Feedback Tracking Control of Continuous Reheating Furnaces

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Abstract: A Lyapunov-based MIMO state feedback controller is developed for slab temperatures in a continuous, fuel-fired reheating furnace. Following an early lumping approach, the computationally simple tracking controller is designed for a nonlinear, switched dynamic model that captures both conductive and radiative heat transfer. The controller modifies reference trajectories of furnace temperatures and is part of a cascade control scheme. Given some nonrestrictive conditions, exponential stability is ensured, even under non-steady state operating conditions. The capabilities of the controller are demonstrated by means of an example problem.

Keywords: Feedback control, Nonlinear systems, MIMO, Two-degrees-of-freedom control, Lyapunov methods, Exponentially stable, Steel industry, Slab reheating furnace.

1. INTRODUCTION

Continuous fuel-fired reheating furnaces are used in the steel industry for heating up steel products, mainly slabs, before they undergo hot working processes. The term continuous means that the slabs are continuously conveyed through the furnace while being reheated. However, the slab movement itself may be discontinuous as in the pusher-type slab furnace shown in Fig. 1. Inside the furnace, the slabs slide on steam-cooled skids so that they can be reheated from both the bottom and the top. The furnace represents a switched, nonlinear, distributed parameter system with multiple inputs, multiple outputs, uncertain physical interdependencies, and sometimes unknown future operating conditions. Hence, control design can be a challenging task.

Chen et al. (2008); Ezure et al. (1997); Hollander and Zuurbier (1982); Steinboeck et al. (2011b) proposed open-loop temperature control schemes for continuous slab reheating furnaces. In most cases, the controller defines reference trajectories of furnace temperatures, which are then feedback controlled by some subordinate controllers. However, these open-loop approaches may be inadequate if the plant exhibits significant uncertainties.

Fig. 1. Continuous slab reheating furnace (not to scale, symbols defined in Sec. 2).

1.1 Control Task

Stimulated by economic reasons and increasing demands in terms of product quality and diversity, furnace control is characterized by the following key performance indicators: energy consumption, reheating quality, processing costs, product throughput, scale formation, and decarburization. The discontinuous nature of the slab reheating process (intermittent movement of slabs, variations in terms of size, steel grade, material properties, initial temperature, desired final temperature, scheduled reheating time) renders tailored control solutions indispensable (cf. Carpenter and Proctor (1987); Hollander and Zuurbier (1982); Roth et al. (1986); Shenvar (1994)). The control performance may be limited by temperature constraints, bounds on control variables (fuel and air supply of the burners), and unforeseen production halts or delays, maybe caused by upstream or downstream process steps.

1.2 Existing Furnace Control Schemes

Dahn and Klima (2002); Doss et al. (1992); Ezure et al. (1997); Facco et al. (1990); Hommer et al. (2001); Knoop and Moreno Pérez (1994); Leden (1986); Marino et al. (2004); Pedersen and Wittenmark (1998); Shenvar (1994); Staalman (2004); Vode et al. (2008); Wang et al. (2004); Westdorp (1988); Yoshitani et al. (1994) reported on feedback control for slab temperatures, which is usually realized as a cascade control scheme. Especially for non-steady state operating conditions, trajectory tracking control may be required. Doss et al. (1992); Leden (1986); Rixin and Baolin (1992); Staalman (2004); Veslocki et al. (1986);...
Vode et al. (2008) described two-degrees-of-freedom control of reheating furnaces. In most cases, the feedback law is based on control errors in terms of slab temperatures. Pedersen and Wittenmark (1998) developed a stabilizing nonlinear feedback control law for a single furnace zone.

The control task may be complicated by the fact that the slab temperature profile cannot be measured. Therefore, estimation algorithms are required for monitoring the slab temperatures (cf. Fitzgerald and Sheridan (1972); Loden (1986); Wick and Köster (1999); Wild et al. (2007); Wild (2010)). Temperature feedback may alternatively be obtained from measurements of the slab surface temperatures (Doss et al. (1992); Honner et al. (2001); Marino et al. (2004); Roth et al. (1986); Shenkar (1994); Staalman (2004); Veslocki et al. (1986); Wang et al. (2004)), e.g., by means of radiation pyrometry, but the achievable accuracy is uncertain.

