Data-driven Adaptive Control: Making Unfalsified Control Work Better

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Abstract: In this paper, a data-driven adaptive control scheme is presented which is based upon Unfalsified Control theory. The presented work extends our previous works on suitable cost functions in Unfalsified Control and the use of optimization to obtain new optimal controllers. A data-driven method is used here to obtain a sufficient condition for robust stability. This condition is used to falsify the non-robust controllers. Additionally it is used along with a signal-to-peak norm constraint in the optimization of the candidate controller parameters, to obtain robust controllers and to avoid the controller saturation. A case study using a well known example from chemical engineering shows the application of the extended scheme.

Keywords: Adaptive control, switching control, online controller optimization, robustness condition.

1. INTRODUCTION

Unfalsified Control, see Safonov and Tsao (1997), is a data-driven control scheme that adapts the parameters of the active controller by switching between finitely many predefined candidate controllers. To compare the candidate controllers, the so-called fictitious signals are used in a cost function to evaluate the performance of the candidate controllers without having to place the candidate controllers in the control loop. However, the originally used cost function, see Safonov and Tsao (1997), Wang et al. (2007), could not indicate the cost of offline destabilizing controllers correctly unless they are placed in the loop. This problem was indicated by Dehghani et al. (2007), Engell et al. (2007), Manuelli et al. (2007). An alternative cost function which can be used to compute the cost of stabilizing or destabilizing controllers correctly was proposed in Engell et al. (2007). To improve the set of candidate controllers, Engell et al. (2007) suggested to use this new cost function and the data recorded from measured signals in an online optimization to search for an optimal candidate controller. This idea has recently been extended to include a robustness constraint, which is only derived from the measurements data, to the optimization of the candidate controller parameters to obtain robust controllers. Also this condition is used to avoid non-robust controllers from been placed in the control loop, see Nabati and Engell (2010).

This paper extends our previous work on using a data-driven method to describe the model uncertainty bounds caused by plant-model mismatch and to derive a sufficient condition for robust stability, by introducing a signal-to-peak norm constraint to restrict the amplitude of output of the controller to a predefined bound for a realistic change in the setpoint. This constraint is used to complement the robustness constraint used in the optimization of the candidate controller parameters, to avoid controller saturation and to obtain robust controllers.

We apply the new scheme to a challenging case study, the control of a Continuous Stirred Tank Reactor (CSTR) with nonlinear dynamics and nonminimum phase behavior, see Engell and Klatt (1993). We here consider the case with constant cooling temperature, i.e. variable reactor temperature and hence varying reaction rates. This simulation shows that our adaptive control scheme is able to control the nonlinear CSTR with nonminimum phase behavior at a wide range of operating points, despite the fact that the initial set of candidate controllers does not contain any well tuned controller and the measurements are delayed and contain noise.

The paper is organized as follows: First Unfalsified Control and the extensions by Engell et al. (2007) are briefly presented. In section 3, a robustness constraint and a signal-to-peak norm constraint are introduced. These constraints are used to extend the controller falsification and the optimization criterion in section 4. Section 5 shows the case study for the CSTR example. Finally, some conclusions are drawn.

2. THE ADAPTIVE CONTROL SCHEME

2.1 Preliminaries

Let \( x := \{x(1), x(2), \ldots \in \mathbb{R} \} \) denote discrete-time signals (sequences). The \( l_2 \)-norm of a sequence \( x \) is defined as
\[
\|x\|_2 := \left( \sum_{k=0}^{\infty} |x(k)|^2 \right)^{1/2}.
\]
The \( l_\infty \)-norm of a sequence \( x \) is defined as
\[
\|x\|_\infty := \max_{0 \leq k \leq \infty} |x(k)|.
\]
The induced 2-norm \( \|\cdot\|_2 \) of a dynamic system \( G : u \rightarrow y \) is defined by
Let $G$ be a dynamic operator which maps $u$ to $y$. Then $y = G(u)$ is written as $y = G \cdot u$ if $G$ is linear. $\pi_k$ denotes the $k$ step discrete time truncation operator defined as $\pi_k : \{x(1), \ldots \} \rightarrow \{x(1), \ldots, x(k)\}$. $\otimes$ denotes the convolution operator.

