Cooperative Tracking of a Moving Target using Integrated Vector Field and Decentralized Extended Information Filter

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Abstract: This paper presents the decentralized target localization and control design for multiple unmanned aircraft to track a moving target. The extended information filter is applied to estimate the position and velocity of the target with additional target information provided by neighbor aircraft. The heading vector field considering both the distance error to the target and the phase spacing error between aircraft is proposed to track the fast target which changes its direction rapidly. Numerical simulation is performed to verify the proposed estimation and tracking scheme using four unmanned aircraft.

1. INTRODUCTION

Many civil and military tasks which are difficult, dangerous, and dull have been recently performed by autonomous unmanned aircraft. Among these tasks, persistent surveillance using multiple unmanned aircraft has been extensively studied to enhance the accuracy and efficiency of detection, observation, and tracking for a target of interest. Recent studies have focused on the cooperative estimation and control of multiple aircraft to observe a moving target while keeping a specific distance from the target.

Fixed-wing aircraft tracking the moving target require specific flight pattern because the aircraft cannot hover near the target but fly forward. Wise and Rysdyk compared several cooperative tracking schemes for various target motions (2006). In addition, Wheeler et al. applied the Good Helmsman logic to cooperative tracking and geo-location using two unmanned aircraft (2006). Also, Campbell and Whitacre proposed decentralized information filter for cooperative tracking (2007). Moreover, Frew et al. (2008) and Summers et al. (2009) designed the Lyapunov guidance vector fields for heading angle and speed separately to achieve the standoff tracking of a moving target.

However, the previous research is restricted to the tracking of a relatively slow target with smoothly changing directions. As the target speed and heading rate increase, the phase spacing between the neighbour aircraft becomes oscillating with the frequent change in the aircraft speed. Moreover, the minimum distance between aircraft decreases due to the phase oscillation. This undesired aircraft motion is resulted from the decoupled heading and speed commands. Instead, in this study, the magnitude of phase oscillation and aircraft speed is reduced by considering the phase spacing error in the design of heading command.

In this paper, cooperative target localization and standoff tracking are studied to accurately estimate the target state and safely track the fast target. The aircraft receives not only the aircraft state but also the target information provided by each front aircraft instead of by all aircraft. Therefore, the extended information filter (EIF) is applied in the decentralized scheme. The vector field guidance is designed to determine the aircraft speed and heading command. Both the distance to the target and the distance to the neighbour aircraft are considered in the design of the heading command.

This paper is organized as follows. Section 2 explains the decentralized filter design based on the measurement of the range and line-of-sight to the moving target. Section 3 shows the vector field design to track the fast target safely. In Section 4, numerical simulation and analysis are described. Concluding remarks are given in Section 5.

2. COOPERATIVE TARGET LOCALIZATION

2.1 Measurement

The planar motion of a fixed-wing aircraft is described as follows. The altitude of the aircraft and target is assumed to be constant.

\[
\begin{align*}
\dot{x}_i &= V_i \sin \psi_i \\
\dot{y}_i &= V_i \cos \psi_i \\
\dot{\psi}_i &= \left( g / V_i \right) \tan \phi_i \\
\dot{V}_i &= -\left( 1 / \tau_v \right) V_i + \left( 1 / \tau_v \right) V_{i+1} \\
\dot{\phi}_i &= -\left( 1 / \tau_\phi \right) \phi_i + \left( 1 / \tau_\phi \right) \dot{\phi}_{i+1}
\end{align*}
\]

where \((x_i, y_i)\) is the aircraft position in the North-East-Down (NED) frame, \(V_i\) is the speed, \(\psi_i\) is the heading angle, \(\phi_i\) is the roll angle, \(g\) is a gravitational acceleration, \(V_{i+1}\) is the speed command, \(\phi_{i+1}\) is the roll angle command, \(\tau_v\) is the time constant of a speed controller, \(\tau_\phi\) is the time constant of a roll angle controller, and \(i\) is the index of the aircraft. The positive heading angle is defined to be clockwise from the north direction. The target motion on the ground is

\[
\begin{align*}
\dot{x} &= V \sin \psi \\
\dot{y} &= V \cos \psi
\end{align*}
\]
Fig. 1 Measurement of range and line-of-sight angle

where \((x_i, y_i)\) is the target position in the NED frame, \(V_i\) is the target speed, and \(\psi_i\) is the heading angle of the target. The relative position of the target with respect to the aircraft is shown in Fig. 1. The aircraft measures the range and the line-of-sight angle from the current position as follows.