1.3 Motivation and Method

Many published furnace temperature controllers are designed for steady-state operation or are based on relatively simple mathematical models, e.g., models that neglect the dynamic interaction between furnace zones. Moreover, most publications do not feature a systematic proof of stability. This work aims at filling this gap with a Lyapunov-based state feedback controller.

The considered mathematical model, which is briefly summarized in Sec. 2, accounts for the transient, nonlinear, switched MIMO behavior of the system. A cascade control system with two-degrees-of-freedom control is outlined in Sec. 3, and Sec. 4 presents the straightforward design of a feedback controller for slab temperatures. The developed control law can cope with time-dependent reference trajectories and captures the thermal interaction within the furnace interior. The positive effect of the feedback controller is demonstrated by means of an example problem in Sec. 5.

2. MATHEMATICAL MODEL

Wild et al. (2009); Wild (2010) developed a comprehensive analytical model of the considered slab reheating furnace. It is based on the energy balance taking into account thermal radiation, bulk flow of gases, heat input from the burners, heat losses through furnace walls, the skids, and the funnel, as well as the slabs, which also represent heat sinks. In the real furnace, the model is implemented in a state observer for estimating slab temperatures. In this work, it will be utilized as a simulation environment for testing the suggested controller. Since the model of Wild et al. (2009) is mathematically too complex as to be used for controller design, a simplified mathematical model published by Steinboeck et al. (2010) will be used in this paper. The model captures thermal radiation in the furnace and heat conduction within the slabs.

Consider that each slab is uniquely identified by an index \( j \in \mathbb{N} \) and that all slabs \( j \in J = \{j_{\text{start}}, \ldots, j_{\text{end}}\} \) are currently reheated in the furnace. For a concise notation, \( N_s = |J| \) and the range \( J = j_{\text{start}}, \ldots, j_{\text{end}} \) will be used. Thus, \( j = J \) is tantamount to \( j = j_{\text{start}}, \ldots, j_{\text{end}} \). The slab \( j \) is inside the furnace during the period \( [t_{j,0}, t_{j,1}] \), and times when the row of slabs is pushed forward are summarized in the series \( \{t_l\} \) with \( l \in \mathbb{N} \). Whenever slabs enter or leave the furnace, \( J \), \( J_s \), and \( N_s \) are updated.

In the coordinate system shown in Fig. 1, the slab \( j \) has the geometric dimensions \( D_j \) and \( W_j \) along the directions \( y \) and \( z \), respectively. The local coordinate \( y \) is \( 0 \) at the middle-plane of the respective slab. The furnace volume is separated into \( N_z = 5 \) zones below the slabs and \( N_Z = 5 \) zones above the slabs.

2.1 Heat Conduction Inside the Slabs

The temperature profile \( T_j(y,t) \) inside the slab \( j \) is described by the 1-dimensional heat conduction problem

\[
\rho_j c_j \frac{\partial T_j}{\partial t} = \lambda_j \frac{\partial^2 T_j}{\partial y^2}, \quad y \in (-D_j/2, D_j/2), \quad t > t_{j,0}
\]

with boundary conditions \( q_j^\top(t) = \mp\lambda_j \partial T_j/\partial y|_{y=-D_j/2} \) and the initial condition \( T_j(y,t_{j,0}) = T_{j,0}(y) \). The mass density \( \rho_j \), the specific heat capacity \( c_j \), and the thermal conductivity \( \lambda_j \) are considered as constant. Here, \( q_j^\top(t) \) and \( q_j^\top(t) \) are the heat fluxes into the bottom and the top surface of the slab. Any quantities associated with \( y \) and the top half of the furnace are labeled by the superscripts \( - \) and \( + \), respectively.