The setup of the control scheme used in our approach, see Safonov and Tsao (1997), Wang et al. (2007) and Wonghong and Engell (2008). The value of the original cost function for the destabilizing controllers remains bounded as a result of pole zero cancellation in the cost function. The issue is solved in Engell et al. (2007) by introducing a new cost function that does not suffer from the pole zero cancellation problem and leads to a correct evaluation of the performance of the both stabilizing and destabilizing candidate controllers. In this new cost function

$$J_{\text{original}}(C_i, k) := \frac{\|\pi_k \hat{e}_i\|_2}{\|\pi_k r_i\|_2 + \rho},$$

(1)

where $\rho$ is a small positive constant. An advantage of this original cost function is that no plant or closed-loop identification was needed to compute the cost values. However, later studies have shown that when using the original cost function the cost of destabilizing candidate controllers can not be computed correctly unless the destabilizing controller is placed in the control loop, see Engell et al. (2007), Dehghani et al. (2007). The value of the original cost function for the destabilizing controllers remains bounded as a result of pole zero cancellation in the cost function. The issue is solved in Engell et al. (2007) by introducing a new cost function that does not suffer from the pole zero cancellation problem and leads to a correct evaluation of the performance of the both stabilizing and destabilizing candidate controllers.

The setup of the control scheme used in our approach, which is a modified version of the scheme in Wonghong and Engell (2008), is shown in Fig. 1. The unknown plant $P$ is a nonlinear and possibly time-varying operator that maps $u$ to $y$. The plant is controlled in a feedback loop. The adaptive controller modifies the active controller $C$ by selecting a controller from a set of candidate controllers defined as $C := \{C_1, C_2, \ldots, C_n\}$. In the presented scheme, the data obtained from the recorded measurements $(r, u, y)$ are used to evaluate the cost of each candidate controller. According to the computed cost values, the best candidate controller is selected by the switching mechanism. Furthermore, optimization is used to obtain optimal candidate controllers if such candidates are not already available in the set $C$.

### 2.2 Calculation of the cost function values

A key feature of the Unfalsified Control technique is to avoid the need to put the candidate controllers in the control loop to evaluate their performance. Also there is no need to have any prior information about the plant. The information obtained from the signals $(r, u, y)$ recorded in the past suffices to evaluate the performance of the candidate controllers. In Unfalsified Control, the so-called fictitious signals are used to derive a measure for the performance of the candidate controllers. For each candidate controller $C_i$, the fictitious reference $\hat{r}_i$ and the fictitious error $\hat{e}_i$ signals are calculated using the setup shown in Fig. 2, where $C_i^{-1}$ is the inverse of the candidate controller $C_i$.

$$J(C_i, k) := \frac{\|\pi_k \hat{e}_i\|_2}{\|\pi_k r_i\|_2 + \rho},$$

(2)

with $\hat{E}_i(z) := S_i(z) \hat{r}_i(z)$ and $E_i(z) := S_i(z) R(z)$. $S_i$ is unknown as the plant $P$ is unknown. However, $S_i$ can be computed approximately by deconvolution of the fictitious signals $\hat{r}_i$ and $\hat{e}_i$. With $\hat{e}_i := \hat{e}_i \otimes r$ and $\hat{e}_i := \hat{e}_i \otimes \hat{r}_i$. This solves the problem of the original cost function, however it involves a kind of identification. The sensitivity function $S_i$ is identified as a FIR model from the fictitious signals $\hat{e}_i$ and $\hat{r}_i$.

The cost function (2) can be used online to evaluate the performance of the candidate controllers and also in an optimization to compute $l_2$-optimal controllers, see Engell et al. (2007) and Wonghong and Engell (2008).

#### 2.3 Switching mechanism

In the Unfalsified Control framework, the adaptation is done by switching the parameters of the active controller to the parameters of a possibly better performing candidate controller, see e.g., Wang et al. (2007). This switching mechanism is based on the $\epsilon$-hysteresis algorithm of Morse et al. (1992) shown below.