\[
z_i = \begin{bmatrix} r_{\text{mi}} \\ \lambda_{\text{mi}} \end{bmatrix} = h(x_i, y_i, x, y) + v_i
\]

\[
= \begin{bmatrix} r_i \\ \lambda \end{bmatrix} + \begin{bmatrix} \sqrt{(x - x_i)^2 + (y - y_i)^2} \\ \tan^{-1}\left(\frac{y - y_i}{x - x_i}\right) \end{bmatrix} + \begin{bmatrix} \sigma_{r_j} \\ \sigma_{\lambda_j} \end{bmatrix}
\]

where \(r_{\text{mi}}\) is the measured range, \(\lambda_{\text{mi}}\) is the measured line-of-sight angle, \(r_i\) is the actual range, \(\lambda\) is the actual line-of-sight angle, and \(v_i\) is the measurement noise with zero-mean and the covariance \(R \in \mathbb{R}^{2 \times 2}\) in the Gaussian distribution of \(v_{r_j} \sim N(0, \sigma_{r_j})\) and \(v_{\lambda_j} \sim N(0, \sigma_{\lambda_j})\). The standard deviation of \(v_{r_j}\) and \(v_{\lambda_j}\) given by

\[
\sigma_{r_j} = k_r r_i
\]

\[
\sigma_{\lambda_j} = k_\lambda \lambda_i
\]

The measurement noise of the range is proportional to the actual range whereas that of the line-of-sight angle is constant. To show the uncertainty in the measurement, the target position errors in the down-range direction and in the cross-range direction are considered as shown in Fig. 1. Using the measurement of (8), the target position in the NED frame is

\[
\begin{bmatrix} \hat{x}_{r_i} \\ \hat{y}_{r_i} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} r_{\text{mi}} cos \lambda_{\text{mi}} \\ r_{\text{mi}} sin \lambda_{\text{mi}} \end{bmatrix}
\]

where \((\hat{x}_{r_i}, \hat{y}_{r_i})\) is the target position measured by the \(i\)th aircraft. Consider the position error in the NED frame as

\[
\begin{bmatrix} v_{r_j} \\ v_{\lambda_j} \end{bmatrix} = \begin{bmatrix} \hat{x}_{r_i} - x_i \\ \hat{y}_{r_i} - y_i \end{bmatrix}
\]

The measurement error \(v_{r_j}\) and \(v_{\lambda_j}\) with the standard deviation given in (9) and (10) yields

\[
\begin{bmatrix} v_{r_j} \\ v_{\lambda_j} \end{bmatrix} = \begin{bmatrix} \sigma_{r_j} \\ \sigma_{\lambda_j} \end{bmatrix}
\]

\[
\begin{bmatrix} v_{r_j} \\ v_{\lambda_j} \end{bmatrix} = \begin{bmatrix} \cos \lambda \\ \sin \lambda \end{bmatrix} \begin{bmatrix} v_{r_j} \\ v_{\lambda_j} \end{bmatrix}
\]

where \(v_{r_j}\) is the down-range position error with the standard deviation of \(\sigma_{r_j}\), and \(v_{\lambda_j}\) is the down-range position error with the standard deviation of \(\sigma_{\lambda_j}\). Consequently, \(\sigma_{r_j}\) and \(\sigma_{\lambda_j}\) satisfies

\[
\begin{bmatrix} \sigma_{r_j} \\ \sigma_{\lambda_j} \end{bmatrix} = \begin{bmatrix} k_r r_i \\ k_\lambda \lambda_i \end{bmatrix}
\]

which indicates that the down-range position error and the cross-range position error are both proportional to the range. Therefore, according to the characteristic of the sensor measurement, the target position measured by the \(i\)th aircraft contains elliptic or circular error as illustrated in Fig. 1. In this study, \(k_r = 0.05\) and \(k_\lambda = 0.005\) are considered which implies that the down-range position error is larger than the cross-range position error.

