Local Self-optimizing Control with Input and Output Constraints

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Abstract: The available methods for selection of controlled variables (CVs) using the concept of self-optimizing control have been developed under the restrictive assumption that the set of active constraints remains unchanged for all the allowable disturbances. To keep the input and output variables within their allowable bounds, the use of cascade controllers, and to track the optimal set of active input constraints, the use of split-range controllers is suggested in literature. In this paper, we propose a different strategy, where CVs are selected as linear combinations of measurements to minimize local average loss, while ensuring that all the constraints are satisfied over the allowable set of disturbances. This result is extended to select a few of the available measurements, whose combinations are used as CVs. The proposed approach offers simpler implementation of operational policy for processes with tight operational constraints. We use the case study of forced-circulation evaporator to illustrate the usefulness of the proposed method.

Keywords: Controlled variables, Control system design, Mixed integer programming, Process constraints, Self-optimizing control.

1. INTRODUCTION

When a process is subjected to disturbances, an ideal optimal controller repeatedly optimizes the process online (Marlin & Hrymak, 1997; Srinivasan et al., 2003). The repeated optimization, however, requires that the states and model parameters be estimated and is also computationally costly (Srinivasan et al., 2003; Alstad et al., 2009). To overcome these drawbacks, Skogestad (2000) introduced the novel concept of self-optimizing control (SOC). In this approach, keeping the selected controlled variables (CVs) at constant setpoints using feedback controller automatically leads the process to acceptable operating condition. In addition to significant reduction in computational load required for optimization, the feedback-based approach offers simpler implementation policy in comparison with the use of ideal optimizing controller. The term ‘acceptable operating conditions’ in accordance to SOC concept is quantified as loss, i.e., the difference between the static cost function, when SOC policy and the ideal optimizing controller are implemented. Here, the loss depends on the selected CVs. Thus, the main issue in SOC is to find appropriate CVs among the possible alternatives that lead to least worst-case or average loss.

CV selection based on direct evaluation of the nonlinear model and cost function requires solving large dimensional nonconvex optimization problems. Thus, local methods, which employ linearized process model and quadratic approximation of the loss function, are instead used to find promising CV candidates. Note that the nonlinear model still needs to be used to assess the suitability of the candidates identified by the local methods to obtain the set of final CVs.

The first local method developed to select CVs is the minimum singular value (MSV) rule (Skogestad & Postlethwaite, 1996). The MSV rule, however, is approximate and may lead to suboptimal set of CVs (Hori & Skogestad, 2008). More recently, exact local methods to select CVs through minimization of worst-case (Halvorsen et al., 2003) and average loss (Kariwala et al., 2008) have been proposed. These methods can be used for selecting CVs as a subset or linear combinations of available measurements, where the latter approach can provide lower loss. Different approaches for finding the locally optimal combination matrix are available in (Kariwala, 2007; Kariwala et al., 2008; Alstad et al., 2009). To make the application of exact local methods viable for large-scale processes, Kariwala and Cao (2009, 2010) have presented efficient branch and bound methods for selecting a subset of available measurements, which can be used directly or combined as CVs.

An assumption involved in the development of exact local methods is that the set of active constraints does not change during the operation. This assumption is not always satisfied in practice, where it may be optimal to keep different sets of variables at their limits depending on the disturbances. To track the optimal set of active input constraints, Lerssamrungrueng et al. (2008) have suggested the use of split range controller. In an alternate approach, Cao (2004) proposed the use of cascade control strategy to keep the input and output variables within their allowable bounds. The use of split range or cascade controllers clearly leads to a more complicated control structure, whereas the goal of SOC is to devise ‘simple’ implementation policy.

This paper takes a fundamentally different viewpoint for handling the possible changes in active constraint set. We aim at finding CVs, whose control ensures that the inputs and outputs are maintained within their allowable bounds for all disturbance scenarios. The resulting ‘passive’ approach maintains the simplicity of the control structure and can be seen as a viable alternative to the use of split-range or...
cascade control strategies, where the penalty (measured in terms of loss) of not tracking the optimal set of active constraint set is not very high.