### Algorithm 1 $\epsilon$-hysteresis algorithm

1: initialize: Let $k := 0$; choose $\epsilon > 0$; let $\hat{C}(0) := C_1$, $C_1 \in C$, be the first controller in the loop
2: $k := k + 1$
3: if $J \left( \hat{C}(k-1), k \right) > \min_{C \in C} J(C, k) + \epsilon$ then
4: $\hat{C}(k) := \arg \min_{C \in C} J(C, k)$
5: else
6: $\hat{C}(k) := \hat{C}(k-1)$
7: end if
8: goto 2

This algorithm selects the best candidate controller as the active controller at each time step. The active controller is adapted only when it does not perform satisfactorily.

Fig. 1. The adaptive Unfalsified Control scheme

Fig. 2. Generation of fictitious signals $\hat{r}_i, \hat{e}_i$. Originally, see Safonov and Tsao (1997), Wang et al. (2007), the $l_2$-norm of the fictitious signals $\hat{r}_i$ and $\hat{e}_i$ were directly used to compute the cost function as

$\|G\|_2 := \sup_{u \neq 0} \frac{\|y\|_2}{\|u\|_2}$. Let $G$ be a dynamic operator which maps $u$ to $y$. Then $y = G(u)$ is written as $y = G \cdot u$ if $G$ is linear. $\pi_k$ denotes the $k$ step discrete time truncation operator defined as $\pi_k : \{x(1), \ldots \} \rightarrow \{x(1), \ldots, x(k)\}$. $\otimes$ denotes the convolution operator.
and if a better candidate controller is available. Thus, the adaptation is not performed continuously but only when necessary. This is an advantage of Unfalsified Control compared to other adaptive control techniques, since it has been observed that the continuous tuning is a major source of instability in adaptive control techniques.

By selecting the threshold \( \alpha \) properly, destabilizing candidate controllers can be detected (falsified) and removed from the set of candidates.

**Definition 1. Falsification:** If the cost of a candidate controller exceeds a threshold \( \alpha \)

\[
J(C_t, k) > \alpha,
\]

for some \( k \), the candidate controller is called a falsified controller, see Safonov and Tsao (1997).

### 2.4 Controller parameter optimization

A restrictive assumption of the original scheme of Unfalsified Control is that the set of candidate controllers is fixed and known a priori, see Safonov and Tsao (1997). If all candidate controllers become falsified at a later time, the control scheme fails.

To remove this restriction, Engell et al. (2007) suggested to use optimization to add new controllers to the set. The information obtained from the measurements is used to optimize the parameters of controller. The optimization was originally triggered by steps of the reference. In Nabati and Engell (2010), we have introduced an error scanning mechanism to activate the optimization. This mechanism monitors the error \( e \) and the rate of change of the error to determine whether the active controller (which is the best available controller in the set) is performing satisfactorily or not. If the active controller is not performing satisfactorily, the optimization is activated.

### 3. CONSTRAINTS FOR THE OPTIMIZATION PROBLEM

If Unfalsified Control is applied to a nonlinear or time varying plant, there will be some plant-model mismatch and hence the optimal controller should also be sufficiently robust. To address this issue, the \( \mathcal{H}_2 \) (integral-quadratic) objective used in the optimization can be complemented by an \( \mathcal{H}_\infty \) constraint. This results in an optimal controller which has the optimal performance and meets a robustness criterion. In this paper also actuator saturation problems are avoided by adding a signal-to-peak norm constraint. This constraint is used to restrict the amplitude of the output of the controller to a predefined bound for realistic changes in the setpoint.

#### 3.1 Robustness constraint

To consider the plant-model mismatch, a data-driven method is used to derive bounds for the model uncertainty. The obtained bounds are then used to define the robustness constraint. The model uncertainty can be described by the model error modeling concept, as discussed by Ljung (1999). The setup of the output-multiplicative model error model is shown in Fig. 3. \( G \) is a linear nominal model of the plant and \( W_u \) is a linear filter. In our approach the linear model \( G \) is obtained from the recorded signals \( (r, u, y) \) using a closed-loop identification technique.

In the setup shown in Fig. 3, the filtered sequence \( \bar{u} \) is obtained as \( \bar{u} = W_u * u \). The operator \( \Delta_c \) which is a possibly nonlinear or time varying operator maps \( \bar{u} \) to \( y \). It is assumed that \( \bar{u} = y_{sim} \) is obtained by the simulation of the linear model \( G \) using the input sequence \( u \). If \( W_u \) is fixed, the gain of the operator \( \Delta_c \) can be determined following Poolla et al. (1994).