Steinboeck et al. (2009, 2010) solved the heat conduction problem using the Galerkin method with the trial functions \( h_{j,1}(y) = 1 \), \( h_{j,2}(y) = 2y/D_j \), and \( h_{j,3}(y) = (2y/D_j)^2 - 1/3 \). The temperature profile inside the slabs is approximated as \( T_j(y,t) = [h_{j,1}(y), h_{j,2}(y), h_{j,3}(y)]x_j(t) \), where the vector \( x_j(t) = [x_{j,1}(t), x_{j,2}(t), x_{j,3}(t)]^T \) accommodates the Galerkin coefficients. The state vector

\[
z_j(t) = \begin{bmatrix} 1 & -1 & 2/3 \\ 1 & 1 & 2/3 \\ 1 & 0 & 0 \end{bmatrix} x_j(t),
\]

which contains the mean temperature \( T_j(t) \), the bottom surface temperature \( T_j(-D_j/2, t) \), and the top surface temperature \( T_j(D_j/2, t) \), respectively, represents the temperature state of the slab \( j \). Choosing the slab surface temperatures as state variables is convenient because they are needed for analyzing the radiative heat exchange. The corresponding initial value problem is obtained as

\[
\dot{z}_j(t) = [a_j \ z_j(t) + b_j^\top q^{-\top}(t) + b_j^\top q^+(t)]^T t > t_{j,0}
\]

with

\[
a_j = \frac{12\lambda_j}{\rho_j c_j D_j^3} \begin{bmatrix} 0 & 0 & 0 \\ -5 & 3 & 2 \\ -5 & 2 & 3 \end{bmatrix}, \quad b_j^\top = \frac{1}{\rho_j c_j D_j} \begin{bmatrix} 1 \\ 6 \pm 3 \\ 6 + 3 \end{bmatrix}
\]

(cf. Steinboeck et al. (2010)) and the initial value \( z_j(t_{j,0}) = z_{j,0} = [T_j(t_{j,0}), T_j(-D_j/2,t_{j,0}), T_j(D_j/2,t_{j,0})]^T \). Steinboeck et al. (2009, 2010) proposed a similar but nonlinear model that captures the influence of temperature-dependent parameters \( c_j \) and \( \lambda_j \).

Summarizing the states and the heat inputs of all slabs \( j \in J \) in the vectors \( z(t) = [z_{j,0}^T, \ldots, z_{j,0}^T]^T \) and \( q^\top(t) = [q_{j,0}^\top(t), \ldots, q_{j,0}^\top(t)]^T \), respectively, yields the sparse system

\[
z(t) = Az(t) + B^- q^-(t) + B^+ q^+(t)
\]

for the whole furnace (cf. Steinboeck et al. (2010)) with
A = \[\delta_{i,j}a_{j}\]_{i=1,\ldots,J,j=1,\ldots,J}, \quad B^\mp = \[\delta_{i,j}b^\mp_{j}\]_{i=1,\ldots,J,j=1,\ldots,J},
and the Kronecker delta \(\delta_{i,j}\). The components and sizes of \(z(t), A, B^\mp,\) and \(q^\mp(t)\) may change at the times \(t^*_i\).

### 2.2 Radiation Heat Transfer in the Furnace

The dynamic subsystems describing the slabs are coupled by thermal radiation inside the furnace. Steinboeck et al. (2010) used the assumption of gray-body radiation in a non-participating gaseous medium and the net radiation method (cf. Baehr and Stephan (2006)) for analyzing the radiation energy balance. The furnace zone temperatures \(T^\mp_{z,j}(t)\) \(i \in \{1,\ldots,N_z\}\), which represent a combination of local flue gas temperatures and surface temperatures of the furnace walls, are considered to be homogeneous distributed within the furnace zone \(i\). They are summarized in the vector \(T^\mp_{z}(t) = [T^\mp_{z,1}(t),\ldots,T^\mp_{z,N_z}(t)]^T\), which serves as model input. Steinboeck et al. (2010) derived the simple, static radiative heat exchange model

\[
q^\mp(t) = P^\mp_{z,s}(t) [T^\mp_{z,s}(t)]^4 + P^\mp_{s,s}(t) [M^\mp(z(t))]^4 \quad (2)
\]

with the matrix \(M^\mp = [\delta_{i,j}+1/2]_{i=1,\ldots,N_s,j=1,\ldots,3N}\). The model furnishes the net heat fluxes \(q^\mp(t)\) separately for the bottom and the top half of the furnace. The 4th powers in (2) are a consequence of the Stefan-Boltzmann law (cf. Baehr and Stephan (2006)) and have to be applied to each component of the respective vector.