We present an exact local method, where linear combinations of measurements are selected as CVs such that the local average loss is minimized subject to constraints on inputs and outputs. It is noted in (Kariwala, 2007; Kariwala et al., 2008; Alstad et al., 2009) that the use of combinations of a few measurements as CVs can often provide similar loss as the case where combinations of all available measurements are used. We extend the proposed approach to identify the locally optimal subset of available measurements, whose linear combinations can be used as CVs. The resulting formulation is a mixed integer cone program and can be solved efficiently by available software. The case study of forced-circulation evaporator (Newell & Lee, 1989; Kariwala et al., 2008) is used to demonstrate the usefulness of the proposed approach.

The rest of this paper is organized as follows: a brief overview of the available exact local method for SOC is presented in Section 2. The exact local method is extended for handling input and output constraints in Section 3. The results of the case study of forced-circulation evaporator are presented in Section 4 and conclusions are drawn in Section 5.

2. LOCAL SELF-OPTIMIZING CONTROL

We consider that the optimal operation of the process requires solving the following steady-state optimization problem:

\[
\begin{align*}
\min J(u,d) \\
\text{s.t. } y &= f_y(u,d) \\
u &\in U, \quad y \in Y, \quad d \in D
\end{align*}
\] (1)

where \( u \in U \subseteq \mathbb{R}^n \), \( d \in D \subseteq \mathbb{R}^m \), and \( y \in Y \subseteq \mathbb{R}^p \) denote the inputs (or degrees of freedom), disturbances, and outputs, respectively. Here, \( U \), \( D \), and \( Y \) denote the allowable or admissible sets for \( u \), \( d \), and \( y \), respectively. \( J \) refers to the scalar (economic) cost function, which needs to be minimized. In the optimization problem given by (1), it is assumed that the internal states of the process have been eliminated using the model equations and that the constraints, which remain active for all \( d \in D \), are controlled. In this sense, \( u \) denotes the ‘remaining’ degrees of freedom; see (Skogestad, 2000) for further details.

For every \( d \in D \), the optimization problem in (1) can be solved online to update \( u \). An alternative and simpler approach to update \( u \) in the presence of disturbances involves the use of a feedback controller to hold the CVs \( c \) at setpoint \( c_s \), i.e.

\[
c = h(y) = c_s
\] (2)

where \( y \) denotes the measured outputs given as \( y = y + e \). Here, \( e \in E \) denotes the implementation error arising due to measurement error. The use of feedback-based policy results in a loss given as

\[
L(d,e) = J_v(d,e) - J_{\text{opt}}(d)
\] (3)

where \( J_{\text{opt}}(d) \) and \( J_v(d,e) \) denote the value of objective function obtained by solving the optimization problem in (1) and by holding \( c \) at \( c_s \), respectively. The loss depends on the choice of \( c \) and the aim of SOC is to find appropriate CVs, which minimize the loss.

In general, the selection of CVs based on direct evaluation of loss in (3) for different CV alternatives is difficult. Instead, local methods are used to identify promising alternatives. In these methods, it is assumed that the inputs and outputs remain within their allowable limits, i.e. the constraints are always satisfied. The process model is linearized around a nominal optimal operating point to obtain

\[
\Delta y = G_u \Delta u + G_y \Delta d
\] (4)

\[
\Delta y = \Delta y + \Delta e
\] (5)

where \( G_u = \partial y / \partial u \) and \( G_y = \partial y / \partial d \) evaluated at the nominal operating point, and \( \Delta \) is used to denote the deviation variables. The deviation in CVs \( \Delta c \) is given as

\[
\Delta c = H \Delta y
\] (6)

with \( H \in \mathbb{R}^{n \times m} \) being a selection or combination matrix. Here, \( HG_u \) is assumed to be non-singular, which is necessary to ensure that \( c \) can be maintained at \( c_s \) by manipulating the inputs using a controller with integral action.

