A bi-level approach for the decentralized optimal control of dangerous goods fleets flowing through a tunnel

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Abstract: An approach to formalize and solve a decision problem related to the management of fleets that transport dangerous goods through a critical infrastructure is here presented. The model introduced takes into account the presence of different decision makers, each of which has access to a specific (partial) information set about the state of the entire system. The problem is also characterized by the presence of several (possibly conflicting) objectives, such as the minimization of the economic costs, and the reduction of hazardous materials transportation risk. The considered model includes an upper-level decision maker that can influence decisions related to the lower level decision makers, taking into account the willingness of the infrastructure users (i.e., the vehicles). Different interacting decision problems (that can be solved through mathematical programming or analytical solutions) are formalized and solved. The decision architecture is tested for a case study characterized by a tunnel and two fleets flowing through a highway traffic system.

Keywords: Optimal Control, Risk, Transportation Control, Traffic Control, Optimization

1. INTRODUCTION

The transportation of hazardous materials (hazmat) on road has an important impact on the overall traffic management (Minciardi et al., 2008). This fact is even more evident when a vehicle must traverse a critical road infrastructure, such as a tunnel or a bridge.

In relation with hazardous material transportation, different decision makers and actors can generally be considered (Public Authority, Fleets managers, Tunnel managers). Moreover, different objectives (risk minimization, economic benefits maximization, etc.) have to be taken into account. In this framework, different decision structures (i.e., hierarchical, decentralized, etc.) may be considered and studied.

In a hierarchical decision model, the upper decision levels, through their decisions, influence the choices of the lower levels. Instead, in a decentralized decision problem, different sub-problems related to specific sub-systems are solved (by different decision makers) and then they interact either in cooperative or in competitive way.

A bi-level mathematical program concerns two optimization problems hierarchically related. One can interpret such problems as referring to two decision makers in which the optimal decisions of the higher level (the leader) constrains the decision of the lower level decision makers (the followers). Each lower level decision maker optimizes his/her objective function within a feasible region that is defined by the higher level decision maker. Within this setting, the latter is assumed to have the capability of forecasting all the possible reactions of the lower level decision makers, and determines their own decisions in order to obtain the best outcome for his/her objective function.

In the literature, the application of such hierarchical and decentralized scheme is presented in several contributions. Some of them are related to hazardous materials transportation.

Bianco et al. (2009) present a bi-level flow model for the network design. The risk minimization is addressed from two points of view (corresponding to two different authorities): regional and local risk minimization. The bi-level model is transformed into a single-level mixed integer linear program by replacing the second level by its KKT conditions and by linearizing the complementary constraints. Brokcorne et al. (2001) propose a bi-level game assuming that the higher level (corresponding to the highway owner) objective is to maximize toll benefits, and the lower level objective is to minimize travel costs. The lower level optimization problem is replaced by its optimality conditions, and a heuristic is developed to solve the overall problem. Chiou (2005) presents a bi-level programming problem for the design of a transportation network. At the upper level, the system performance index is the sum of travel times and investment costs for capacity expansion. At the lower level, the user equilibrium flow is determined by Wardrop’s principle and can be formulated as an equivalent minimization problem. Hou (2001) deals with dynamic large scale hierarchical systems and propose a neural network for its optimization. The upper level in the hierarchical structure provides some input to the lower level, that, on the basis of the optimization of its objective function, provides to the upper level the state and the control variables. A stability analysis is provided that allows evaluating whether the neural network is asymptotically stable and the stable state corresponds to the optimal solution of the optimization problem.
Semsar-Kazerooni and Khorasan \cite{SemsarKazerooni2008} focus attention on team cooperation, namely consensus, for both leaderless and modified leader-follower architectures. Consensus problems deal with the agreement of a group of agents upon specific quantities of interest (for example velocity and positions). In this configuration, the agents try to decide and agree among themselves upon what the final state should be. One of the main challenges that arise in the development of an optimal cooperation in a team of unmanned systems is the lack of complete information and the presence of uncertainties, faults and unpredictable events in the team.

In the case of non-cooperative players, Carraro and Sgorbbi \cite{CarraroSgorbbi2008} present a game theory approach in which different players bargain to attain optimal strategies and equilibrium agreements, taking into account the effects of possible uncertainties.

