Decentralized output-feedback control of large-scale nonlinear systems with sensor noise

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Abstract: This paper studies decentralized output-feedback control of large-scale nonlinear systems in the presence of sensor noise. The measurement errors are dealt with by set-valued maps in the recursive control design of the subsystems. With appropriately designed decentralized controllers, the subsystems are rendered input-to-state stable (ISS) with respect to the outputs of the other subsystems and the disturbances. The robust stability of the closed-loop decentralized system is guaranteed by a recently developed cyclic-small-gain theorem.

Keywords: Decentralized control, large-scale nonlinear systems, output feedback, sensor noise, input-to-state stability (ISS), small-gain.

1. INTRODUCTION

Decentralized control problems arise from various engineering applications, such as power systems, transportation networks, water systems, chemical engineering, telecommunication networks. Among the main characteristics of decentralized control are the dramatic reduction of computational complexity and the enhancement of robustness against interacting operation failures. Significant contributions have been made to the development of decentralized control theory (see Šiljak [1991] and a recent review in Bakule [2008]). In this paper, we study the decentralized output-feedback control problem of large-scale nonlinear systems in disturbed output-feedback form. Beyond the external disturbances influencing the system dynamics, we are also concerned about measurement errors which may be nonsmooth signals with respect to time.

There have not been many results on nonlinear control design with nonsmooth measurement errors. We just review some results closely related to our paper. In Freeman & Kokotović [1996], a controller was designed with set-valued maps and “flattened” Lyapunov functions following the backstepping methodology such that the control system is input-to-state stable (ISS) with respect to the measurement errors. The “flattened” Lyapunov function appears to be a powerful tool for dealing with the nonsmoothness of the measurement errors. However, in this result, the influence of the measurement errors grows with the order of the system. Jiang, Mareels, & Hill [1999] studied nonlinear systems composed of two subsystems, one is input-to-state stable and the other one is input-to-state stabilizable with respect to the measurement noise. In Jiang, Mareels, & Hill [1999], the ISS of the control system was guaranteed by the gain assignment technique introduced in Jiang, Teel, & Praly [1994], Praly & Wang [1996], Jiang & Mareels [1997] and the nonlinear small-gain theorem proposed in Jiang, Teel, & Praly [1994], Jiang, Mareels, & Wang [1996]. The problem studied in our paper is technically challenging because the measurement error attenuation problem has not been well addressed for large-scale nonlinear systems.

In this paper, we will introduce the Lyapunov-based cyclic-small-gain theorem in Jiang & Wang [2008], Liu, Hill & Jiang [2009] as a new tool for the modern design of decentralized controllers. The result in Liu, Hill & Jiang [2009] using system graphs to construct ISS-Lyapunov functions is more intuitive than the matrix-small-gain theorem developed earlier in Dashkovskiy, Rüffer & Wirth [2007, 2010]. It will be shown that the cyclic-small-gain theorem is really effective for the design of robust measurement feedback controllers for large-scale nonlinear systems.

Motivated by the celebrated backstepping methodology (see Krstić, Kanellakopoulos & Kokotović [1995]) and its small-gain based extension in Jiang & Mareels [1997], we design decentralized controllers through a recursive design approach. To deal with the nonsmooth measurement noise, we employ set-valued maps to construct Lyapunov functions and to design the virtual control laws. The gain assignment technique in small-gain based control design (see Jiang, Teel, & Praly [1994], Praly & Wang [1996], Jiang & Mareels [1997]) is used as a basic ingredient. Under the designed controllers, the outputs of the closed-loop system are driven arbitrarily close to the levels of their corresponding measurement noise.

The main result of this paper is a significant extension of the previous results to the decentralized case. Interestingly, when reduced to the single (centralized) system, our result appears to be new and more general than previously known results.
Notations: A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is positive definite if $\gamma(s) > 0$ for all $s > 0$ and $\gamma(0) = 0$. $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is a $\mathcal{K}$ function (denoted by $\gamma \in \mathcal{K}$) if it is continuous, strictly increasing and $\gamma(0) = 0$; it is a $\mathcal{K}_\infty$ function (denoted by $\gamma \in \mathcal{K}_\infty$) if it is a $\mathcal{K}$ function and also satisfies $\gamma(s) \to \infty$ as $s \to \infty$. $I_n$ represents the identity function. $I_n$ represents the identity matrix of size $n$. $\lambda_{\text{max}}(P)$ means the largest eigenvalue of a real and symmetric matrix $P$.