Let \( \Delta d = W_d \Delta \hat{c}, \Delta e = W_e \Delta \hat{e} \), where the diagonal matrices \( W_d \) and \( W_e \) contain the expected magnitudes of disturbances and measurement errors, respectively. For uniformly distributed \( d \) and \( e \) over the set

\[
\| \begin{bmatrix} \hat{d} \\ \hat{e} \end{bmatrix} \|_F \leq 1
\] (7)

the local average loss is given as (Kariwala et al., 2008)

\[
L_{\text{average}}(H) = \frac{1}{6} \| M_d \|_F \| M_e \|_F
\] (8)

where \( \| \cdot \|_F \) denote the Frobenius norm and

\[
M_d = -J_{uu}^{-1} \left( (HG_u)^{-1} HG_y - J_{ud} J_{ud}^{-1} \right) W_d
\]

\[
M_e = -J_{ee}^{-1} (HG_u)^{-1} HW_e
\] (9)

Here, \( J_{uu} = \partial^2 J / \partial c \partial c \) and \( J_{ud} = \partial^2 J / \partial c \partial d \) evaluated at the nominal operating point. Note that \( J_{uu}^{-1} \) is guaranteed to exist as \( J_{uu} \) is positive definite. The expression for local worst-case loss is available in (Halvorsen et al., 2003; Kariwala, 2007). We suggest selection of CVs through minimization of average loss, as the worst case may not occur frequently in practice (Kariwala et al., 2008).

When individual measurements are used as CVs, the
elements of $H$ are limited to 0 and 1 and $HH^T = I$. When combinations of measurements are used instead, the elements of $H$ are allowed to take any value provided that the condition $\text{rank}(H) = n_H$ is satisfied. An explicit expression to obtain $H$ based on average loss minimization is given as (Alstad et al., 2009)

$$H^r = (YY^T)^{-1}G^r((G^r)^T(YY^T)^{-1}G^r)^{-1}J^{-1/2}_{u,u}$$

where

$$Y = [(G^r J^{-1} J_{ad} - G^r J^{-1}) W_d W_r]$$

(10)

The assumption that the set of active constraints does not change with disturbances often limits the application of the available exact local method. In general, it may be optimal to keep different sets of variables at their limits for different disturbance scenarios. For heat exchanger networks described using linear models, Lerbachungsk et al. (2008) suggested the use of split range controllers to track the optimal set of active input constraints. Cao (2004) has proposed a cascade control strategy, where the goal is instead to keep the input and output variables within their allowable bounds. In this approach, the CV identified based on the concept of SOC is placed in the outer loop and the variable likely to violate the constraint in the inner loop; see Figure 1. Clearly, the use of split-range or cascade controllers leads to a control structure with increased complexity. In the next section, we propose an alternate approach to handle input and output constraints, which maintains the simplicity of the control structure.

In this section, we extend the available exact local method for CV selection to account for the presence of constraints. We consider that the admissible sets for input and output are

$$U = \{u : B_i u \leq b_i\},$$

$$Y = \{y : B_i y \leq b_i\},$$

(12)

where $B_i \in \mathbb{R}^{n_{in} \times n_{in}}$, $b_i \in \mathbb{R}^{n_{in}}$, $B_e \in \mathbb{R}^{m_{out} \times n_{out}}$ and $b_e \in \mathbb{R}^{m_{out}}$. These constraints can be equivalently expressed in terms of $\Delta u$ and $\Delta y$ as

$$\Delta U = \{\Delta u : B_i \Delta u \leq b'_i\},$$

$$\Delta Y = \{\Delta y : B_i \Delta y \leq b'_i\},$$

(13)

where $b'_i = b_i - B_i u'$ and $b'_i = b_i - B_i y'$, where $u'$ and $y'$ are the values of $u$ and $y$ at nominal operating point.