**Theorem 1.** Poolla et al. (1994). Given input and output sequences \( \bar{u}, y \) of length \( N \), there exists a stable causal dynamic system \( \Delta_c \) with

\[
\|\Delta_c\|_2 \leq \gamma,
\]

iff

\[
\|\pi_k y\|_2 \leq \gamma \|\pi_k \bar{u}\|_2, \quad \forall k \in [1, N].
\]

Theorem 1 gives an upper bound for the gain of the mapping operator \( \Delta_c \). The linear filter \( W_u \) required to compute the uncertainty bound can be selected arbitrarily. However it is desired to select \( W_u \) such that a tight bound is obtained. To obtain the filter \( W_u \), we follow the ideas of Hindi et al. (2002) and Völker and Engell (2005), where the goal is to use optimization to compute \( W_u \) such that a tight uncertainty model is obtained which is consistent with the measurements (6). Völker and Engell (2005) formulate the optimization problem as

\[
\min_{W_u \in \mathcal{H}_\infty} \|W_u\|_2,
\]

s.t.

\[
\|\pi_k y\|_2 \leq \|\pi_k (W_u \cdot \bar{u})\|_2, \quad \forall k \in [1, N].
\]

To solve this optimization problem a model structure with orthonormal basis functions for the filter \( W_u \) is selected. The coefficients of the basis functions are optimized. The objective function is evaluated in the frequency domain over a finite grid of frequencies and the constraints are evaluated in the time domain. After solving the optimization problem the obtained filter \( W_u \) is scaled according to algorithm 2. Thus \( \|\Delta_c\|_\infty \leq 1 \) holds.

**Algorithm 2 \( \Delta_c \) consistency test**

1. **initialize:** Let \( k := k_0 \) be the time when Alg. 2 is called for the first time; let \( y := \{y(0), \ldots, y(k)\} \) be measurements
2. \( k := k + 1 \)
3. \( \bar{u} := y - G \cdot u \)
4. \( \bar{u} := W_u \cdot G \cdot u \)
5. \( \gamma := \max (\|\pi_k y\|_2/\|\pi_k \bar{u}\|_2, 1) \)
6. \( W_u := \gamma \cdot W_u \)
7. **goto 2**

According to the small gain theorem, the stability of the system shown in Fig. 3 for all possible uncertainties described by the output-multiplicative uncertainty setup is guaranteed, if \( ||T_{\delta y}||_\infty \cdot ||\Delta_c||_\infty < 1 \), where \( T_{\delta y} \) is the transfer function between \( y \) and \( \delta y \). Since \( ||\Delta_c||_\infty \leq 1 \) is guaranteed by scaling of \( W_u \), a sufficient condition for robust stability is

\[
\left\| \frac{W_u G C_1}{1 + G C_1} \right\|_\infty < 1.
\]

This inequality has to be evaluated in the frequency domain. It is used as a constraint in the optimization to obtain optimal robust controllers.
Fig. 3. Output-multiplicative uncertainty description.

3.2 Signal to peak norm constraint

Actuator saturation can cause a control systems to fail due to wind-up problems. Therefore, it is necessary to prevent the manipulated variables from exceeding their physical limits to avoid actuator saturation problems. One possibility to prevent the manipulated variables from exceeding a predefined bound for a realistic change in the setpoint is to consider a signal-to-peak norm constraint in the optimization of the controller parameters. A signal-to-peak norm constraint is defined using $T_u r$, the transfer function from the reference $r$ to manipulated variable $u$, as

$$||T_u r||_1 < \gamma_l,$$

where $\gamma_l := [-u_{\text{min}}, u_{\text{max}}]^T$ is the bound for the controller output $u$ and $||T_u r||_1$ is defined by

$$||T_u r(s)||_1 := \max_k (w T_u r(k))$$

where the row vector $T_u r(k)$ is the step response of $T_u r(s)$ and $w$ is a constant vector $w := [-1, 1]^T$. According to the setup shown in Fig. 3, (10) can be written as

$$\left\| \frac{C_0}{1 + GC_0} \right\|_1 < \gamma_l.$$  

This inequality has to be evaluated in the time domain using the step response of $\frac{C_0}{1 + GC_0}$ to the reference $r$ and is used as a constraint in the optimization.