2. PROBLEM FORMULATION

Consider a class of large-scale systems composed of $N$ subsystems with the $i$-th subsystem ($1 \leq i \leq N$) in the disturbed output-feedback form:

\begin{align}
\dot{x}_{ij} &= x_{i(j+1)} + A_{ij}(y_i, w_i) , \quad 1 \leq j \leq n_i \\
y_i &= x_{i1} \\
x_{i(n_i+1)} &\overset{\text{def}}{=} u_i \\
w_i &= [a_{i1}y_1, \ldots, a_{iN}y_N, d_i^m]^T \\
y_{i}^m &= y_i + d_i^m
\end{align}

where $x_i = [x_{i1}, \ldots, x_{in_i}]^T \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$, and $y_i \in \mathbb{R}$ are the state, control input, and output of the $i$-th subsystem ($x_i$-subsystem), respectively, $y_{i}^m \in \mathbb{R}$ is the measurement of the output, $d_i^m \in \mathbb{R}^{n_i}$ and $d_i^m \in \mathbb{R}$ are external disturbance and measurement error, respectively, $a_{ij}$’s ($1 \leq j \leq n_i$) are real constant parameters and $a_{i0} = 0$, and $\Delta_{ij}$’s ($1 \leq j \leq n_i$) are uncertain locally Lipschitz functions.

Assumptions 1 and 2 are made on system (1)–(5).

Assumption 1. For each $\Delta_{ij}$ ($1 \leq i \leq N$, $1 \leq j \leq n_i$), there exists a known $\nu_{\Delta_{ij}} \in \mathcal{K}_\infty$ such that for all $y_i, w_i$,

\[ |\Delta_{ij}(y_i, w_i)| \leq \nu_{\Delta_{ij}}(|[y_i, w_i]^T|^\tau). \]

Assumption 2. For each $1 \leq i \leq N$, there exist constants $\bar{d}_i, \underline{d}_i, \bar{d}_i^m, \underline{d}_i^m \geq 0$, such that

\[ |d_i^m| \leq \bar{d}_i; \quad |d_i^m| \leq \underline{d}_i^m. \]

Remark 1. The disturbed output-feedback form (1)–(5) is a measurement-disturbed version of a system considered in Jiang, Repperger, & Hill [2001] and exists in some mechanical systems, e.g., the interconnected system of cart–inverted double pendulum. Differently from centralized systems, the input-output feedback linearization cannot be implemented in the decentralized system due to the dependence of the $\Delta_{ij}$ on $w_i$ and the inaccurate measurement caused by the sensor noise $d_i^m$.

Our objective is to design decentralized controllers for system (1)–(5) using the output measurements $y_i^m$’s, such that the plant outputs $y_i$’s ($1 \leq i \leq N$) are steered to within some small neighborhoods of the origin.

3. MAIN RESULT

The main result will be presented in Theorem 1 after we introduce a novel decentralized controller for system (1)–(5).

3.1 A Technical Lemma

In this subsection, we present a modified gain assignment lemma, which is an extension of the gain assignment techniques used in Jiang, Teel, & Praly [1994], Praly & Wang [1996], Jiang & Mareels [1997], Jiang, Mareels, & Hill [1999].

Consider the following first-order system:

\[ \eta = \phi(\eta, a_0, \ldots, a_{n-2}) + \kappa \]

where $\eta \in \mathbb{R}$ is the state, $\kappa \in \mathbb{R}$ is the control input, $a_0, \ldots, a_{n-2} \in \mathbb{R}$ represent disturbances, the nonlinear function $\phi(\eta, a_0, \ldots, a_{n-2})$ is locally Lipschitz and satisfies

\[ |\phi(\eta, a_0, \ldots, a_{n-2})| \leq \eta \phi(|\eta, a_0, \ldots, a_{n-2}|) \]

with $\phi \in \mathcal{K}_\infty$ known. Define $\alpha_\eta(s) = \frac{1}{2}s^2$ for $s \in \mathbb{R}^+$.

Lemma 1. Consider system (8). For any specified $0 < \varepsilon < 1$, $\delta > 0, \alpha > 0$ and $\chi_{n-1}, \ldots, \chi_{n-2} \in \mathcal{K}_\infty$, one can find a continuously differentiable, odd, strictly decreasing and radially unbounded $\kappa$ such that if

\[ \kappa \in \{\kappa(\eta + \delta a_{n-1} + \text{sgn}(\eta) a_0) : |\delta| \leq 1 \} \]

with $a_{n-1}, a_0 \in \mathbb{R}^+$, then $V_{\eta}(\eta) = \alpha(\eta)$ satisfies

\[ V_{\eta}(\eta) \geq \max_{k=1,\ldots,n-2} \left\{ \chi_{n-1}(\alpha_\kappa), \alpha_{\varepsilon}(\frac{a_{n-1} - 1}{c}), \varepsilon \right\} \]

\[ \Rightarrow \nabla V_{\eta}(\eta) \eta \leq -\nu V_{\eta}(\eta). \]

The proof of Lemma 1 is in Appendix A.