Based on (4)-(6), maintaining $c = c$ ( $\Delta c = 0$ ) requires

$$\Delta u = -(HG_r)^{-1}H \left[G^r W_d W_r \right] \frac{d}{d}$$

(14)

Similarly, by substituting for $\Delta u$ in (4), we have

$$\Delta y = \left[(G^r - G_r (HG_r)^{-1}) W_d - G_r (HG_r)^{-1}HW_r \right] \frac{d}{d}$$

(15)

Thus dropping the scalar term in (8), the local average loss minimization problem with input and output constraints can be formulated as:

$$\min_{M_r} \| M_r \|_{\infty}$$

s.t.,

$$B_i \left[(G^r - G_r (HG_r)^{-1}) W_d - G_r (HG_r)^{-1}HW_r \right] \frac{d}{d} \leq b'_i,$$

$$\| B_i \|_{\infty} \leq b_i,$$

(16)

The optimization problem in (16) is nonlinear in $H$ and thus difficult to solve directly. To overcome this difficulty, we perform a transformation to obtain an equivalent convex problem. The transformation is based on an observation made earlier in (Alstad et al., 2009).

The observation is that if $H^0$ is a feasible (not necessarily optimal) solution to the optimization problem in (16), so will be $QH^0$ for any nonsingular $Q \in \mathbb{R}^{n_r \times n_r}$. Since $H^0 G_r$ is nonsingular (which is always assumed in the formulation of SOC), it is always possible to force $QH^0 G_r = I$ with certain nonsingular $Q$. Let us denote $H = QH^0$. A solution to the optimization problem in (16) can be obtained by equivalently solving:

$$\min_{M_r} \| M_r \|_{\infty}$$

s.t.,

$$B_i H \leq b_i,$$

$$HG_r = I,$$

(17)

where $\| \|$ denotes the vector norm computed as the sum of the absolute values of the elements of the vector,

$$M'_d = -J_{u,u}^{-1/2}(HG_d - J_{u,u}^{-1} J_{ad}) W_d$$

$$M'_e = -J_{u,u}^{-1/2} HW_e$$

and $B_i$ and $b_i$ represent the $i$th rows of $B$ and $b$, respectively, which are defined as

$$B = \left[ -B_i H \left[G^r W_d W_r \right] \left[b'_i - b'_i \right] \right].$$

(19)

The solution to (17) gives the optimal combination matrix $H'$, based on which the CVs can be selected as $c = QH' \hat{y}$, where $Q$ is any nonsingular matrix with compatible dimensions.
As mentioned previously, combinations of fewer measurements as CVs, which give similar loss in comparison
with combinations of all measurements, are preferable as they allow simpler implementation. Such a measurement subset can be selected by including the following constraints in the optimization problem in (16):

$$\sum_{i=1}^{n} \sigma_{y_i} = n, \sigma_{y_i} \in \{0, 1\},$$

$$-M \sigma_{y_i} \leq H_y \leq M \sigma_{y_i}, \text{ for } \forall i \in \{1, 2, \ldots, n\}$$

(20)

where $\sigma_{y_i} = 1$ if $y_i$ is included and 0 otherwise. The integer $n$ is the specified size of the subset, and $M$ is a large number satisfying $M \geq |H_y|$ for $\forall i, j$. The constraints in (20) are motivated by the work in (Sierskma, 2002; Yelchuru et al., 2010). Now, the overall problem can be written as

$$\min_{\bar{y}} \|M'_y \bar{y} - M'_{\bar{y}}\|^2$$

s.t. $$HG_y = I_n,$$

$$\sum_{i=1}^{n} \sigma_{y_i} = n, \sigma_{y_i} \in \{0, 1\},$$

$$-M \sigma_{y_i} \leq H_y \leq M \sigma_{y_i}, \text{ for } \forall i \in \{1, 2, \ldots, n\}.$$  

The optimization problem in (21) is a mixed integer program and can be solved efficiently using available software. In this paper, we use Sedumi (Sturm, 1999) interfaced to Matlab through YALMIP (Lofberg, 2004) for solving this problem.

4. EVAPORATOR CASE STUDY

Problem description. We consider forced-circulation evaporation process (Newell & Lee, 1989; Kariwala et al., 2008) to demonstrate the usefulness of this approach. In this process, dilute solution is pumped upwards through the vertical heat exchanger, while steam flows in counter-current direction as the heating fluid to evaporate the solvent, thus increasing the concentration of the solution. A part of this concentrated solution is circulated back to the evaporator, while the rest is drawn as product.