4. THE EXTENDED SCHEME

4.1 Constraint optimization

In the new approach, the data available from the measurements are divided into two parts when the optimization is activated. From the first part, a nominal model of the plant $G$ is identified. The nominal model along with the second part of the measurements are used to obtain the linear filter $W_u$. The plant model and linear filter are used to solve the following optimization problem to find the optimal controller parameters.

$$\min_{\theta} \left\| \frac{1}{1 + GC_0} \right\|_{\infty},$$

subject to

$$\left\| \frac{C_0}{1 + GC_0} \right\|_1 < \gamma_l,$$

$$\left\| \frac{W_u GC_0}{1 + GC_0} \right\|_{\infty} < \gamma_\infty,$$

and $\theta \in \mathbb{P}_1 \times \mathbb{P}_2 \times \ldots$.

4.2 $l_2 / H_\infty$ controller falsification

In the original Unfalsified Control approach, the only criterion used to falsify the candidate controllers is the condition (4). Here, the candidate controller falsification conditions are extended by an $H_\infty$ robustness criterion. Thus, not only the destabilizing candidate controllers are falsified and removed from the set of candidate controllers but also the non-robust candidate controllers. The extended falsification conditions are

$$J(C_i, k) > \alpha,$$

or

$$\left\| \frac{W_u GC_0}{1 + GC_0} \right\|_{\infty} > 1.$$  

The candidate controller $C_i$ is falsified if one of these conditions hold. Both conditions (16) and (17) are evaluated at each time step for each candidate controller. The cost function $J(C_i, k)$ in condition (16) is evaluated using an FIR model of $\hat{S}_i$ obtained by deconvolution. This cost value reflects the performance of the candidate controller at the current operating conditions accurately, since $\hat{S}_i$ is updated at each time step. In contrast, the condition (17) is calculated using the fixed linear plant model $G$ and uncertainty filter $W_u$, that were obtained during the last call of the optimization mechanism. This condition is calculated in the frequency domain. The only term in condition (17) which changes by time is the filter $W_u$ which is rescaled at each time step according to the measurements $(u, y)$ via algorithm 2. The condition (17) is used to determine if a candidate controller is robust against the plant model mismatch or not. This additional criterion increases the stability of the presented adaptive controller scheme.

5. A CASE STUDY

In this case study, a nonlinear model of a continuous stirred-tank reactor (CSTR) is considered, see Engell and Klatt (1993). The scheme of a CSTR is presented in Fig. 4. The reaction mechanism in this figure is assumed to be of the van de Vusse type.

$$\begin{align*}
A & \xrightarrow{k_1(T_R)} B \xrightarrow{k_2(T_R)} C, \\
2A & \xrightarrow{k_3(T_R)} D,
\end{align*}$$

A $(\mu, \kappa, \lambda)$-evolution strategy is used to solve this optimization problem. Details on evolution strategies can be found in Schwefel (1995). After activation of the evolution strategy, the controller parameters in the current set of candidate controllers constitute the initial population of individuals. The new offspring are generated by recombination and mutation with adapted mutation strengths. The objective (12), which has to be evaluated in time domain using the step response of $\frac{1}{1 + GC_0}$, is used to evaluate the fitness of the individuals. The constraints (13) and (14) are implemented by adding a large penalty if the constraints are violated. The best individuals are selected according to the $(\mu, \kappa, \lambda)$ selection as the next population. The loop of evolution continues until no further improvement is observed. The obtained optimal controller is set as the active controller and is placed in the set of candidate controllers. The diversity of the set is maintained by only replacing a falsified candidate controller or, if neither of candidates are falsified, by replacing an inactive candidate controller with the optimal controller.
where $A$ is the raw material, $B$ is the desired product, $C$ and $D$ are unwanted by-products and $k_1(T_R)$, $k_2(T_R)$, $k_3(T_R)$ are the rate coefficients which depend on the reaction temperature $T_R$. This reaction scheme gives rise to nonminimum phase behavior of the reactor. The balance equations for the concentrations of materials and energy along with the parameters of the model are taken from Engell and Klatt (1993), Klatt and Engell (1998). The balance equations are