Remark 2. Recall $V_{\eta}(\eta) = \alpha(\eta)$, Property (11) implies that the ISS gain from $a_{n-1}$ to $\eta$ is $1/c$, which is arbitrarily close to one if we choose $c$ arbitrarily close to one.

Remark 3. The gain assignment lemmas in Jiang, Teel, & Praly [1994], Praly & Wang [1996], Jiang & Mareels [1997] considered the effects of input and state dynamic uncertainties on stabilization, assuming nevertheless that the system in question is free of measurement noise. Proposition 4.1 in Jiang, Mareels, & Hill [1999] developed a gain assignment technique to guarantee the ISS of the control system with respect to the measurement noise and the gain from the measurement noise to the corresponding output was assigned to be of $\mathcal{K}_\infty$. Lemma 1 above considered a more general case when the control law as in (10) is influenced by $\text{sgn}(\eta) a_0$.

3.2 Decentralized Reduced-order Observers

For each $i$-th subsystem ($1 \leq i \leq N$), the following implementable reduced-order observer is constructed:

\[ \dot{\hat{\xi}}_{ij} = \xi_{ij(j+1)} + L_{ij}y_i + L_{ij}Q_{ij}^m, \quad 2 \leq j \leq n_i - 1 (12)\]

\[ \xi_{i0} = u_i - L_{i0}Q_{i0}^m, \quad 2 \leq j \leq n_i - 1 (13) \]

where $\xi_{ij}$ is an estimate for the unmeasured state $x_{ij} - L_{ij}y_i$ for each $2 \leq j \leq n_i$. With $\gamma_i^m = y_i + d_i^m$, observer (12)–(13) can be equivalently represented as:

\[ \dot{\hat{\xi}}_{ij} = \xi_{ij(j+1)} + L_{ij}y_i + L_{ij}(Q_{ij}^m + L_{ij}d_i^m), \quad 2 \leq j \leq n_i - 1 (14) \]

\[ \hat{\xi}_{i0} = u_i - L_{i0}(Q_{i0}^m + L_{i0}d_i^m), \quad 2 \leq j \leq n_i - 1 (15) \]

Define $e_0 = [x_j^2 - L_{ij}y_j, \ldots, x_{n_i} - L_{ij}y_i - \xi_{i0}]^T$ as the observation error. Then, we obtain the observation error system:

\[ \dot{e}_0 = A_i e_0 + \phi_0(y_i, w_i, d_i^m) \]

where
In this subsection, we will construct a new $[e^T_0, e_1, \ldots, e_{ni}]^T$-system consisting of ISS subsystems obtained through a recursive design of the $[e^T_0, e_1, \xi_2, \ldots, \xi_{ni}]^T$-system. Define $\alpha_0(s) = \frac{1}{s^2}$ for $s \in \mathbb{R}_+$. All the $e_j$-subsystems $(1 \leq j \leq ni)$ will be designed with the ISS-Lyapunov function candidate

$$V_{ij}(e_j) = \alpha_0(|e_j|). \tag{23}$$

In the following discussions, we simply use $V_{ij}$ instead of $V_{ij}(e_j)$. Denote $\dot{e}_ij = [e^T_0, e_1, \ldots, e_{ij}]^T$ and $\delta_{ij} = [\xi_2, \ldots, \xi_{ij}]^T$

Define the directed distance from a point $z \in \mathbb{R}$ to a closed convex set $\Omega \subset \mathbb{R}$ as:

$$d(z, \Omega) = \begin{cases} 
z - \max \Omega, & \text{if } z > \max \Omega; \\
\z - \min \Omega, & \text{if } z < \min \Omega; \\
0, & \text{otherwise.}
\end{cases} \tag{24}$$

The $e_0$-subsystem Define $\gamma^e_0(0) = \mathcal{X}_{e_0} \circ \alpha_0^{-1}$. Then, from (18),

$$V_0 \geq \max \{\gamma^e_0(0), \mathcal{X}_{e_0}(|w_1|), \mathcal{X}_{e_0}(|d^m|)\} \tag{25}$$

The $e_1$-subsystem Write the $e_1$-subsystem as:

$$\dot{e}_1 = \ddot{e}_2 - e_2 + (\phi_1(e_0, e_1, w_1) + e_2) \tag{26}$$

Then, we have $\dot{e}_2 - e_2 = \phi_1(e_0, e_1, w_1)$ and $\psi_{e_0}^{e_2} = \psi_{e_1}^{e_2}$. Define $\alpha_0(s)$ s.t.