The aim of this work is to deal with dynamic systems, real time optimization and transportation operational management, and to provide an innovative architecture for the multiple-objective and multi-decision maker problem of fleets flowing through a tunnel, and, more generally, for transportation problems in which there is an upper-level that can take decisions influencing the lower decision makers, but taking into account their willingness. The bi-level optimization problem includes the tunnel manager objective of minimizing the overall transport risk and the fleets economic objectives.

In this scheme, each fleet solves its own economic problem and provides reference values for the control variables of the tunnel manager (that has to minimize the overall risk). The tunnel manager solves his/her own risk assessment problem and finds his/her own optimal results. If the optimal solutions of the fleets provide a cost value (for the tunnel manager) that is acceptable from the risk analysis point of view, thus a satisfactory solution is found. Otherwise, the tunnel manager has to modify the parameters that influence the behavior of the fleets. In this work, he finds and applies the tolls which motivate the fleets to change their plans.

2. THE SYSTEM MODEL

The system model is a simplified version of a previous work (Minciardi et al., 2008). Fig. 1 shows the considered system model. A given number of vehicles transporting hazardous material and belonging to different fleets has to use a highway and to reach one critical infrastructure (e.g. a tunnel). They can stop in a park before the highway entrance and start their travel according to the exigencies of a decision maker that can be identified as the tunnel and highway manager. The park may be viewed as an inventory whose level (i.e., the number of vehicles) represents the system state, at a specific time instant. In order to represent the flow dynamics of the hazardous material vehicles, highway directed towards the critical infrastructure is modelled as divided in tracts. Two highway tracts and two fleets are considered in this paper. The first tract goes from the park area to the infrastructure, whereas the second tract is the infrastructure itself. The external inputs of the whole system are the quantities $V_1^\ell$ and $V_2^\ell$, i.e., the (known) number of vehicles entering the park near the highway entrance in time interval $(t, t+1)$, $t = 0, \ldots, T-1$ for fleet 1 and 2, respectively.

The control variables correspond to the number of vehicles that enter the highway for each fleet ($U_1^t$ and $U_2^t$) in a specific time interval $(t, t+1)$, while the state variables correspond to the number of vehicles in the inventory/queue for each fleet ($I_1^t$ and $I_2^t$), and the number of vehicles (per unit length) of each tract of the highway ($N_1^{1,1}$, $N_2^{1,1}$), where tract 1 corresponds to a highway tract, and tract 2 corresponds to the tunnel. The number of vehicles in each tract are composed by the vehicles numbers for each fleet, i.e., $N_1^{1,1} = N_1^{1,1} + N_1^{1,2}$, $N_2^{1,1} = N_2^{1,1} + N_2^{1,2}$). Moreover, it could be convenient to consider also the overall number of vehicles entering the tunnel ($Y^t = Y_1^t + Y_2^t$), and the overall number of vehicles going out from the tunnel ($Z^t = Z_1^t + Z_2^t$), in time interval $(t, t+1)$. $Y_1^t$, $Y_2^t$, $Z_1^t$ and $Z_2^t$ are functions of the state variables, as it will be clarified later on.

The inventory model

The two state equations of the inventories are

\begin{align}
I_1^{t+1} &= (I_1^t + V_1^t - U_1^t) \quad t = 0, \ldots, T-1 \tag{1} \\
I_2^{t+1} &= (I_2^t + V_2^t - U_2^t) \quad t = 0, \ldots, T-1 \tag{2}
\end{align}

where $I_1^t$ ($I_2^t$) is the number of vehicles stored for fleet 1 (fleet 2), at time instant $t$, in the park, and variables $V_1^t$, $V_2^t$, $U_1^t$, and $U_2^t$ have already been defined.

The overall inventory state equation can be written as

\begin{equation}
I^{t+1} = (I^t + V^t - U^t) \quad t = 0, \ldots, T-1 \tag{3}
\end{equation}

where $I^t = I_1^t + I_2^t$, $V^t = V_1^t + V_2^t$, and $U^t = U_1^t + U_2^t$.