2.2 Decentralized Extended Information Filter

In each aircraft, the position and velocity of the target are estimated using the EIF which is the inverse covariance form of the Kalman filter (Mutambara, 1998). The information filter makes the measurement update simpler than the Kalman filter does, and therefore it is convenient to exchange and integrate the target information among multiple aircraft. Consider the discrete target model in the NED frame as

\[
X(k+1) = f(X(k), k) + w(k)
\]

\[
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Delta t \begin{bmatrix} x_i(k) \\ y_i(k) \end{bmatrix} + \begin{bmatrix} w_x(k) \\ w_y(k) \end{bmatrix}
\]

\[
\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_x(k) \\ V_y(k) \end{bmatrix} + \begin{bmatrix} w_{v_x}(k) \\ w_{v_y}(k) \end{bmatrix}
\]

\[
X(k) = \begin{bmatrix} x_i(k) \\ y_i(k) \\ V_x(k) \\ V_y(k) \end{bmatrix}
\]

where \(X(k)\) is the target state, \(\Delta t\) is a unit time, \(V_{r_i}\) and \(V_{\lambda_j}\) are the x-directional velocity and μ-directional velocity in the NED frame, and \(w\) is the process noise with zero-mean and the covariance \(Q \in \mathbb{R}^{4 \times 4}\) in the Gaussian distribution of \(w_x, w_y, w_{v_x}\), and \(w_{v_y}\), respectively.

In the EIF of the \(i\)th aircraft, the information state vector \(Y_i(k)\) is defined using the information matrix \(Y_i(k) = P_i^{-1}(k)\) which is the inverse of the covariance matrix \(P(k)\) as follows.

\[
Y_i(k) = \begin{bmatrix} Y_i(k) \\ \dot{Y}_i(k) \end{bmatrix}
\]

\[
\dot{Y}_i(k) = \begin{bmatrix} \dot{x}_i(k) \\ \dot{y}_i(k) \end{bmatrix}
\]

where \(\dot{X}_i(k)\) is the target state estimated by the \(i\)th aircraft. At \((k+1)\) step, the prediction of the information matrix and state is described as

\[
\begin{bmatrix} Y_i(k+1|k) \\ \dot{Y}_i(k+1|k) \end{bmatrix} = \begin{bmatrix} \nabla f_i(k+1|k) Y_i(k|k) \nabla f_i(k+1|k) + Q(k+1) \end{bmatrix}^{-1}
\]

\[
\dot{Y}_i(k+1|k) = Y_i(k+1|k) \dot{f}_i(\dot{X}_i(k|k))
\]

where \(\nabla f_i\) is the Jacobian of \(f\) in (18) evaluated at \(X_i(k) = \dot{X}_i(k|k)\). Then, the measurement update of the information matrix and state is described as

\[
Y_i(k+1|k+1) = Y_i(k+1|k) + I_i(k+1)
\]

\[
\dot{Y}_i(k+1|k+1) = \dot{Y}_i(k+1|k) + I_i(k+1)
\]
where $i_j(k+1)$ is the information state contribution from the measurement $z_j(k+1)$, and $I_j(k+1)$ is its associated information matrix defined as follows:

$$i_j(k+1) = \nabla h_j^T(k+1)R_k^{-1}(k+1)[y_j(k+1) - \nabla h_j(x_j(k+1))]$$

$$I_j(k+1) = \nabla h_j^2(k+1)R_k^{-1}(k+1)\nabla h_j(x_j(k+1))$$

where $\nabla h_j$ is the Jacobian of $h_j$ in (8) evaluated at $x_j(k+1) = \hat{x}_j(k+1|k)$. Finally, the estimated target state and the covariance matrix at $(k+1)$ step are

$$\hat{x}_j(k+1) = Y_j^{-1}(k+1|k+1)\hat{x}_j(k+1|k) + P_j(k+1|k+1)$$

When the single target is estimated by multiple aircraft, the target information of each aircraft can be shared to improve the estimation accuracy. In the Kalman filter, the exchanged information is the target state $\hat{x}_j(k)$ including $P_j(k)$ and $\nabla h_j(k)$ evaluated at the aircraft position of $(x_j(k), y_j(k))$. On the other hand, in the information filter, $i_j(k)$ and $I_j(k)$ are exchanged regardless of the aircraft position. For this reason, the information filter can be easily applied to the multi-sensor system. In the fusion of the measurement provided by multiple sensors or aircraft, the decentralized information filter is easy to implement and communicate. Hence, in the decentralized information filter, (24) and (25) are replaced with (31) and (32), respectively.