$$\alpha_0(s) = \frac{s}{\max_{\Omega}} \text{ for } s \in \mathbb{R}_+. \tag{27}$$

3.3 Decentralized Recursive Control Design

Denote $e_1 = y_i$. For each $1 \leq i \leq N$, the $i$th combined controller/observer subsystem is described by the following $[e^T_0, e_1, \xi_2, \ldots, \xi_{ni}]^T$-system:

$$\dot{e}_0 = A_0 e_0 + \phi_0(e_1, w_1, d^m) \tag{19}$$
$$\dot{e}_1 = \ddot{e}_2 + \phi_1(e_0, e_1, w_1) \tag{20}$$
$$\dot{\xi}_j = \ddot{\xi}_j + \phi_j(e_1, \xi_{j-1}, d^m), \quad 2 \leq j \leq ni - 1 \tag{21}$$
$$\dot{\xi}_{ni} = u_i + \phi_{ni}(e_1, \xi_{ni}, d^m) \tag{22}$$

where

$$\phi_1(e_0, e_1, w_1) = L_{e_0} e_1 + (x_{e_0} - L_{e_0} e_1 - \xi_2) + \Delta_1(e_1, w_1),$$
$$\phi_j(e_1, \xi_{j-1}, d^m) = L_{e_0} e_1 - L_{e_0} e_1 + L_{\xi_{j-1}}(\xi_{j-1} + L_{e_0} e_1) + (L_{\xi_{j-1}} - L_{e_0} e_1) d^m,$$
$$\phi_{ni}(e_1, \xi_{ni}, d^m) = -L_{\xi_{ni}}(\xi_{ni} + L_{e_0} e_1) - L_{\xi_{ni}} L_{e_0} e_1.$$

Notice that in the definition of $\phi_1(e_0, e_1, w_1)$ we used the fact that $x_{e_2} - L_{e_2} e_1 - \xi_2$ is the first element of vector $e_0$.
The e;j-subsystem (2 ≤ j ≤ ni) By convention, S_{j}(ei_{1}, \xi_{i_{1}}) := S_{i_{1}}(ei_{1}). When 3 ≤ j ≤ ni, for each 2 ≤ k ≤ j − 1, a set-valued map S_{k}(ei_{1}, \xi_{i_{k}}) is defined as

\begin{align*}
S_{k}(ei_{1}, \xi_{i_{k}}) &= \{ \kappa_{k}(\xi_{i_{k}} - \mu_{i_{k}}) : \mu_{i_{k}} \in S_{i_{k}}(ei_{1}, \xi_{i_{k}}(l_{i_{k}})) \}
\end{align*}

with \kappa_{k} continuously differentiable, odd, strictly decreasing and radially unbounded, and ei_{k+1} is defined as

\begin{align*}
ei_{k+1} &= d_{k}(\xi_{i_{k+1}}, S_{k}(ei_{1}, \xi_{i_{k}})).
\end{align*}

**Lemma 2.** Consider the [\xi_{i_{1}}^{T}, ei_{1}, \xi_{i_{2}}, ..., \xi_{in_{1}}^{T}]-system in (19)–(22). Given \xi_{i_{k}} and ei_{k+1} as designed in (27), (28), (31) and (32) for 1 ≤ k ≤ j − 1, then for any variable ei_{j+1}, when ei_{j} ≠ 0, the ei_{j+1}-subsystem can be represented as

\begin{align*}
ei_{j+1} &= \xi_{i_{j+1}} - ei_{j+1} + \phi_{j}^{i_{1}}(ei_{j+1}, w_{i_{j}}, d^{m}_{i_{j}}, \xi_{i_{j}})
\end{align*}

where

\begin{align*}
|\phi_{j+1}^{i_{1}}(ei_{j+1}, w_{i_{j}}, d^{m}_{i_{j}}, \xi_{i_{j}})| &\leq \psi_{j}^{i_{1}}(|ei_{j+1}|, w_{i_{j}}^{T}, d^{m}_{i_{j}}^{T})
\end{align*}

with \psi_{j_{1}}^{i_{1}} ∈ \mathcal{K}_{\infty} known. Specifically, \xi_{i_{1}}(ni_{1}+1) = u_{i}.

The proof of Lemma 2 is in Appendix B.

Define a set-valued map S_{j} as in (31) and ei_{j+1} as in (32) with k = j.