The highway tract state equations

These state equations describe the evolution over time of state variables that represent the number of vehicles (carrying hazardous materials) per each fleet, and per unit length, present in a specific tract of the highway. In agreement with the literature dealing with traffic models, it is assumed that

\begin{align*}
N_1^{1,1} &= N_1^{1,1} + N_1^{1,2} \\
N_2^{1,1} &= N_2^{1,1} + N_2^{1,2}
\end{align*}
the (average) vehicle speed is never so high to allow the complete covering of a highway tract within a single time interval. The corresponding state equations are given by

\[ N_{i+1} = N_i + U_i - Y_i \quad t = 0, \ldots, T-1 \quad i = 1,2 \]  \hspace{1cm} (4)

\[ N_{i+1} = N_i - Z_i + Y_i \quad t = 0, \ldots, T-1 \quad i = 1,2 \]  \hspace{1cm} (5)

where it is assumed that

\[ Y_i = N_i \text{vel}_i \Delta t \quad t = 0, \ldots, T-1 \quad i = 1,2 \]  \hspace{1cm} (6)

\[ Z_i = N_i \text{vel}_i \Delta t \quad t = 0, \ldots, T-1 \quad i = 1,2 \]  \hspace{1cm} (7)

and where:

- \( L_1, L_2 \) are the highway tract and tunnel lengths respectively;
- \( \Delta t \) is the time discretization interval;
- \( \text{vel}_i \) are the (average) velocities in the tract 1 and the tunnel in time interval \((t, t+1)\), which is assumed to be imposed by the ordinary traffic (i.e., non hazmat), assuming that the hazmat vehicle flow is only a negligible part of the overall traffic flow; such velocities are assumed to be known;

Thus, (4) and (5) may be written as

\[ N_{i+1} = N_i + \frac{U_i}{L_1} - \frac{N_i \text{vel}_i \Delta t}{L_1} \quad t = 0, \ldots, T-1 \quad i = 1,2 \]  \hspace{1cm} (8)

\[ N_{i+1} = N_i - \frac{Z_i}{L_2} + \frac{N_i \text{vel}_i \Delta t}{L_1} \quad t = 0, \ldots, T-1 \quad i = 1,2 \]  \hspace{1cm} (9)

The following conditions are assumed to hold in order to ensure the fulfillment of the above mentioned physical condition about the vehicle speed: \( \text{vel}_1 \Delta t < L_1, \text{vel}_2 \Delta t < L_2 \).

3. THE OVERALL DECISION ARCHITECTURE

In a centralized setting, one can state the following tunnel manager decision problem, aiming at minimizing the overall risk in the system.

\textbf{Tunnel Manager Problem (TMP)}

\[
\min \sum_{i=0}^{T} \eta_{HAZ1}^i N_1^i + \eta_{HAZ2}^i N_2^i + \eta_{HAZ}^i I^i
\]

subject to constraints (1)-(10) plus non negativity constraints for \( U_1^i, U_2^i, I_1^i, I_2^i \). Note that, non negativity constraints for the other state variables \( (N_{i,j}, i=1,2) \) are not necessary, owing to state equations (8), (9). Of course, the initial values of the state variables should be specified. Moreover, the following constraints have been added to restrict the number of hazmat trucks in the tunnel to a maximum \( \bar{T} \):

\[
\sum_{i=1}^{2} N_{i,j}^i L_2 \leq \bar{T} \quad t = 0, \ldots, T-1
\]

A more reasonable formalization of the tunnel manager decision problem (TMP2) is one that takes into account the lower-level decision makers, each one associated to a fleet, and provided with her/his own utility function. To this end, two lower level decision problems, each one corresponding to a fleet manager (namely PCF1 and PCF2) are considered. In the next section, the structure of such problems will be introduced and discussed. At this moment, we assume that PCF1 and PCF2 can be influenced by choosing the values of parameters acting as incentives (or prices) over the fleet managers.

Thus, one can evaluate the overall risk deriving from the optimal solutions of PCF1 and PCF2, namely, \( HAZ_{PCF} \), calculated through a definition analogous to (10). Then, one can compare the value \( HAZ_{TMP} \) and \( HAZ_{PCF} \) and, decide, in case the inequality \( |HAZ_{TMP} - HAZ_{PCF}| \leq \varepsilon \) is fulfilled (being \( \varepsilon \) a suitably defined threshold), to retain the solutions obtained by PCF1 and PCF2 as satisfactory.