The operational objective of this process involves minimizing

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) + 0.2F_1 - 4800F_2$$  

(22)

which denotes negative profit. In (22), the first four terms are related to the costs of steam, water, pumping and raw material. The last term is related to the revenue obtained by selling the product. The following constraints need to be satisfied:

$$X_2 \geq 35.5$$

$$40 \leq P_2 \leq 80$$

$$P_{100} \leq 400$$

$$0 \leq F_{200} \leq 400$$

$$0 \leq F_1 \leq 20$$

$$0 \leq F_2 \leq 100$$

(23)

This process has eight degrees of freedom (DOF), among which three ($X_2$, $T_1$ and $T_{200}$) are disturbances. The remaining five variables $F_1$, $F_2$, $P_{100}$, $F_3$, and $F_{200}$ are manipulated variables. The case where $X_2 = 5\%$, $T_1 = 40^\circC$, and $T_{200} = 25^\circC$ is taken as the nominal operating point. Solving the optimization problem in (22)-(23) for these nominal disturbances results in optimum negative profit of –582.233 $/h. The corresponding nominal optimal values of different variables are shown in Table 1.

DOF analysis. The constraints on $X_2$ and $P_{100}$ remain active over the entire set of allowable disturbances. In addition, separator level ($L_2$), which has no steady-state effect, needs to be stabilized at its nominal setpoint, which consumes one DOF. After control of active constraints and $L_2$, two inputs $(u)$ remain. Without loss of generality, they are taken as $F_1$ and $F_{200}$. For these $u$, we consider that 2 CVs are to be chosen as a subset or combinations of the following available measurements:

$$\bar{y} = [P_2, T_2, T_3, F_2, F_{100}, T_{201}, F_3, F_{200}, F_1]^{T}$$  

(24)

Note that the pump circulation flow ($F_1$) is not included in $\bar{y}$, as the linear model for this measurement results in large plant-model mismatch due to linearization (Kariwala et al., 2008).

Local analysis. The allowable disturbance set corresponds to ±5% variation in $X_2$ and ±20% variation in $T_1$ and $T_{200}$ around their nominal values. Based on these variations, we have $W_d = \text{diag}(0.25, 8, 5)$. The measurement errors for the pressure and flow measurements are taken to be ±2.5% and ±2%, respectively, of the nominal operating values. For temperature measurements, this error is considered to be ±1°C. Accordingly, we have $W_e = \text{diag}(1.285, 1, 1, 0.027, 0.189, 1, 0.163, 4.355, 0.189)$. The Hessian and gain matrices for this process are available in (Kariwala et al., 2008).

We first consider the selection of CVs through the available exact local methods, i.e. the constraints are not explicitly taken into account. For this approach, the best individual measurements are $c'_1 = [F_{100}, F_{200}]^{T}$ with average local loss being 3.899 $/h. When linear combinations of all the 9
measurements are used, the average local loss decreases to 0.250 $/h. A similar trend is observed, when the input and output constraints are taken into account during CV selection. With the proposed approach, the best individual measurements \( c_2 = [F_{100} T_{201}] \) result in an average local loss of 22.160 $/h, which reduces to 10.852 $/h when all measurements are included. Results from both approaches signify that controlling combinations of measurements can lead to substantial reduction in loss.

From practical point of view, combining fewer measurements as CVs, which gives similar loss as the loss obtained using combinations of all the available measurements, is desirable. The combinations of \( n \) out of 9 measurements, which give the smallest average local loss for available exact local method are found using branch and bound method (Karirawa & Cao, 2010). A similar analysis is carried out for the proposed approach by solving the optimization problem in (21) for different values of \( n \). The results are presented in Figure 3.