From the definition of ei_{j}, in the case of ei_{j} ≠ 0, for all p_{i_{j}} ∈ S_{j}(ei_{j}, \xi_{i_{j}}), we can observe that |\xi_{i_{j}} - ei_{j}| ≤ |ei_{j}| and \textit{sgn}(\xi_{i_{j}} - ei_{j}) = \textit{sgn}(ei_{j}) which implies \textit{sgn}(\xi_{i_{j}} - ei_{j}) = \textit{sgn}(ei_{j}) and thus \xi_{i_{j}} - ei_{j} = ei_{j} + (\xi_{i_{j}} - ei_{j}) = ei_{j} + \textit{sgn}(ei_{j})(\xi_{i_{j}} - ei_{j} - ei_{j}).

From the definition of ei_{j+1} in (32), there always exists a p_{i_{j+1}} satisfying p_{i_{j+1}} ∈ S_{j+1}(iei_{j+1}, \xi_{i_{j+1}}) such that \xi_{i_{j+1}} - ei_{j+1} = \kappa_{j}(\xi_{i_{j}} - ei_{j}) = \kappa_{j}(ei_{j} + \textit{sgn}(ei_{j})(\xi_{i_{j}} - ei_{j} - ei_{j})).

With Lemma 1, for any ei_{j} > 0, ei_{j} > 0, \gamma_{i_{j+1}}, ..., \gamma_{i_{j+1}}^{j}, \gamma_{i_{j+1}}^{j+1} \textit{X}_{ei_{j}}(\gamma_{i_{j+1}}^{j+1}), \textit{X}_{ei_{j}}(\gamma_{i_{j+1}}^{j+1}) ∈ \mathcal{X}_{\infty}, we can design a continuously differentiable, odd, strictly decreasing and radially unbounded \kappa_{j} such that the ei_{j+1}-subsystem is ISS with V_{j} satisfying

\begin{align*}
V_{j} &\geq \max_{k=0,...,j+1,i_{j+1}} \left\{ \phi_{j}^{i_{1}}(V_{k}^{i_{1}}), \textit{X}_{ei_{j}}^{j+1}(w_{i_{j}}), \textit{X}_{ei_{j}}^{j+1}(d^{m}_{i_{j}}), ei_{j} \right\}
\end{align*}

\begin{align*}
\Rightarrow V_{j}ei_{j+1} ≤ -i_{j}V_{j}.
\end{align*}

In the case of j = ni, we define ei_{(ni+1)} = 0 and V_{i_{(ni+1)}} = 0.

**Decentralized Controllers** All the ei_{j}-subsystems can be rendered ISS if we can find a u_{i} always satisfying u_{i} ∈ S_{in_{1}}(ei_{1}, \xi_{in_{1}}). This is achieved by defining

\begin{align*}
p_{i_{2}} &= \kappa_{i_{1}}(ei_{1} + d^{m}_{i_{1}})
p_{i_{j}} &= \kappa_{i_{j-1}}(\xi_{i_{j-1}} - p_{i_{j-1}}), \quad 3 ≤ j ≤ ni
u_{i} &= \kappa_{i_{ni}}(\xi_{in_{1}} - p_{in_{1}}).
\end{align*}

We can prove

\begin{align*}
d^{m}_{i_{1}} ≤ d_{i_{1}} \Rightarrow p_{i_{2}} ∈ S_{i_{1}}(ei_{1}) \Rightarrow \cdots \Rightarrow p_{in_{1}} ∈ S_{in_{1}}(ei_{1}, \xi_{in_{1}}) \Rightarrow \cdots \Rightarrow u_{i} ∈ S_{in_{1}}(ei_{1}, \xi_{in_{1}}).
\end{align*}

**3.4 Synthesis of the Subsystems**

For each 1 ≤ i ≤ N, denote \bar{e}_{i} = \bar{e}_{in_{i}}. With the recursive control design, the e_{bar}i-subsystem is an interconnection of ISS subsystems. We employ the cyclic-small-gain theorem to guarantee the ISS of the e_{bar}i-subsystem and to construct an ISS-Lyapunov function.

According to the recursive design, given the e_{bar}(i-1)-subsystem, by designing the set-valued map S_{j} for the ei_{j}-subsystem, we can assign the ISS gains \gamma_{i_{j+1}}^{j+1}(0 ≤ k ≤ j−1) such that

\begin{align*}
\gamma_{i_{j+1}}^{j+1} : \cdots : \gamma_{i_{j+1}}^{j+1} < Id, \quad 0 ≤ k ≤ j−1.
\end{align*}

Applying this reasoning repeatedly, we can guarantee (39) for all 1 ≤ j ≤ ni. In this way, the ei_i-subsystem satisfies the cyclic-small-gain condition in Jiang & Wang [2008].