$$Y_j(k+1|k+1) = Y_j(k+1|k) + I_j(k+1) + \sum_{j:k \neq j} I_j(k+1-d)$$

$$\hat{x}_j(k+1|k+1) = \hat{x}_j(k+1|k) + I_j(k+1) + \sum_{j:k \neq j} i_j(k+1-d)$$

where $N$ is the number of aircraft tracking the same target, $i_j(k)$ is the information state contribution transmitted by the $j$th aircraft, $I_j(k)$ is its associated information matrix of the $j$th aircraft, and $d$ is the communication delay between the aircraft. For example, $d=0$ implies that there is no time delay in the target information provided by other aircraft. If the communication link is restricted to neighbor aircraft, the sum of $i_j(k)$ and $I_j(k)$ for all aircraft in (31) and (32) are replaced with the sum of those for neighbor aircraft. Consequently, the position and velocity of the single target are accurately estimated by multiple aircraft using the decentralized EIF.

### 3. COOPERATIVE STANDOFF TRACKING

The roll angle and velocity of the unmanned aircraft are commanded to track the moving target continuously based on the estimated target information. In the tracking of an unknown or unfriendly target, the standoff distance is defined to maintain a specific gap between the target and the aircraft because of the safety and mission requirements. At the same time, when multiple aircraft track the same target, the phase angle between aircraft should be controlled to improve target localization and to prevent collision between aircraft.

In this approach, the roll angle command of the aircraft is designed to regulate the standoff distance $r_j$ to the desired radius $r_d$, and the aircraft speed command is designed to keep a constant phase spacing $\theta_j$ between aircraft. Instead, in this study, the phase spacing between aircraft with respect to the target is controlled mainly by the aircraft speed and additionally by the aircraft heading. The standoff distance and phase angle are shown in Fig. 2.

When the phase spacing reference is the phase angle of front aircraft, the phase spacing error is given by

$$e_{\theta_j} = (\theta_j - \theta_d) - \theta_d$$

where $\theta_j$ is the phase angle of the $j$th aircraft, $\theta_d$ is the phase angle of the $i$th aircraft, and $\theta_d$ is the desired phase spacing angle between two aircraft. Note that if all phase spacing angles are controlled to be $2\pi/N$ for $N$ aircraft, the target will be monitored equivalently at all directions.

Using the Lyapunov function $L_i = e_{\theta_j}^2$, the time derivative of $L_i$ yields

$$L_i = 2(\dot{\theta}_i - \theta_d)$$

The angular rate of the $i$th aircraft with respect to the target is selected as

$$\dot{\theta}_i = \dot{\theta}_d + K_i(\theta_i - \theta_d)/(r_j^2/r_d^2)$$

where $K_i$ is a positive gain. Substituting (35) into (34) gives $L_i \leq 0$. Therefore, the phase spacing angle between two vehicles converges to $\theta_d$ according to the Lyapunov stability theorem. Finally, the speed command of the $i$th aircraft is

$$V_{\theta_j} = r_d \dot{\theta}_d + K_i(\theta_i - \theta_d)/(r_j^2/r_d^2)$$

Next, the standoff distance from the target to each aircraft is controlled by the aircraft heading. In the previous approach of Frew et al. (2008), the heading command is designed to be independent of the phase spacing error using the Lyapunov function $L_2 = (r_j^2 - r_d^2)^2$. Let $x_{ij} = x_i - x_j$ and $y_{ij} = y_i - y_j$ denote the relative position of the $i$th aircraft with respect to the target. The time derivative of $L_2$ yields

$$\dot{L}_2 = 4y_{ij}(r_j^2 - r_d^2)[\dot{x}_{ij} \dot{y}_{ij}]$$

![Fig. 2 Geometric configuration between aircraft and target](image-url)
By substituting (38) and (39) into (37) gives $L_i \leq 0$.