The matrices \(P^\mp_{z,s}(t)\) and \(P^\mp_{s,s}(t)\) depend on the geometry and the radiative properties of the slab surfaces and the furnace walls. Their computation is particularly simple if a 2-dimensional configuration \((yz\text{-plane})\) is assumed (cf. Steinboeck et al. (2010) for a justification of this assumption). Because the slabs change their position, \(P^\mp_{z,s}(t)\) and \(P^\mp_{s,s}(t)\) are piecewise constant with changes occurring only at the times \(t^*_i\). In the sequel, the argument \(t\) is omitted whenever confusion is improbable.

Consider that \(S_j\) is the base area of slab \(j\). As a consequence of the reciprocity relations of radiation heat transfer (cf. Baehr and Stephan (2006)), \(\text{diag}([S_{j,j=1,\ldots,J}] P^\mp_{z,s})\) is symmetric. By analyzing the monotonicity properties of (2), Steinboeck et al. (2010) showed why the proposed radiation model is in line with the second law of thermodynamics. They also demonstrated that \(P^\mp_{z,s} = [P^\mp_{s,s}]\) is a Hurwitz matrix — a fact that will be utilized for controller design.

### 2.3 Assembled Model

Eqs. (1) and (2) constitute a nonlinear state-space model. Steinboeck et al. (2010) reported that this model is exponentially stable if \(P^\mp_{s,s} = P^\mp_{z,s}\) and a rather weak condition on the relation between \(\rho_j, c_j, \lambda_j,\) and \(D_j\) are satisfied. Therefore, the furnace system can be open-loop controlled (cf. Steinboeck et al. (2011a,b)). However, it is expected that the control performance (mainly the reheating quality of the slabs) can be improved by feedback control, as proposed in the following.

### 3. CASCADE TEMPERATURE CONTROL

The primary objective of controlling a slab reheating furnace is that the slabs reach their desired final temperature state \(z_j(t_{j,end})\) upon being discharged from the furnace. Moreover, there are several physical and safety limits on both the slab and the furnace zone temperatures. In view of the complexity of the control task, a cascade control scheme as outlined in Fig. 2 seems appropriate.

In accordance with the requirements of other process steps like the rolling mill, the superordinate plant controller defines the order, the movement, and the desired final temperatures \(T_{j,end}\) of slabs. The outer control loop generates reference signals \(\tilde{T}^\mp_{z,j}(t)\) for the zone temperatures, which are feedback controlled by individual zone controllers (inner loop). In the inner loop, PI controllers regulate the combustion air and fuel supply to the burners. Since the furnace zone temperatures \(T^\mp_{z,j}(t)\) exhibit a fast response characteristic and are measured by thermocouples, they are a good choice for interfacing between the two control loops. As usual for cascade control, the assumption \(T^\mp_{z,j}(t) = \tilde{T}^\mp_{z,j}(t)\), i.e., an ideal inner loop, is made when designing hierarchically higher controllers.

The outer control loop is responsible for the slab temperatures and features a two-degrees-of-freedom control structure. Its control task may be demanding because of the non-linear, switched dynamics of the system and the fact that sometimes more than 30 slabs are concurrently reheated, while the number of control inputs is just \(2N_z = 10\). The feedforward block defines reference trajectories \(\tilde{z}_j(t)\) \((j = 1,\ldots,N^s)\) with \(N_s \ll N^s < \infty\) and \(\tilde{T}^\mp_{z,j}(t)\). The block can be realized in form of an iterative planning and optimization algorithm or dynamic optimization as proposed by Steinboeck et al. (2011a,b). Neither the feedback controller nor the inner control loop will be further discussed in this work. It is therefore assumed that useful reference signals \(\tilde{z}_j(t)\) and \(\tilde{T}^\mp_{z,j}(t)\), which are solutions of (1) and (2) with the initial values \(z_{j,0}\) \((j = 1,\ldots,N^s)\), are available.

The feedback controller aims at minimizing the deviation \(e(t) = z(t) - \tilde{z}(t)\). The corresponding control law will be derived in the following section. Since slab temperatures can normally not be measured in the furnace, they need to be estimated, e. g., by a state observer. In the considered furnace control system, an extended Kalman filter developed by Wild et al. (2007) is used for this purpose.