With the method in Liu, Hill & Jiang [2009], we construct an ISS-Lyapunov function for the ei_i-subsystem as the maximum influence from V_{0}(ei_{1}, ..., V_{ni}(ei_{ni})) to V_{i_{1}}(ei_{1}):
the $\varepsilon_i$ in (44) arbitrarily small. By designing the $\sigma_{ij} \circ \chi_{ei}^m$'s ($0 \leq j \leq n_i, j \neq 1$) small enough, from the definitions of $\chi_{ei}^m$ and $\chi_{eij}^m$ (see (30)), we can achieve

$$\chi_{ei}^m(s) = \sigma_{i1} \circ \chi_{eij}^m(s) = \chi_{ei}^m(s) = \alpha_{\theta} \left( \frac{s}{c_{i1}} \right)$$

(45)

where $c_{i1}$ can be chosen to be any value satisfying $0 < c_{i1} < 1$.

3.5 Analysis of the Closed-loop Decentralized System

The closed-loop decentralized system is an interconnection of ISS $\tilde{e}_i$-subsystems ($1 \leq i \leq N$) with ISS-Lyapunov functions (44). As the discussion in Subsection 3.4 shows, we can design all the ISS gains $\chi_{eij}^m$'s ($1 \leq i, l \leq N, i \neq l$) arbitrarily small. Thus, $\chi_{eij}^m$'s ($1 \leq i, l \leq N, i \neq l$) can be tuned such that the closed-loop decentralized system satisfies the cyclic-small-gain condition. In this way, the ISS of the closed-loop decentralized system is achieved.

To analyze the effects of the disturbances to each the $\tilde{e}_i$-subsystem ($1 \leq i \leq N$), with the method in Liu, Hill & Jiang [2009], we construct an ISS-Lyapunov function of the interconnected system composed of the $\tilde{e}_i$-subsystems with $r \in RS(i) \subseteq \{1, \ldots, N\}$ as:

$$V_l(\tilde{e}_i) = \max_{r \in RS(i)} \{\rho_l(V_r(\tilde{e}_r))\}$$

(46)

where $\tilde{e}_r$ consists of all the $\tilde{e}_i$'s ($r \in RS(i)$), $\rho_l = 1d$, and the $\rho_l$'s ($r \in RS(i) \setminus \{i\}$) are compositions of $\chi_{eij}^m$'s ($r', r \in RS(i)$, $r \neq r'$) which are smooth on $(0, \infty)$ and slightly larger than the corresponding $\chi_{eij}^m$'s.

Correspondingly, we can represent the maximum influence from the disturbances to $V_l(\tilde{e}_i)$ as

$$\theta_l = \max_{r \in RS(i)} \{\rho_l \circ \chi_{eij}^m(\tilde{e}_r), \rho_l \circ \chi_{eij}^m(d_{m_i}), \rho_l(\tilde{e}_r)\}.$$  

(47)

Using the cyclic-small-gain theorem in Liu, Hill & Jiang [2009], we have

$$V_l(\tilde{e}_i) \geq \theta_l \Rightarrow V_l(\tilde{e}_r) \tilde{e}_i \leq -\alpha_l V_l(\tilde{e}_r)$$

(48)

with $\alpha_l$ positive definite, wherever $V_l(\tilde{e}_i)$ exists.

Note that, in (47), $\rho_l$ ($r \in RS(i) \setminus \{i\}$) can be designed arbitrarily small by designing the $\chi_{eij}^m$'s (and of course the $\chi_{eij}^m$'s) arbitrarily small, the $\chi_{eij}^m$ and the $e_i$ can be designed arbitrarily small, and the $\chi_{eij}^m$ can be designed to be $\alpha_l V_l(\tilde{e}_r)$. Thus, through an appropriate design, we arrive at

$$\theta_l = \rho_l \circ \alpha_l \left( \frac{d_{m_i}}{c_{i1}} \right) = \alpha_l \left( \frac{d_{m_i}}{c_{i1}} \right).$$

(49)

From (48) and (49), we can see $V_l(\tilde{e}_i)$ ultimately converges to within the region $V_l(\tilde{e}_i) \leq \alpha_l \left( \frac{d_{m_i}}{c_{i1}} \right)$. Using the definitions of $V_l(\tilde{e}_i)$, $V_l(\tilde{e}_i)$, and $\alpha_l(\tilde{e}_i)$ (see (23), (40) and (46)), we have $\alpha_l(\tilde{e}_i) \leq V_l(\tilde{e}_i) \leq V_l(\tilde{e}_i)$, which implies $y_i := e_{i1}$ ultimately converges to within the region $|y_i| \leq d_{m_i}/c_{i1}$. By choosing $c_{i1}$ arbitrarily close to one, the output $y_i$ can be driven arbitrarily close to the region $|y_i| \leq d_{m_i}$.