$$\dot{x}_{ij} = -[V_i/(r_i^2 + r_j^2)] [x_j - r_j^2 - r_i^2 + 2r_j r_i]$$

(38)

$$\dot{y}_{ij} = -[V_i/(r_i^2 + r_j^2)] [y_j - r_j^2 - x_j^2 - 2r_j r_i]$$

(39)

The heading command of the previous approach is determined as

$$\psi_{ij} = \tan^{-1} \left( \frac{x_{ij} y_{ij} + V_{ij} \phi_{ij}}{x_{ij} x_{ij} + y_{ij} y_{ij}} \right)$$

(40)

where $a_i$ is a scaling factor that adjusts the magnitude of aircraft velocity using the speed command given in (38)-(39) and the target speed given in (29) (Frew et al., 2008).

In this study, to improve the phase oscillation and frequent speed change, the phase spacing error is included additionally in the computation of the heading command. Alternative approach controlling the phase angle is to change the standoff distance by steering the aircraft heading towards the radial direction. In a circular motion, the angular velocity is inversely proportional to the radius. Therefore, if the phase spacing error is negative, the aircraft circling around the target steers outwards to decrease the angular velocity. If the phase spacing error is positive, the aircraft steers inwards to increase the angular velocity. In this respect, the radial speed of $i$th aircraft is defined to be proportional to the phase spacing error.

$$V_{ij} = -K_1 \dot{\psi}_{ij} = -K_1 \left( \dot{\theta}_i - \dot{\theta}_j \right)$$

(41)

where $K_1$ is a positive gain. Then, the desired heading given in (40) is modified as follows.

$$\psi_{ij} = \tan^{-1} \left( \frac{a_i \dot{y}_{ij} + \dot{V}_{ij}}{a_i \dot{x}_{ij} + \dot{V}_{ij}} \right)$$

(42)

$$\dot{x}_{ij} = V_{ij} \cos \theta_i, \quad \dot{y}_{ij} = V_{ij} \sin \theta_i$$

(43)

where $a_2$ is a scaling factor using the aircraft speed, the distance but also to reduce the phase spacing error at the same time, the aircraft measures the range and the line-of-sight angle of the target to estimate the target position and velocity using the decentralized filter.

To compare the estimation and tracking of the target, four cases are considered for the tracking of the identical target motion: A) decoupled vector field and stand-alone EIF; B) decoupled vector field and partially decentralized EIF; C) integrated vector field and partially decentralized EIF; and D) integrated vector field and fully decentralized EIF. If the decoupled vector field, the heading command is determined by (40) whereas that of the integrated vector field is determined by (42). In the stand-alone EIF, no target information is communicated between aircraft. In the partially decentralized EIF, the 2nd aircraft receives the target information of the 1st aircraft only, and other aircraft do not share the target information. In the fully decentralized EIF, the $i$th aircraft receives the target information of the $(i-1)$th aircraft $(i = 2, 3, 4)$, and the 1st aircraft receives that of the 4th aircraft.

For the case A (the decoupled vector field and the stand-alone EIF), the aircraft and target trajectory is shown in Fig. 3. All aircraft continuously circle around the moving target while keeping the constant standoff distance of 300 meter from the target. The target position and the target velocity estimated by the 2nd aircraft are shown in Figs. 4 and 5, respectively. The target position is accurately estimated within the standard deviation of 6.2 meter using the measurement which standard deviation is larger than 13.3 meter. In addition, the target velocity is estimated within the standard deviation of 3.1 meter using the range and line-of-sight angle measurement.

The aircraft speed is shown in Fig. 6, and the phase spacing angle and distance between the neighbour aircraft are shown in Fig. 7, respectively. Since the target moves fast and changes its direction perpendicularly, the phase spacing angle and distance are not maintained constantly and the aircraft speed is controlled periodically.