### 4. FEEDBACK CONTROLLER

An exponentially stabilizing control law for the block feedback controller in Fig. 2 is developed. Its control performance in terms of the norm ||\(e(t)\)|| will turn out to be superior to that of pure open-loop control.
4.1 Furnace System with Immobile Slabs

Consider a situation where the slabs do not move, e.g., during the interval \( [t_{s}, t_{s+1}] \), which implies that \( P_{r}^{\top} \) and \( P_{s}^{\top} \) are constant. Moreover, the following proposition requires \( P_{s}^{\top} = P_{s}^{*} \). This condition is approximately satisfied by the considered system because the furnace geometry is almost symmetric with respect to the mid plane of the slabs.

**Proposition 1.** Given that the slabs are immobile and that \( P_{s}^{*} = P_{s} \) as well as \( \sum_{j=0}^{l} P_{s,j} d_{j} < -2P_{r}^{\top} d_{i} \forall i \in J \) with \( d_{j} = \lambda_{j}/(\rho_{j}c_{j}D_{j}^{2}) \forall j \in J \), the control law (3) ensures uniform stability of the system.

Proof. Consider the error dynamics

\[
\frac{d}{dt}(\mathbf{z}^{\top}(t)) = (A + B^{T}P_{r}^{\top}(\mathbf{T}_{r}^{\top} + d_{s})^{4}(G - M - P_{r}^{\top}(\mathbf{T}_{r}^{\top} + d_{s})M + s^{4} - z^{4})e + (B^{T}P_{s}^{\top} - M + P_{r}^{\top}(\mathbf{T}_{r}^{\top} + d_{s})^{4}M + s^{4} - z^{4}),
\]

where \( \mathbf{z} \) is a small perturbation of \( \mathbf{z} \) and \( G \) is a user-defined gain vector \( G^{\top} \in (R^{+})^{N_{s}} \). The Lyapunov function candidate

\[
V(e) = e^{\top}P_{e}e
\]

with the positive definite matrices

\[
P_{s,j} = \begin{bmatrix} 36 & -3 & -3 \\ -3 & 4 & -1 \\ -3 & -1 & 4 \end{bmatrix}, \quad P_{r} = \begin{bmatrix} \mathbf{P}_{r}^{\top} \end{bmatrix}_{i,j} = \begin{bmatrix} \mathbf{P}_{r}^{\top} \end{bmatrix}_{i,j} - \text{diag}[\rho_{j}c_{j}D_{j}^{2}]^{4}M^{4} - \text{diag}[\rho_{j}c_{j}D_{j}^{2}]^{4}M^{4} + \mathbf{P}_{r}^{\top} - M^{4} + \mathbf{P}_{r}^{\top}M^{4} + s^{4} - z^{4}.
\]

Thus, the control law (3) ensures uniform stability of the system.

**Proposition 2.** The assumptions of Proposition 1 are adopted but for a furnace with moving slabs. Given that \( t_{i+1} > t_{i} + \Delta t_{\text{min},l} \forall l \in N \) with \( \Delta t_{\text{min},l} = k_{2}/k_{1} \), the control law (3) ensures uniform stability of the system.

so that \( \bar{V}(e) \) is negative definite (cf. Steinboeck et al. (2010)). Since \( p \) is negative semidefinite, \( PA + A^{T}P \) is negative semidefinite if

\[
[P_{r,j}(d_{j} + d_{s})]_{i,j} = \mathbf{P}_{r}^{\top} - M^{4} + \mathbf{P}_{r}^{\top}M^{4} + s^{4} - z^{4},
\]

and \( \bar{V}(e) \) is negative definite because the furnace system is exponentially stable (cf. Vidyasagar (1992)) with respect to the equilibrium \( e(t) = 0 \forall t \). Given some initial error \( e(t_{0}) \), the control error \( e(t) \) will decrease according to

\[
\frac{\|e(t)\|^{2}}{\|e(t_{0})\|^{2}} = \exp(-\delta t_{0}(k_{1}/k_{2})^{2}k_{1}/k_{2} \forall t \geq t_{0} \text{ with } k_{1} = \mu_{\min}(P), k_{2} = \mu_{\max}(P),
\]

where \( \mu_{\min}(\cdot) \) and \( \mu_{\max}(\cdot) \) are the minimum and maximum eigenvalue of the respective matrix.