\[
\begin{align*}
dc_A &= \frac{V_{in}}{V_R} (c_{A,in} - c_A) - k_1 \cdot c_A - k_2 \cdot c_A^2, \\
dc_B &= \frac{V_{in}}{V_R} \cdot c_B - k_1 \cdot c_A - k_2 \cdot c_B, \\
dT_R &= \frac{(k_1 \cdot c_A \cdot \Delta H_{RAD} + k_2 \cdot c_B \cdot \Delta H_{RBC} + k_3 \cdot c_A^2 \cdot \Delta H_{RAD})}{\rho_R \cdot c_p} + \\
&\quad \frac{V_{in}}{V_R} (T_0 - T_R) + \frac{k_{B1} \cdot A_R}{\rho_R \cdot c_p \cdot V_R} (T_K - T_R),
\end{align*}
\]

where the rate coefficients $k_1$, $k_2$, $k_3$ are given by

\[
k_1(T_R) = (1.29 \pm 0.4) \cdot 10^{12} \cdot e^{-\frac{3778}{T_R-273}} \; h^{-1},
\]

\[
k_2(T_R) = k_1(T_R),
\]

\[
k_3(T_R) = (9.0 \pm 0.3) \cdot 10^{9} \cdot e^{-\frac{8560}{T_R+273}} \frac{l}{m^{4} \cdot h}.
\]

As shown in Fig. 4, the reactor has a cooling jacket which removes the heat produced by the reaction. The temperature of the coolant $T_K$ is regulated by a separate controller in the plant and it is kept constant. The reaction changes according to the above differential equation for $T_R$.

![Fig. 4. Schematic of CSTR.](image)

The presented CSTR works in overflow, thus the volume of the liquid inside reactor $V_R$ is constant. The parameters of the model are given in table 1.

| Table 1. Parameters used in the simulation |
| --- | --- | --- | --- | --- | --- |
| param. | value | unit | param. | value | unit |
| $\Delta H_{RAD}$ | 4.2 | $\frac{kJ}{mol}$ | $V_R$ | 10 | l |
| $\Delta H_{RBC}$ | $-11$ | | $K_p$ | 1, 100 | - |
| $\Delta H_{RAD}$ | $-41.85$ | $\frac{kJ}{mol}$ | $T_n$ | 0.01, 1 | - |
| $T_{R,D}$ | 130 | $^oC$ | $\gamma_t$ | 0.9 | - |
| $T_K$ | 128.95 | | $\gamma_t$ | [0.35] | - |
| $C_R$ | 3.01 | $\frac{l}{mol}$ | $\rho$ | 0.01 | - |
| $\rho_R$ | 0.9342 | $\frac{mol}{l}$ | | | |
| $k_W$ | 4032 | $\frac{mol}{l \cdot h}$ | $\alpha$ | 1000 | - |
| $A_R$ | 0.215 | $m^2$ | | | |

In this application, the concentration of product $B$ in the product stream is measured and controlled, $y := c_B$. The manipulated variable is the inflow to the reactor $u := V_{in}/V_R$. It is important to keep this input in the range $0 \; h^{-1} \leq u \leq 30 \; h^{-1}$, because of the physical limitations. During the reaction, the raw material enters the reactor with a concentration of $c_{A,in}$. The nominal value of $c_{A,in}$ is $5.1 \; mol_A \cdot h^{-1}$.

The selected controller to control the CSTR is of PI type and is defined as $C(s) := k_p \left(1 + \frac{1}{T_n s}\right)$. The parameters $\theta := [k_p, T_n]^T$ have to be adapted during the reaction according to the switching mechanism presented in section 2.3. The plant is under operation before the adaptive controller begins to adapt the parameters of the active controller. The parameters of the initially active controller are $[18, 0.2]^T$. This controller is a stabilizing controller but it is not well tuned. Additionally three other candidate controllers are available. These controllers are presented in the first row of table 2. Intentionally neither of the initial candidate controllers are well tuned as the goal of this case study is to show that our algorithm is able to tune the active controller even in the case that initially no good candidate controller is available.

In order to collect enough measurements before the optimization is performed for the first time, the error scanning mechanism is turned off for a period of 6 hours. The sampling time is 10 sec and the measurements have a delay of 20 sec and are corrupted by white noise. A GBN signal with mean values of [0.82, 0.9, 0.97] and standard deviation of 0.05 is used as the reference signal. Other selected parameters are given in table 1. Immediately after turning the error scanning mechanism on at 6.0 hours, the optimization is activated by the error scanning mechanism since the active controller is not performing satisfactorily.

