F \overline{H}) C_{2} - (G - F \overline{H}) A \right] \sigma_{k} + (RB_{2} - HC_{2}) w_{k} + (RB_{1} - G_{0} u_{k}.
\]

(7)

From (7) the observer design conditions, namely, state estimation independence from unknown input disturbance (so called “unknown input invariance conditions”) follows in the form:

\[
(R - HC_{2}) A - F (R - HC_{2}) = \overline{G} C_{2},
\]

\[
RB = HC_{2} B_{2}, \quad RB_{1} = G_{0}.
\]

(8)

If rank \( S_{22} = q_{2}, q_{2} \geq m_{2} \) (invertibility conditions, which may be treated as UIO structural synthesis problem solvability), the linear matrix equation (8) solution may be obtained as

\[
F_{2} = R \Pi_{2} A Q_{2}, \quad \overline{G}_{2} = R \Pi_{2} A (H + P \Omega_{2}),
\]

\[
\overline{H}_{2} = R B_{2} S_{22}^{+}, \quad G_{0} = R B_{1},
\]

where \( \cdot^{+} \) denotes Moore-Penrouze generalized inverse, and projection matrices \( \Pi_{2} = I_{n} - B_{2} S_{22}^{+} C_{2} \), \( \Omega_{2} = I_{q} - S_{22} S_{22}^{+} \) while \( C_{2} \Pi_{2} = \Omega_{2} C_{2} \).

Taking an input vector \( w_{k} \) estimate as \( w_{k} = B^{+} (x_{k+1} - A x_{k}) \) and considering it as IM output vector, the inverse model of channel “D-M” equations may be obtained from (6), (9):

\[
x_{k+1} = R \Pi_{2} A Q \sigma_{k} + R \Pi_{2} A P \sigma_{k}^{2} + R B_{2} S_{22}^{+} y_{k+1}^{2} + R B_{1} u_{k},
\]

\[
w_{k} = (S_{22}^{+} + B_{2}^{+} P R) (y_{k+1}^{2} - C_{2} A Q \sigma_{k} - C_{2} A P \sigma_{k}^{2}).
\]

From (10) follows, that IM dynamics matrix \( A^{I} = F_{2} = R \Pi_{2} A Q \) depends from the arbitrary aggregate matrix \( R \) of given rank \( n - q_{2} \), which may be considered as an IM based DO tuning matrix.

### 3.2 Disturbance Observer Parameterization

Using the special block form of object (1) model (for notation simplicity indices at \( q \) are omitted),

\[
A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad C_{2} = \begin{pmatrix} I_{q} & 0 \\ 0 & I_{n-q_{2}} \end{pmatrix}, \quad B_{2} = \begin{pmatrix} B_{21} \\ B_{22} \end{pmatrix}_{n-q},
\]

(11)

which may be obtained by non-singular state-space transformation, and concretely define the matrices \( P \), \( Q \) choice, one can admit:

\[
(P \ Q) = \begin{pmatrix} P_{1} & Q_{1} \\ P_{2} & Q_{2} \end{pmatrix}_{n-q}, \quad P_{1} = I_{q}, \quad Q_{1} = 0_{n-q_{2}-q}.
\]

(12)

In such a case, for any \( Q_{2} \) such that \( \det Q_{2} \neq 0 \), aggregate matrix may be found in the form \( R = Q_{2}^{-1} (P_{2} I_{n-q}) \), and consequently, the IM matrices for plant representation (11) are the following:

\[
R \Pi_{2} A Q_{2}^{-1} (\tilde{A}_{22} - P_{2} \Omega B_{21} A_{21}) Q_{2} = \tilde{Q}_{2}.
\]

(13)

Thus, in fact, the nonsingular matrix \( Q_{2} \) specifies the similarity transformation and doesn’t change the \( A^{I} \) spectrum, which, as follows from (13), completely determined by only arbitrary tuning matrix \( P_{2} \). The last one may be found by any type of pole-placement method, and appropriate modal control problem will be solvable if matrix pair \( (\tilde{A}_{22}, \tilde{A}_{21}) \) is observable. In particular, in such a way the designed IM model may be stabilized for nonminimum-phase plant. It may be shown, that such a condition is equivalent to the well-known UIO design solvability condition, namely input observability (Hou M., Muller P.C., 1992). Therefore, the aggregate matrix \( R \) is determined up to an arbitrary nonsingular matrix \( Q_{2} \).