The main result of the paper is summarized in Theorem 1.

Theorem 1. Consider the large-scale system (1)–(5) with Assumptions 1 and 2 satisfied. The decentralized controller (36)–(38) with the decentralized reduced-order observer (12)–(13) can steer each output $y_i$ ($1 \leq i \leq N$) arbitrarily close to the region $|y_i| \leq d_{m_i}$.

4. CONCLUSIONS

In this paper, we have designed a new class of decentralized controllers with robustness to sensor noise for a class of large-scale nonlinear systems composed of uncertain output-feedback subsystems by using set-valued maps and the cyclic-small-gain theorem. The outputs of the subsystems can be driven arbitrarily close to the levels of their corresponding measurement noise. More complex large-scale systems with sensor noise, such as interconnected strict-feedback nonlinear systems, will be further explored.

REFERENCES


Appendix A. PROOF OF LEMMA 1

With (9) satisfied, one can find $\psi_1^n, \psi_2^n, \ldots, \psi_{n-2}^n \in C_{\infty}$ such that

$$|\phi(\eta, \omega_1, \ldots, \omega_{n-2})| \leq \psi_1^n(|\eta|) + \sum_{k=1}^{n-2} \psi_k^n(\omega_k).$$

(A.1)

Because $\psi_1^n(s) + \sum_{k=1}^{n-2} \psi_k^n \circ (\chi_k^n)^{-1} \circ \alpha_v(s) + \frac{s^2}{2}$ is a $C_{\infty}$ function of $s$, from Lemma 1 in Jiang & Mareels [1997], for any $0 < c < 1$, $\epsilon > 0$, one can find a $v : \mathbb{R}^+ \to \mathbb{R}_+$ which is positive, nondecreasing and continuously differentiable on $(0, \infty)$, and satisfies

$$(1-c)v((1-c)s)s \geq \psi_1^n(s) + \sum_{k=1}^{n-2} \psi_k^n \circ (\chi_k^n)^{-1} \circ \alpha_v(s) + \frac{s^2}{2}$$

(A.2)

for all $s \geq \sqrt{2\epsilon}$. With the $v$ satisfying (A.2), define $\kappa(a) = -v(|a|)a$ for $a \in \mathbb{R}$. Then, $\kappa$ is continuously differentiable, odd, strictly decreasing and radially unbounded.

Recall $V_q(\eta) = \alpha_v(|\eta|) = \frac{1}{2} \eta^2$. Consider the case of $V_q(\eta) \geq \max_{k=1, \ldots, n-2} \{ \chi_k^n (|\omega_k|), \alpha_v(\omega_k), \epsilon \}$. In this case, we have

$$|\omega_k| \leq (\chi_k^n)^{-1} \circ \alpha_v(|\eta|), \quad 1 \leq k \leq n-2$$

$$|\omega_{n-1}| \leq c\alpha_v(V_q(\eta)) = c|\eta|$$

$$|\eta| \geq \sqrt{2\epsilon}$$

$$\eta \neq 0$$

(A.3) (A.4) (A.5) (A.6)

The $K$ satisfying (10) can be represented as $K = K(\eta') = -v(|\eta'|)|\eta'|$ with $\eta' = \eta + \text{sgn}(\eta)\alpha_v + \delta\omega_{n-1}$ satisfying $|\delta| \leq 1$. With $0 < c < 1$ and (A.4), when $\eta \neq 0$, we have $\text{sgn}(\eta') = \text{sgn}(\eta), |\eta'| \geq |\eta| + \delta\omega_{n-1} \geq (1-c)|\eta|$ and $v(|\eta'|)|\eta'| \geq (1-c)v((1-c)|\eta|)|\eta|$. Using (A.1), (A.2)–(A.5) and the discussion above, for any $K$ satisfying (10), we obtain

$$\nabla V_q(\eta) \phi(\eta, \omega_1, \ldots, \omega_{n-2} + \tilde{\kappa})$$

$= \eta \phi(\eta, \omega_1, \ldots, \omega_{n-2}) - v(|\eta'|)|\eta'|$ |\eta|

$\leq |\eta|\phi(\eta, \omega_1, \ldots, \omega_{n-2}) - |\eta|v(|\eta'|)|\eta'|$

$\leq |\eta|$$

$$\leq |\eta|\left( |\psi_1^n(|\eta|) + \sum_{k=1}^{n-2} \psi_k^n(\omega_k) - (1-c)v((1-c)|\eta|)|\eta| \right)$

$\leq |\eta|\left( |\psi_1^n(|\eta|) + \sum_{k=1}^{n-2} \psi_k^n(\omega_k) - (1-c)v((1-c)|\eta|)|\eta| \right)$

$\leq \frac{1}{2} |\eta|^2 - v(\eta).$ (A.7)

This ends the proof.