For both approaches, the use of combinations of 3 or 4 measurements as CVs provides a reasonable trade-off between the simplicity of control system and operational loss. The CV candidates comprising of 3 and 4 measurements obtained using the available exact local method, i.e. without considering constraints, are:

\[
\begin{align*}
c_3' &= [-49.437F_2 + 6.216F_{100} + 0.073F_{200}\] 
&\quad -98.863F_2 + 16.156F_{100} - 0.017F_{200} \\
c_4' &= [-51.287F_2 + 4.649F_{100} + 2.336F_5 + 0.072F_{200}\] 
&\quad -104.599F_2 + 11.298F_{100} + 7.246F_5 - 0.021F_{200} 
\end{align*}
\]

(25) (26)

with average local losses of 0.652 and 0.474 $/h, respectively. The corresponding CVs, obtained using the proposed approach, are

\[
\begin{align*}
c_3 &= \begin{bmatrix} 3.843P_2 - 318.658F_2 + 1.358F_{200} \\
0.159P_2 - 6.100F_2 + 0.015F_{200}\end{bmatrix} \\
c_4 &= \begin{bmatrix} 1.526P_2 - 730.119F_2 + 98.946F_5 + 1.142F_{200} \\
0.098P_2 - 12.443F_2 + 1.875F_5 + 0.009F_{200}\end{bmatrix}
\end{align*}
\]

(27) (28)

The average local loss for \( c_3 \) and \( c_4 \) are 16.414 and 11.112 $/h, respectively. These alternatives are further evaluated using the nonlinear process model.

Nonlinear analysis. The promising CV candidates identified using local analysis are evaluated for 100 scenarios with randomly generated \( d \) and \( e \). Cascade control is required for the implementation of \( c_3' \) and \( c_4' \), otherwise \( P_2 \) can violate the constraints in (23) for some disturbances and measurement errors. For implementation of cascade control strategy, the lower and upper bounds on \( P_2 \) are revised to 41.285 and 78.715 kPa, respectively, to account for measurement errors. The resulting average losses for the four CV candidates are presented in Table 2.

Table 2 shows that the results from nonlinear analysis are in good agreement with the results from local analysis. It can be seen that for CV candidates synthesized using the proposed approach, which minimizes average loss subject to input and output constraints, the loss is higher in comparison with the losses seen with CV candidates synthesized using the available exact local method and implemented using cascade controller. Nevertheless, these losses (15.131 and 8.840 $/h, respectively) are relatively small in comparison to the optimal cost at nominal conditions, i.e. 583.23 $/h. Thus, the resulting implementation can still be considered acceptable economically. An advantage of using the CVs found using the proposed approach is that their implementation does not require additional controllers since all input and output variables remain within their bounds for all the disturbance scenarios. This can be confirmed from Figure 4, where the variation of \( P_2 \) with the use of different CV alternatives is shown. In this sense, the proposed approach provides a trade-off between the loss and the simplicity of implementation strategy.
Table 2. Average local and nonlinear losses for the self-
optimizing CV candidates

<table>
<thead>
<tr>
<th>CV candidate</th>
<th>Measurements</th>
<th>Average Loss [$/h$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_3'$</td>
<td>$F_2, F_{100}, F_{200}$</td>
<td>0.652, 3.967</td>
</tr>
<tr>
<td>$c_4'$</td>
<td>$F_2, F_{100}, F_3, F_{200}$</td>
<td>0.474, 3.363</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$P_2, F_2, F_{200}$</td>
<td>16.414, 15.131</td>
</tr>
<tr>
<td>$c_4$</td>
<td>$P_2, F_2, F_3, F_{200}$</td>
<td>11.112, 8.840</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

We have proposed a method for systematic selection of CVs in the framework of self-optimizing control, for processes with tight operating constraints. In this approach, linear combinations of measurements are selected as CVs such that maintaining the CVs at constant setpoints minimizes the local average loss and keeps the inputs and outputs within their constraint limits over the allowable set of disturbances. In comparison with existing approaches, which involve the use of split-range (Lersbamrungsuk et al., 2008) and cascade controllers (Cao, 2004), the proposed approach is conservative, but allows for simpler implementation strategy. The use of the proposed approach is attractive, when the penalty (measured in terms of loss) of not tracking the optimal set of active constraints is not very high. The case-study of forced-circulation evaporator is used to show that the proposed approach can provide a good trade-off between the loss and complexity of the control system.

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