### 3.3 Inverse Model Regularization

The input observability condition is obviously violated in the case when \( m_{2} = q_{2} \). At that \( \Omega B_{1} = 0 \) and \( A^{I} \) doesn’t depend from \( P_{1} \). In such a case for the tuning properties guarantee it is possible to use the so-called “regularized” UIO (Lyubchyk L., Grinberg G., 2007), which ensure the approximately invariance with respect the unknown disturbance:

\[
\|R B_{2} - H C_{2} B_{2}\|^{2} + \|H\|^{2} \rightarrow \min_{H}
\]

(14)

where \( \varepsilon > 0 \) – regularization parameter.
In such a case, the solution of minimization problem (14) is
\[ \mathcal{H}_2(\varepsilon) = R B_2 S_{22}^T (\varepsilon_2 q + S_{22} S_{22}^T)^{-1} \] and solution regularized IM structural synthesis problem may be obtained in the form:
\[ \mathcal{F}_2(\varepsilon) = \mathcal{A}_{22}(\varepsilon) - P_2 \Omega_{21}(\varepsilon) \mathcal{A}_{12} \] \[ \mathcal{A}_{22}(\varepsilon) = \mathcal{A}_{22} - B_2 \Psi_{21}(\varepsilon) A_{22} \] \[ \Psi_{21}(\varepsilon) = B_2^T (\varepsilon_2 q + S_{22} S_{22}^T)^{-1} B_{21} \] \[ \Omega_{21}(\varepsilon) = I_q - B_{21} \mathcal{B}_{21}(\varepsilon) (\varepsilon_2 q + S_{22} S_{22}^T)^{-1} \].

(15)

It is obvious, that provided \( q_2 = m_2 \), matrix \( \Omega_{21}(\varepsilon) \neq 0 \) for any \( \varepsilon > 0 \), and corresponding modal design problem may be solved, if observability condition for matrix pair \( (\mathcal{A}_{22}(\varepsilon), \mathcal{A}_{12}(\varepsilon)) \), \( \mathcal{A}_{12}(\varepsilon) = \Omega_{21}(\varepsilon) A_{12} \), takes place.

The estimation errors for designed disturbance observer based on regularized IM are given by:
\[ e^w_{k+1} = \mathcal{F}_2(\varepsilon) e_{k+1} + \varepsilon R B_2 (\varepsilon I_q + S_{22}^T S_{22})^{-1} w_k \]
\[ e^w_k = -B_2^* (H_2(\varepsilon) + \varepsilon \Omega(\varepsilon) C_2 Q \xi^e_{k+1}) + \varepsilon B_2^* (I - PC_2) (\varepsilon I_q + S_{22}^T S_{22})^{-1} w_k \] (16)

At the proposed IM regularization ensure desired dynamic properties of disturbance observer in singular case, but the estimation errors are depended from the real disturbance, though the factor level is determined by regularization parameter \( \varepsilon \geq 0 \), which in turn should be specified by the trade-off between disturbance observer dynamic performance and additional estimation error, component, caused by inverse model regularization

4. DISTURBANCE COMPENSATOR DESIGN

4.1 Disturbance Compensator Structural Synthesis

The disturbance estimates may be used for disturbance rejection control law construction. Consider the combined control law as the sum of stabilizing and compensation components \( u_k = u^s_k + u^c_k \), though control stabilizing component \( u^s_k \) is taken as \( u^s_k = K \cdot e^c_k \), where \( e^c_k = \gamma^c_k - y_k \) - control error, \( K \) - linear output feedback matrix, determined by output modal control methods (Wolovich W.A., 1995).

Compensative control component \( u^c_k \) is formed by multivariable dynamic disturbance compensator (DC) constructed in the form of plant’s channel “C-O” inverse model with special input signal \( r_k \), generated by the predictive model (PM)
\[ x^p_{k+1} = A r_k + B_j w_k, \quad y^p_k = C x^p_k, \quad r_k = y^c_k - y^p_k \] (17)
where \( x^p_k \in \mathbb{R}^n \) - PM state vector, \( y^p_k \in \mathbb{R}^q_i \) - predicted plant reaction on input disturbance.