Appendix B. PROOF OF LEMMA 2

We simply use $S_k$ instead of $S_k(\xi_1, \xi_2, \xi_3)$ for $1 \leq k \leq j-1$. We only consider the case of $e_{ij} > 0$. The other cases can be treated similarly. Consider the definition of $S_2$ in (27) and the iteration-type definitions of $S_k$’s in (31). The strictly decreasing properties of the $\kappa_k$’s imply

$$\max S_1 = \kappa_1(\epsilon_1 - d_{\epsilon_1}^m)$$

$$\max S_k = \kappa_k(\xi_k - \max S_{k-1} - 1), \quad 2 \leq k \leq j-1.$$ (B.2)

The continuous differentiability of the $\kappa_k$’s implies the continuous differentiability of $\max S_1$ with respect to $\epsilon_1$ and the continuous differentiability of $\max S_k$ with respect to $\xi_k$ and $\max S_{k-1}$ for $2 \leq k \leq j-1$. By the chain rule, we can see $\max S_{j-1}$ is continuously differentiable with respect to $[\epsilon_{j-1}, \xi_{j-1}, \xi_{j-1}]^T$ and thus $\max S_1$ is continuous with respect to $[\epsilon_1, \xi_1]^T$. In the case of $e_{ij} > 0$, the dynamics of $e_{ij}$ can be derived as

$$e_{ij} = \xi_{i+j} - V \max S_{j-1}(\xi_1, \xi_{j-1}, \xi_{j-1})^T$$

$$= -\xi_{i+j} + e_{i+j} + \phi_{ij}(\xi_1, \xi_{j+1}, \xi_{j+1})$$

$$- V \max S_{j-1}(\xi_1, \xi_{j-1}, \xi_{j-1})^T$$

(B.3)

Specifically, $\xi_{i+1}$ is obtained by solving (20), (21) and (22) to establish the last equality. Also from (20), (21) and (22), one sees that $|\phi_{ij}(\epsilon_1, \xi_{j+1}, d_{\epsilon_1}^m)|$ is bounded by a $C_{\infty}$ function of $|\epsilon_1| + |\xi_{j+1}| + |d_{\epsilon_1}^m|$, and $|\phi_{ij}(\xi_1, \xi_{j+1}, d_{\epsilon_1}^m)|$ is bounded by a $C_{\infty}$ function of $|\epsilon_1| + |\xi_{j+1}| + |d_{\epsilon_1}^m|$. Thus, we can conclude that $|\phi_{ij}(\xi_{i+j}, d_{\epsilon_1}^m)|$ is bounded by a $C_{\infty}$ function of $|\epsilon_1| + |\xi_{j+1}| + |d_{\epsilon_1}^m|$. To prove (34), we show that for each $1 \leq k \leq j-1$, $\xi_k$ is bounded by a $C_{\infty}$ function of $|\epsilon_1| + |\xi_{k+1}| + |d_{\epsilon_1}^m|$. From the definitions of $e_{ij}$ in (28) and $e_{i+j+1}$ for $2 \leq k \leq j-1$ in (32), we have $\xi_{i+k+1} - e_{i+j+1} < S_k$ and thus

$$\xi_k < \max |\max S_1, |\max S_k| + |e_{i+k+1}|.$$ (B.4)

Define $\kappa_k^m(\eta) = |\kappa_k(\eta)|$ for $\eta \in \mathbb{R}$. Since $\kappa_k^m$ is odd, strictly decreasing and radially unbounded, $\kappa_k^m \in C_{\infty}$ From (B.1) and (B.2), we have

$$\max S_1 \leq \kappa_1^m(|\epsilon_1| + d_{\epsilon_1}^m)$$

$$\max S_k \leq \kappa_k^m(|\xi_k - e_{i+k+1} - \max S_{k-1}|)$$

$$\leq \kappa_k^m(|\max S_{k-1}| + |\max S_{k-1}| + |e_{i+k+1}|),$$ (B.5) (B.6)

Similarly, we can also obtain

$$\min S_1 \leq \kappa_1^m(|\epsilon_1| + d_{\epsilon_1}^m)$$

$$\min S_k \leq \kappa_k^m(|\max S_{k-1}| + |\max S_{k-1}| + |e_{i+k+1}|),$$ (B.7) (B.8)

Using (B.4) along with a repeated application of (B.5)–(B.8), we can prove that for each $1 \leq k \leq j-1$, $\xi_{j+k-1}$ is bounded by a $C_{\infty}$ function of $|\epsilon_1| + d_{\epsilon_1}^m|$. This ends the proof.