The selected measurements are used to obtain a model of the plant $G$ and a filter $W_u$. The selected model of plant $G$ is a fifth order output error (OE) model with second order Padé delay approximation. The parameters used in the evolution strategy are $\mu = 3$, $\lambda = 21$ and $\kappa = 100$. The initial strategy parameters are set to 10% of the ranges of the variables. The frequency response of the obtained filter $W_u$ is shown in Fig. 7. The result of first optimization is $[18.37, 0.023]^T$. The set of candidate controllers after the optimization is shown in table 2. Table 2 also shows the computation time (Intel® E2180, 2GB RAM). This time includes all steps from the model estimation to obtaining the filter $W_u$ and solving the optimization problem presented in section 4.1.

<p>| Table 2. Optimization start time, computation time and controller set after optimization |
| --- | --- | --- | --- |</p>
<table>
<thead>
<tr>
<th>Nr.</th>
<th>Start</th>
<th>Comp.</th>
<th>Controller set after optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.01 hrs</td>
<td>160 sec</td>
<td>[10, 0.1], [5, 0.2], [18, 0.2]</td>
</tr>
<tr>
<td>2</td>
<td>9.98 hrs</td>
<td>189 sec</td>
<td>[18.37, 0.023], [10, 0.1], [5, 0.2]</td>
</tr>
<tr>
<td>3</td>
<td>10.39 hrs</td>
<td>186 sec</td>
<td>[50.67, 0.022], [46.28, 0.032], [10, 0.1]</td>
</tr>
<tr>
<td>4</td>
<td>12.6 hrs</td>
<td>176 sec</td>
<td>[29.75, 0.026], [46.28, 0.032], [50.67, 0.02]</td>
</tr>
</tbody>
</table>

Fig. 5 shows the system output, where the system performance is improved after the first optimization at 6.0 hours. It can be seen from Fig. 5, that $u$ is always in the range $[0, 0.35]$ therefore the requirement on the physical limitations of the manipulated variable is satisfied.

The error scanning mechanism continues to monitor the control performance. At times 9.98, 10.39 and 12.6 hours, it activates the optimization again since the control performance is not satisfactory. After each optimization, the optimal controller $\theta_{opt}$ is set as the active controller and is placed in the set of candidate controllers. The results are shown in table 2, where the controllers in the first column
are results of optimizations. During the whole plant operation the $\epsilon$-hysteresis algorithm switches between the candidate controllers. This is shown in Fig. 6.

![Fig. 5](image)

**Fig. 5.** System reference $r$ (top, dashed), system output $y$ (top, line). Manipulated variable $u$ (bottom).

![Fig. 6](image)

**Fig. 6.** Adaptation of the active controller gain $k_p$ (top) and time constant $T_n$ (bottom).

![Fig. 7](image)

**Fig. 7.** Freq. responses of $W_u$ (line) and $T_{ny} \cdot W_u$ (dashed) for each optimization.

Fig. 8 shows the evaluated cost and the left hand side of the robustness criterion (17) for the candidate controllers. The evaluated costs are used in the switching mechanism and the candidate controller with the lowest evaluated cost is selected as the active controller. After the optimization, the obtained plant model $G$ and the filter $W_u$ are used to evaluate the condition (17) for each candidate controller at each time step. As shown in Fig. 8, the candidate controller $[18, 37, 0.023]^T$ fails to meet the robustness criterion and thus is falsified at time 8.1 hours.

![Fig. 8](image)

**Fig. 8.** Cost of the candidate controllers (top). Left hand side of (17) (bottom).

6. CONCLUSION

The presented adaptive control scheme extends our previous work on accounting for uncertainties caused by plant-model mismatch in the unfalsified control framework by considering an additional signal to peak norm constraint in the optimization of the controller parameters. The realistic case study presented here includes a nonlinear model of a chemical reaction with noisy and delayed measurements. The simulation results show that our adaptive controller fulfills the performance, robustness and physical limitations requirements. Due to the feasible computation times of the optimizations, the approach can be applied to a real-time operation. Our current research focus is to extend the presented scheme to MIMO systems.

REFERENCES