Using the approach described above, the multivariable DC equation may be obtained as a combination of “C-O” control channel IM and disturbance channel “D-O” PM. Consider the one-step PM input signal prediction
\[ r_{k+1} = y^c_{k+1} - C_1 A r^p_k - C_1 B_2 w_k \], it is easily to obtain DC equation in the form:
\[ \xi^c_{k+1} = \mathcal{F}_1(\varepsilon, P) \xi^c_k + R(\varepsilon, P) \Omega(\varepsilon) B_2 w_k \]
\[ u^c_k = -\mathcal{P}(\varepsilon) (C_1 A Q \xi^c_k + C_1 B_2 w_k) \] (18)

where \( \xi^c_k = Y^c_k + R x^c_k \) - dynamic compensator state vector.

As a result compensator matrices
\[ \mathcal{F}_1(\varepsilon) = R \mathcal{P}(\varepsilon) \mathcal{Q}, \quad \mathcal{F}_1(\varepsilon) = R B_2 \mathcal{S}_1(\varepsilon), \]
\[ \mathcal{B}(\varepsilon) = B_2^* (H_2(\varepsilon) + P \Omega(\varepsilon)), \quad \mathcal{P}(\varepsilon) = I_n - H(\varepsilon) C_1 \]
\[ \Omega(\varepsilon) = I_q - S_{11} \mathcal{S}_1(\varepsilon) \]
are depended from the appropriate IM tuning parameters \( \varepsilon, P \).

Matrix \( S_{11}(\varepsilon) = S_{11} (\varepsilon I_q + S_{11}^T S_{11}^{-1}) \) is \( S_{11} \) generlized inverse, though \( S_{11}(\varepsilon) \) \( \varepsilon=0 = S_{11}^{-1} \) and \( S_{11}(\varepsilon) \) \( \varepsilon=\infty = 0 \).

4.2 Combined Control System Dynamics

Consider the dynamic properties of closed-loop combined feedback-feedforward control system with control plant (2) and designed DO (10) and DC (18), (19). In order to obtain the control error equation, consider at first equation for output controlled signal for system with DC, with used a direct measured (ideal feedforward control).

For the purpose of simplicity consider the case when reference signal \( y_0 = 0 \) and \( m_1 = q_1 \).
\[ y_{k+1} = C_1 A x_k + C_1 B_1 u_k + C_1 B_2 w_k = A x_k + S_{11} (-K y_k + \xi^c_k) + C_1 B_2 w_k = C_1 A x_k + S_{11} K y_k - S_{11} B \mathcal{B}(\varepsilon) C_1 A Q \xi^c_k - (I_m - S_{11} \mathcal{B}(\varepsilon)) C_1 B_2 w_k \]

Using the matrix identity
\[ I - S_{11} \mathcal{B}(\varepsilon) = (I_q - S_{11} B_1^* P) \mathcal{Q} \mathcal{A} \mathcal{B}(\varepsilon), \]
\[ \mathcal{B}(\varepsilon) = I_q - (I_q - S_{11} B_1^* P) \mathcal{Q} \mathcal{A} \mathcal{B}(\varepsilon), \]
and taking into account that \( x_k = Q \xi^c_k = Q \bar{y}_k - P e_k \), where \( \bar{y}_k = R x_k - \xi^c_k \), after some algebraic transformation it is possible to get the output control error equation:
\[ e_{k+1} = C_1 (A P - B_1 K) e_k - C_1 A Q \bar{y}_k + \Lambda_1(\varepsilon) \pi_{k+1} \]
\[ \bar{y}_k = R (B_1 K - A P) e_k + R A Q \bar{y}_k + \Lambda_2(\varepsilon) \pi_{k}, \]
\[ \pi_k = -C_1 B_2 w_k + C_1 A Q \xi^c_k, \]
where \( \Lambda_1(\varepsilon) = (I_q - S_{11} B_1^* P) \mathcal{Q} \mathcal{A} \mathcal{B}(\varepsilon), \Lambda_2(\varepsilon) = R B_2 \mathcal{B}(\varepsilon) \mathcal{Q} \mathcal{A} \mathcal{B}(\varepsilon), \)