Note that the period

\[
\Delta t_{\text{min}} = k_{2}/k_{1} \ln \left( k_{2}/k_{1} \right)
\]

is sufficient to ensure

\[
\|e(t)\|^{2} \leq \|e(t_{0})\|^{2} \forall t \geq t_{0} + \Delta t_{\text{min}},
\]

Clearly, if \( g^{\top} = 0 \), i.e., in case of open-loop control, \( k_{3} \) is smaller, implying that the control error \( e(t) \) decreases slower.

4.2 Furnace System with Moving Slabs

Normal furnace operation requires that slabs are regularly pushed forward. Because of sparse and uncertain knowledge about ambient conditions before and after the furnace, Steinboeck et al. (2010) suggested \( \tilde{z}_{j}(t) = \tilde{z}_{j}(t_{0}) = 0 \forall j \notin J \). Therefore, the furnace system with moving slabs cannot be asymptotically stable.

Consider that the reheating process is to be analyzed for a time interval where \( N^{s} = N \approx N^{s} < \infty \) slabs are reheated. Despite \( N^{s} < \infty \), the considered interval may extend to infinity because the slab movement can be stopped at some distant point in the future. The control errors in terms of the slabs \( j \in \{1, \ldots, N^{s}\} \) are summarized in the vector \( e^{s}(t) = [e_{1}^{s}(t), \ldots, e_{N}^{s}(t)]^{\top} \).

**Proposition 2.** The assumptions of Proposition 1 are adopted but for a furnace with moving slabs. Given that \( t_{i+1} > t_{i} + \Delta t_{\text{min},l} \forall l \in N \) with \( \Delta t_{\text{min},l} = k_{2}/k_{1} \), the control law (3) ensures uniform stability of the system.
Proof. It follows from the Lyapunov function candidate \( V_s(e^s) = \|e^s\|^2 \) and (6) that \( V_s(e^s(t_{l+1})) \leq V_s(e^s(t_l)) \). Consequently, the discrete-time error system with the state trajectory \( e^s(t_l) \) is uniformly stable (cf. Vidyasagar (1992)). Moreover, the continuous-time error \( e^s(t) \) remains finite within each interval \( [t_l, t_{l+1}] \). □

5. EXAMPLE PROBLEM

The feedback controller is tested in a simulation environment using the validated mathematical model presented by Wild et al. (2009); Wild (2010) and an emulator of the inner control loop from Fig. 2. The considered furnace is 35 m long and has a nominal throughput of 280 t/h.

5.1 Problem Formulation

In this scenario, the furnace contains \( N_z = 18 \) slabs. All slabs are \( D_y = 400 \) mm thick and stay in the furnace for 5 h, i.e., \( t_{j,1} = t_{j,0} + 5 \) h. The corresponding path-time diagram is shown in Fig. 3.

The slab \( j \) should reach the homogeneously distributed final temperature \( T_{j,\text{end}} \). The final slab temperature should not fall below \( T_{j,\text{end, min}} = T_{j,\text{end}} - 15 \) K and should not exceed \( T_{j,\text{end, max}} = T_{j,\text{end}} + 15 \) K. For the representative slab \( j = 10 \), some additional temperature constraints are indicated by gray bars in Fig. 4.

5.2 Results

Reference trajectories \( z(t) \) and \( T^z_j(t) \), which obey relevant constraints, have been generated by dynamic optimization (see, for instance, Steinboeck et al. (2011a)). Open-loop control is used in a first simulation run, i.e., the reference trajectories are directly applied to the simulation model. The resulting minimum, mean, and maximum temperatures of the slab \( j = 10 \) are shown as dashed lines in Fig. 4. Moreover, the resulting final temperature profiles \( T_j(y, t_{j,1}) \) of some slabs are indicated by circles in Fig. 5. With open-loop control based on an inaccurate model (intentionally degraded) most slabs do not reach their temperature goals and violate some constraints.

Therefore, feedback tracking control is added in a second simulation run. The gain vector \( g^z \), which is critical to the performance of the controller, has been empirically found. For the period \( [t_{j,0}, t_{j,1}] \), the (predominantly positive) correcting values \( T^z_j(t) - T^z_j(t) \) added by the feedback controller are given in Fig. 6. The respective zone numbers are given on the right-hand side of Fig. 6. Zone 1 is omitted because there are no burners, i.e., this zone is not controllable. The correcting values \( T^z_j(t) - T^z_j(t) \) for the bottom half of the furnace are similar.