Otherwise, the tunnel manager has to modify the parameters which influence the fleet decision managers cost functions. Fig. 2 describes the iterative process by which the finally retained solution \( \{ U_1^i, U_2^i, t = 0, \ldots, T-1 \} \) may be found. In the figure, the parameters \( C_{U_i}^i, i=1,2, t = 0, \ldots, T-1 \), appear, whose meaning will be clarified in the next section.
5. TUNNEL MANAGER PROBLEM 2 (TMP2)

The Tunnel Manager Problem 2 (TMP2) has the following structure:

**Tunnel Manager Problem 2 (TMP2)**

The variation of values of the tolls $C_{u,t}^i$ may be carried out by the higher level decision maker according to some heuristic rule. Alternatively, one can think of defining a new decision problem for the higher level decision maker, as follows.

Let $\widetilde{U}_i = \text{col} \left[ \widetilde{U}_{i,t}^1, t = 0, \ldots, T-1 \right]$ and $\widetilde{U}_2 = \text{col} \left[ \widetilde{U}_{2,t}^2, t = 0, \ldots, T-1 \right]$ be the optimal solution of decision problem TMP, and $J_i(\widetilde{U}_i)$ the objective function of each fleet decision problem, calculated in $\widetilde{U}_i$, $i=1,2$. That is,

$$J_i(\widetilde{U}_i) = \sum_{t_i=1}^{T_i} C_{i,t}^T T_{i,t}^T + \sum_{t_i=0}^{T_i-1} C_{i,t}^I T_{i,t}^I + C_{i,t}^F T_{i,t}^F$$

where $\widetilde{U}_i^1$, $i=1,2$, $t=0,\ldots,T$ are the inventory values corresponding to the solution $\widetilde{U}_i$, $i=1,2$, $t=0,\ldots,T-1$.

Then, one can introduce the following decision problem: find the optimal "modified" tolls $\overline{C}_{u,t}^i = C_{u,t}^i + dc_{t}^i$ $t=0,\ldots,T-1$ $i=1,2$

being $dc_{t}^i$, $i=1,2$, $t=0,\ldots,T-1$, the variations of the tolls, constrained within the intervals

$$c_{\text{min},i} \leq dc_{t}^i \leq c_{\text{max},i} \quad t=0,\ldots,T-1 \quad i=1,2$$

More specifically, the statement of this problem is the following

$$\min_{t=0,\ldots,T-1} \left( J_i(\overline{U}_i) - J_i(\overline{C}_{u,t}^i) \right)^2$$

where $\overline{C}_{u,t} = \text{col} \left[ \overline{C}_{u,t}, t = 0, \ldots, T-1 \right]$

$$J_i(\overline{C}_{u,t}) = \min_{t=0,\ldots,T-1} \left( \sum_{t_i=1}^{T_i} C_{i,t}^T T_{i,t}^T + \sum_{t_i=0}^{T_i-1} C_{i,t}^I T_{i,t}^I + C_{i,t}^F T_{i,t}^F \right)$$

s.t.

$$I_{t+1} = (I_t + T_{t+1} - \overline{U}_t) \quad t=0,\ldots,T-1 \quad i=1,2$$

$$N_{1,t}^{i+1} = N_{1,t}^i + \overline{U}_t - N_{1,t}^i \text{vel}_1 \Delta t \quad t=0,\ldots,T-1 \quad i=1,2$$

$$N_{2,t}^{i+1} = N_{2,t}^i - N_{2,t}^i \text{vel}_2 \Delta t + N_{1,t}^i \text{vel}_1 \Delta t \quad t=0,\ldots,T-1 \quad i=1,2$$

4. THE TWO FLEETS DECISION PROBLEM (PCF1 and PCF2)

Each fleet manager has to minimize an economic cost given by the costs of highway, inventory and tunnel. It is supposed that the unit [€/vehicle] costs are time-varying and that they are known parameters imposed by the higher level decision maker. The considered decision problem is assumed to be the same for each fleet $i$, $i=1,2$. That is,

$$\min_{t=0,\ldots,T-1} \sum_{t_i=1}^{T_i} C_{i,t}^T T_{i,t}^T + \sum_{t_i=0}^{T_i-1} C_{i,t}^I T_{i,t}^I + C_{i,t}^F T_{i,t}^F$$

where $C_{u,t}^i$ represents the toll paid by a vehicle which enters the highway in time interval $(t, t+1)$. This toll is assumed to take globally into account the highway trac and the tunnel traversal, and is just the variable through which the higher level decision maker influences the fleets managers. Instead, $C_{i,t}^I$ is the inventory (unit) cost and it is assumed not to depend on the time interval and the fleet. Finally, $C_{i,t}^F$ is the (final) inventory cost.