From the properties of introduced matrices follows that
\[ \mathcal{O}_1(\varepsilon = 0) = 0, \quad S_{11} \mathcal{B}_0(\varepsilon = 0) = I_q, \quad S_{11} \mathcal{B}_\infty(\varepsilon = \infty) = S_{11} B_1^* P. \]
Using the notation $x_k^0 = Q\theta_k - P\epsilon_k$, the control error equation (22) may be represented in equivalent form:

$$x_k^0 + (A - B_1K)\sqrt{x_k^0} + \nabla\epsilon = -C_1x_k^0,$$

where $\nabla\epsilon(\epsilon, P_2) = (I_m - B_1B_1^*)P\Omega_1(\epsilon)$, though $\nabla\epsilon(\epsilon = 0) = 0$.

Note, that $x_0^k$ is a closed-loop system state vector, and $\overline{w}_k$ may be considered as some equivalent disturbance.

From equation (23) follows, that including in control law additional compensation component, formed by the DC, leads to the effect, which equivalent to initial disturbance transformation by the some dynamic filter. At that the equivalent disturbance $\overline{w}_k$ influence on control error is determined by matrix $\nabla\epsilon(\epsilon, P_2)$, depended from the DC tuning parameters.

4.3. Disturbance Compensator Parametric Synthesis

The problem of DC parametric design is to find the compensator tuning parameters $\epsilon, P_1$ based on the trade-off between the requirements to the compensator dynamic properties (degree of stability, dynamic performance and so on) and degree of disturbance compensation.

In the case when disturbance is immeasurable, the DC input signal should be formed by the real-time disturbance estimates $w_k$, obtained by DO (10). This situation may be considered as indirect disturbance measurement with some measurement noise $\overline{w}_k = w_k + \xi_k$, which must be taken into account under DC parametric design.

Using the proposed technique, the equations for control error dynamics for control system with DO and DC in the presence of measurement noise have been obtained as:

$$e_{k+1} = C_1\left[A(P - B_1K)e_k - C_1AQ\theta_k + \Lambda_1(\epsilon)\pi_k - S\beta_1B_2\xi_k\right],$$

$$\overline{w}_{k+1} = R(B_1K - AP)e_k + RAQ\theta_k + \Lambda_2(\epsilon)\pi_k - R(\Pi_1(\epsilon) + B_1B_1^*)B_2\xi_k,$$

where $\pi_k = -S_1w_k + C_1AQ\theta_k^0$,

$$\Lambda_1(\epsilon, P_2) = (I_q - S_1B_1^*P)\Omega_1(\epsilon), \quad \Lambda_2(\epsilon, P_2) = RB_1B_1^*P\Omega_1(\epsilon).$$

From (24) follows the equation of closed-loop feedback-forward system with indirect disturbance measurement, realized by DO:

$$x_k^0 + (A - B_1K)\sqrt{x_k^0} + \nabla\epsilon = -C_1x_k^0,$$

where $\nabla\epsilon(\epsilon, P_2) = (I_m - (I_m - B_1B_1^*)P\Omega_1(\epsilon)C_1)B_2$, $\nabla\epsilon(\epsilon = 0) = B_2$.

As it follows from closed-loop control system equation (25), combined control realization by the inverse model-based DO and DC is equivalent to some type of dynamic disturbance transformation and has a selected invariance properties with respect to unmeasurable and unknown disturbance.

From (25) follows that for nonminimum-phase object there exist the reachable control accuracy level depended from both object and disturbance characteristics and compensator parameters.

In order to DC parametric synthesis problem formalization, stated at first relationship between discrete $Z$ -transformations of control error $e(\epsilon)$ and disturbance and measurement noise $w(\epsilon), \xi(\epsilon)$:

$$e(\epsilon) = G_w(z)w(\epsilon) + G_\xi(\epsilon)\xi(\epsilon),$$

where discrete matrix transfer functions $G_f(z), G_\xi(z)$ are the following:

$$G_w(z) = C_1(J_n - A + B_1K)^{-1}\nabla\epsilon(\epsilon)C_1,$$

$$G_\xi(z) = C_1(J_n - A + B_1K)^{-1}T.$$

The DC design problem formalization should be used $a priori$ information concern disturbance estimation distortion.