4. SIMULATION RESULT AND ANALYSIS

Numerical simulation is performed to verify the proposed estimation and tracking scheme. Four unmanned aircraft are considered to track the unknown moving target which turns perpendicularly with the speed of 15 m/s. The desired standoff distance to the target is $r_d = 300$ meter, and the desired phase spacing angle between aircraft is $\phi_{ij} = \pi / 2$.

The roll angle is constrained within ±40°, and the aircraft speed is controlled within 20–30 m/s. The aircraft shares its phase angle and angular velocity with neighbor aircraft. The $i$th aircraft receives the phase state of the $(i-1)$th aircraft $(i = 2, 3, 4)$, and the 1st aircraft receives that of the 4th aircraft.

![Fig. 3 Flight trajectory: decoupled vector field and stand-alone EIF (Case A)](image)
Note that the desired phase spacing angle is 90°, and the desired distance is 424.3 meter. In Fig. 7, the phase angle between the 2nd aircraft and the 3rd aircraft becomes 265.3° at 293 seconds while the phase spacing angles between the rest aircraft are 40.9°, 34.3°, and 18.1°, respectively. The minimum distance between the 3rd aircraft and the 4th aircraft decreases to 76.0 meter at 216 sec. During last 300 seconds, the minimum distance which is smaller than 150 meter was found three times: 111.1 meter at 293.5 sec, 114.3 meter at 393.4 sec, and 106.4 meter at 503.2 sec. Moreover, as shown in Fig. 6, the aircraft speed is controlled excessively to decrease the phase oscillation, and therefore the aircraft speeds of all aircraft are saturated frequently.

The simulation result of the case B (the decoupled vector field and the partially decentralized EIF) is similar to the result of case A except that the standard deviation of the target position estimated by the 2nd aircraft is decreased. On the other hand, the simulation result of cases C and D (the integrated vector field) is different from that of the cases A and B in that the phase spacing error and the aircraft speed change is reduced positively.

For the case D (the integrated vector field and the fully decentralized EIF), the phase spacing angle and distance are shown in Fig. 8, and the aircraft speed is shown in Fig. 9, respectively. In Fig. 8, the phase oscillation still appears. Even so, the minimum distance is larger than that of Fig. 6 because the amplitude of the phase oscillation is reduced. During last 300 seconds, the minimum distance which is smaller than 150 meter was found just once: 125.3 meter at 508.5 seconds. In addition, the initial phase spacing error shown in Fig. 8 decreases earlier than that of Fig. 6. This fast convergence to the desired phase spacing is the result of integrating the phase spacing error into the heading command. As shown in Fig. 9, the frequent variation and saturation of the aircraft speed are improved remarkably compared with the result of Fig. 7. The heading command in (42) enables the aircraft speed to be less sensitive by steering inwards or outwards whereas the heading command in (40) does not account for the phase spacing error or distance error between the neighbour aircraft even in the probable collision situation.

In Table 1, the estimation and tracking results of four cases are summarized. In the comparison of filters, the accuracy of
the target position and velocity estimation is improved using the decentralized filter. The standard deviation of $x$-directional target position of the 2nd aircraft decreases from 6.17 to 3.09 because of the additional target information transmitted by the 1st aircraft. In the fully decentralized filters, the overall standard deviations decrease due to the information exchange. Similarly, the information which is defined as the square sum of the determinant of the information matrix increases as the aircraft receives the target information of other aircraft.

In the comparison of vector fields, the phase spacing error is reduced using the integrated vector field. The phase spacing error during the last 300 seconds is compared in Table 1. Using the integrated vector field, the phase spacing error of the 1st, 2nd, and 4th aircraft decreases regardless of the filter type. In addition, the minimum distance between the neighbour aircraft increases overall as shown in Fig. 9. Thus, the phase oscillation is relaxed by considering the phase spacing error in the design of the heading vector field.

5. CONCLUSION

Cooperative target localization and tracking schemes are designed to track the fast target. The decentralized extended information filter accurately estimates the target position and velocity using the range and line-of-sight measurement. The integrated vector field composed of the standoff distance error and the phase spacing error reduces the phase oscillation. The strict stability analysis of the integrated vector filed is remained as a further study.

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REFERENCES


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<th>Integrated vector field</th>
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