The state equations that constrain the minimum value of (13) are

$$I_{t+1} = (I_t + T_{t+1} - \overline{U}_t) \quad t=0,\ldots,T-1 \quad i=1,2$$

Also the non negativity constraints $I_{t}^i \geq 0$, $U_{t}^i \geq 0$, $t=0,\ldots,T-1$, $i=1,2$, are to be taken into account in the minimization (13).
together with the non negativity constraints \( t_i' \geq 0, \bar{U}_i' \geq 0 \), 
\( t=0,\ldots,T-1, i=1,2 \). 

Note that the higher level decision maker has to solve two (independent) problems having the structure (18), and that each of such problems has an objective function that is defined through the minimization of another objective function. As this second minimization can be hardly thought to provide an optimal value which depends analytically on the decision variables, the use of some heuristics to change the values of the toll parameters may be considered as a viable approach to implement the considered decision scheme. This is exemplified in the next section, together with other solution approaches: dynamic programming and mathematical programming techniques.

6. SOLUTION ALGORITHMS AND OPTIMAL RESULTS

In the following, different solution approaches are discussed. Specifically, the possible approaches are: a simple heuristics rule for tolls assignment, dynamic programming for the fleet sub-problem, mathematical programming techniques.

A simple heuristic rule can be applied to change the toll parameters by the higher level decision maker. Specifically, once the optimal values of \( \bar{U}_i' \) from PCF and \( \bar{U}_i' \) from TMP have been determined and the risk has been assessed, the following rules is used to change the toll values.

Determine the value \( A_i, i=1,2 \), defined as
\[
A_i = \max \left| \tilde{U}_i' - \bar{U}_i' \right|
\]
Let \( \bar{t} \) the value of \( t \) for which \( A_i \) takes place. Suppose, for the sake of simplicity, that there is only one of such \( \bar{t} \).

Then, if \( \bar{U}_i' > \tilde{U}_i' \), set
\[
C_{u,i} = C_{u,i}' + a
\]
with \( a \) known parameter >0, whereas, if \( \bar{U}_i' < \tilde{U}_i' \), set
\[
C_{u,i} = \max(C_{u,i}' - a,0)
\]
Please note that the constraints over the tolls expressed by equation (17) should be taken into account in the iterative process. In case many equivalent time instants \( \bar{t} \) occur, the tolls corresponding to such time intervals are kept constants, if it turns out \( \bar{U}_i' > \tilde{U}_i' \) (or \( \bar{U}_i' < \tilde{U}_i' \)) for all such time instants. In case \( \bar{U}_i' < \tilde{U}_i' \) for some \( \bar{t} \), and the reverse inequality holds for the other, an arbitrary choice can be made.

Data. The following numerical example clarifies the application of the proposed procedure. The external inputs for each fleet are:

### Table 1. Optimal results for decision problem TMP.

<table>
<thead>
<tr>
<th>time</th>
<th>( \bar{U}_1' )</th>
<th>( \bar{U}_2' )</th>
<th>( \bar{t}_1' )</th>
<th>( \bar{t}_2' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.0</td>
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<tr>
<td>2</td>
<td>2.4</td>
<td>0.0</td>
<td>6.4</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>1.4</td>
<td>1.1</td>
<td>7.6</td>
<td>12.2</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>0.0</td>
<td>7.6</td>
<td>10.9</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>1.8</td>
<td>6.1</td>
<td>9.9</td>
</tr>
<tr>
<td>6</td>
<td>1.9</td>
<td>0.6</td>
<td>5.4</td>
<td>8.1</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
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<td>0.0</td>
<td>14.1</td>
</tr>
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<td>14.1</td>
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<td>0.0</td>
<td>5.0</td>
<td>14.1</td>
</tr>
</tbody>
</table>

The overall risk in this case is 0.522881. Using the initial toll values, the optimal solution of problems PCF1 and PCF2 are calculated. Results are reported in Table 2.