Consider the problem solution in the case when $a priori$ information is given in non-statistic form. In such a case as a model of disturbance and measurement noise the signals of restricted energy are used:

$$\|w\|_2 = \sqrt{\sum_{\epsilon(k)}|w_k|^2} \leq E_w < \infty, \quad \|\xi\|_2 = \sqrt{\sum_{\epsilon(k)}|\xi_k|^2} \leq E_\xi < \infty.$$

Disturbance compensation level will be characterized by control error energy $E_e = \sum_{\epsilon(k)}|e_k|^2$ with upper bond

$$E_e \leq E_c = \|G_w(z)\|_2 E_w + \|G_\xi(z)\|_2 E_\xi,$$

as it follows from norm relationship theorem Francis B.A., 1987), where $H^\infty$ norm of matrix transfer function defined as follows:

$$\|G(z)\|_\infty = \sup_{|\epsilon| = 2\pi} |G(\epsilon)|_2 = \sup_{0 \leq \omega \leq 2\pi} \lambda_{\max}^{1/2}[G^T(e^{-j\omega})G(e^{j\omega})].$$

In accordance with general purposes of $H^\infty$ norm, $\overline{E}_u$ criterion minimization is equivalent to control error energy minimization for worst input signals (disturbance and noise) of restricted energy (Francis B.A., 1987, Zhou, K., Doyle, J. C., and Glover, K., 1996).

The appropriate problems of DC parametric $H^\infty$ optimization with additional dynamic compensator performance constraint, namely, transfer process duration and variation, may be stated as:

$$E_e(\epsilon, P_2) \rightarrow \min, \quad T(\epsilon) \leq T^*, \quad \sigma(\epsilon) \leq \sigma^*.$$
where $\lambda(\varepsilon) = \|F(\varepsilon)\|_F = \lambda_1^{1/2}(F^T(\varepsilon)F(\varepsilon))$, $\delta$ – predetermined level, which characterized the DC transfer processes duration.

The problem (31) may be solved by numerical methods, which makes it possible to find optimal values of DC tuning parameters.

Hereby the proposed method of DC design ensures its internal stability and desired dynamic properties and robustness with respect to uncertain inaccuracy of disturbance indirect measurement by IM based DO.

Thus the reachable control accuracy level, depended from disturbance and measurement noise properties (28), is defined by (29), (30). Such a level may be estimated using proposed technique and characterized accessible control accuracy in the presence of unmeasurable disturbance.

6. CONCLUSIONS

Following the inverse model approach, the reduced-order regularized multivariable DO and DC has been developed and relevant structural and parametric synthesis as well as design pole-placement methods has been proposed.

The properties of closed-loop feedback/feedforward combined control system with DO and DC have been also investigated and it was shown, that for nonminimum-phase object there exist the reachable control accuracy depended from both object and disturbance characteristics and compensator parameters. So the design procedure should be based upon trade-off between DO/DC dynamic requirements and disturbance rejection level. Proposed approach ensures the possibility of DO/DC parameters optimization taking into account a priori information concern disturbance and measurement noise.

IMC approach to selective invariance properties ensuring in linear multivariable systems leads to the decomposition of the problem on the disturbance estimation and compensation. As it has been shown, both problems may be solved by the IM approach utilization. Therefore, the IMC control method is seemed to be one of the most practical approaches to the disturbance rejection problem in multivariable systems and may be consider as a basis for high accuracy control system design. Moreover, the UIO theory may be used as a basis for IM design. It is essentially that such an approach gives a simple criterion of inverse model design problem solvability.

The IMC approach may be also successfully used in dynamic identification problems (Corless, M., Tu, J. F. 1998), such as real-time unknown structure input signal reconstruction and blind separation (Lyubchik L., Grinberg G., 2007) using only current measurements of system output variables. In such a case the problem of simultaneous signal state and model parameter estimation is appeared, so the problem under the conventional approach comes to nonlinear adaptive filtering problem. At that the IMC approach ensures the decomposition of the identification problem into the separate signal state and model linear estimation realized by the well-developed linear recurrent algorithms.

REFERENCES