### Table 2. Optimal results for decision problems PCF1 and PCF2.

<table>
<thead>
<tr>
<th>time</th>
<th>( \bar{U}_1' )</th>
<th>( \bar{U}_2' )</th>
<th>( t_1' )</th>
<th>( t_2' )</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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</tbody>
</table>
The overall risk in this case is about 1.0824 (0.57144 for fleet 1 and 0.51094 for fleet 2). Then, the heuristic procedure is applied, with α=2. In this case, similar results to Table 2 can be found.

Then, the fleets problem can be solved through dynamic programming, to find an analytical solution to be inserted in equation (18). This is of course an advantage. However, the optimal control law is piecewise constant and the number of analytical functions increases exponentially with the number of stages. Thus, TMP2 becomes

\[
\min_{C_i} \sum_{t=0}^{T-1} \left( J_i(\tilde{U}_t) - \sum_{i=1}^{C} \lambda_{i,c} \tilde{J}_{i,c} \right)^2 \quad i=1,2
\]

s.t.

\[
c_{\min,j} \leq c_{u,j} \leq c_{\max,j} \quad t=0,\ldots,T-1 \quad i=1,2
\]

where \(C\) is the overall number of sub-cases, and \(\lambda_{i,c}\), \(i=1,2\), \(c=1,\ldots,C\), is a binary parameter that is set equal to 1 when a specific condition \(c\) occurs, and 0 otherwise. Thus, the following additional constraints should be taken into account

\[
\lambda_{i,c} = \begin{cases} 
1 & \text{if } f_i(C_{u,N-k,i}) \geq 0 \\
0 & \text{otherwise}
\end{cases} \quad c=1,\ldots,C \quad i=1,2
\]

where \(f_i(C_{u,N-k,i})\), \(k=1,\ldots,K\), \(c=1,\ldots,C\), \(i=1,2\), is the specific condition for case \(c\), and \(K\) is the overall number of stages. Though for each stage an analytical solution is obtained, the limits of this approach lie in the number of cases that increases exponentially. Moreover, in absence of noise, dynamic programming is not advantageous with respect to explicit enumeration. Thus, one can think of using dynamic programming to solve decision problem (19), and to finally solve with mathematical programming techniques problem TMP2 for a lower optimization horizon, but adopting the analytical solution for sub-problem (19) found through dynamic programming. Then, a receding horizon control scheme can adopted to derive the optimal solution for the simulation time horizon equal to 15.

Finally, TMP2 can be completely solved through mathematical programming techniques.

7. CONCLUSIONS

An architecture for the optimal control of fleets transporting hazardous materials that takes into account both risk and economic objectives has been presented. Different decision makers (i.e., the tunnel manager and the fleets managers) have been taken into account and interact to reach a solution.

Different possible solution methods (based on the application of a heuristic rule, dynamic programming and mathematical programming techniques) have been discussed and results reported. Specifically, dynamic programming allows obtaining analytical solutions for the fleets decision problems, and, thus, having the optimal cost expressed as a function of the state and of the parameters (among which there are the tolls that have to be optimized by tunnel manager decision problem). However, dynamic programming, due to the presence of constraints over the control variables, has, in this case, the limit of having a number of sub-cases (and thus analytical solutions) that increases exponentially with the number of stages. Instead, in the case the fleets decision problems can be expressed as a LQ decision problem without non negativity constraints, techniques based on the Ricatti equation can be used to find a recursive optimal control law. Future developments may regard the formalization of optimization problems for tolls definition that can be solved analytically, exploiting a LQ structure of the “inner sub-problem”. Then, further developments can be related to the improvement of the decision architecture and its solution, for example with specific reference to the fleet manager problems, taking into account other realistic requirements like delivery schedule/deadlines, etc. Finally, possible conditions related to equilibrium criteria or consensus among decision makers, and the influence of the known information on the optimal solution can be taken into account.

REFERENCES


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