Abstract: This paper presents a jitter control design for a platform-mounted laser. The controller reduces the transmission of platform jitter, modeled as a multi-tonal signal plus broadband noise, to the laser beam. The design utilizes the well-known \( H_\infty \) loop shaping design procedure to ensure a balance between robust stability requirements and disturbance rejection specifications. Numerical simulation demonstrated at least a 20 dB (or, equivalently, > 90%) reduction at the tonal frequencies. Comparison to a previously implemented controller provides verification of a successful implementation of the proposed controller.

1. INTRODUCTION

The rejection of disturbances, such as vibration or jitter, to a laser beam presents a fundamental challenge to applications requiring laser accuracy such as communication between satellites in space [Skormin 1997] or between devices near the ground [Sergeyev 2010], osteotomy (bone cutting) [Busack 2010], and directed energy weapons [Beerer 2009, Orzechowski 2008]. For the current project, an experimental platform, shown in Fig. 1, provides a test bed for examining the effect of platform jitter on the laser’s beam position at an external target [Barton 2008]. This paper summarizes the development of a controller to reduce the transmission of platform jitter to the laser beam and, in turn, to maximize the energy density (i.e., energy per unit of surface area) of the beam delivered to a target.

The proposed controller development utilizes \( H_\infty \) controller design methods due to their inherent robustness. Specifically, the \( H_\infty \) loop shaping design procedure of McFarlane and Glover allows the inclusion of frequency-domain performance specifications and provides a straightforward measure of the compatibility of these specifications with the robust stability requirements [McFarlane 1992]. For jitter/vibration rejection, frequency-domain design methods provide an intuitive approach to the controller design [O’Brien 2000]. Recently, a group from UCLA presented an adaptive, frequency-domain jitter control design using \( L_2 \) techniques [Perez 2009]. \( L_2 \) techniques provide excellent performance but poor robustness [Doyle 1981] and the adaptive nature of their controller mitigates the lack of robustness.

The remainder of the paper is organized as follows. Section 2 presents the problem statement. Section 3 presents the model of the laser platform in [Barton 2008]. Section 4 provides an overview of the McFarlane-Glover \( H_\infty \) loop shaping design procedure. Section 5 presents the specific design for the problem stated in section 2. Section 6 provides simulation results to demonstrate the effectiveness of the proposed controller. Section 7 presents an indirect experimental verification of the proposed controller using a previously designed and verified controller. Section 8 provides conclusions and recommendations for future work.

2. PROBLEM STATEMENT

This project addresses a scenario where a laser is mounted on a mobile platform. The objective is to maximize the energy density of the laser beam delivered to a target. The control system should prevent, as much as possible, the transmission of platform jitter (i.e., vibration) to the target. For this project, the disturbance consists of several tones of known frequency stemming from the platform motion plus broadband noise. The control system utilizes feedback information about the beam position on the target to steer the beam using a mirror.

3. LASER SYSTEM MODEL

3.1 Experimental Apparatus

Fig. 1: Laser table layout with source, disturbance (shaker) fast steering mirror (FSM), and target.
The laser system shown in Fig. 1 supports the project described in this paper and other related projects. The source apparatus and target receiver sit on separate tables that use a combination of springs, dampers, and compressed nitrogen to eliminate external vibration [Barton 2008]. A physical shell and tunnel surrounds the two tables and blocks disturbances such as air movements and some vibrations such as voices and machinery.

The laser sits atop another similarly stabilized platform on the source table. Three sensing modules, consisting of a laser and detector pair, measure the position of this platform with respect to the source table. On the laser platform, a dynamic shaker sits at a 45° angle to the platform and generates simultaneously vibration disturbances in the horizontal and vertical directions. The fast steering mirror (FSM in Fig. 1) directs the laser beam and serves as the actuator in the control system.

3.2 FSM Model

The laser system model describes the relationship between the FSM’s angular position and the position of the beam on the target. For this paper, a single-axis FSM model is used where the input is the desired angular position and the output is the achieved angular position. Based on the given documentation and specifications [Barton 2008], the FSM model admits an underdamped 2\textsuperscript{nd} order model of the form

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where \(\zeta\) represents the damping ratio for the mirror response, and \(\omega_n\) represents the undamped natural frequency in rad/sec.

### Table 1. Identified FSM model parameters

<table>
<thead>
<tr>
<th>Rotation</th>
<th>(\zeta)</th>
<th>(\omega_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth</td>
<td>0.591</td>
<td>1570</td>
</tr>
<tr>
<td>Elevation</td>
<td>0.576</td>
<td>1300</td>
</tr>
</tbody>
</table>

As part of his undergraduate thesis project [Shreffler 2010], the parameters of the azimuth and elevation rotation models were estimated from the manufacturer-provided frequency response data (i.e., Bode plots). Table 1 shows the identified parameters. Furthermore, Shreffler verified these parameters by comparing experimental and simulated responses of the FSM for step, impulse, and sinusoidal voltage inputs.

To construct a state space representation of the FSM model, the state variables and the inputs are defined as

$$\dot{x} = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T, u = \theta_d$$

where \(\theta\) represents the angular position of the FSM and \(\theta_d\) represents the desired angular position. In practice, the desired angular position of the FSM is specified by an applied voltage between +10 and -10 V and the angular displacement is limited to \(\pm 23\times 10^{-2}\) rad. The state space representation of the FSM model is given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} x + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} u$$

(1)

3.3 Target Position Model

Obtaining feedback about the laser’s position on the target presents an additional challenge to the jitter rejection problem. Since the targets may be unknown beforehand, it may not be possible to place sensors on the target especially in an adversarial situation. For this project, the controller generates the feedback signal by predicting the position on the target from the azimuth and elevation of the FSM.

The relationship between the FSM angles and the laser position on the target depends on orientation of the supporting platform [Roberts 2010]. This relationship simplifies under small angle assumption. For this project, the position of the target is given by

$$y = \begin{bmatrix} 2d & 0 \end{bmatrix} x + v$$

(2)

where \(d\) is the distance from the mirror to the target and \(v\) is the disturbance and represents changes in the measured target position due to the platform motion, atmospheric conditions, and sensor noise.

For the purposes of the controller design, (1) and (2) define the laser system model.

4. \(H_\infty\) LOOP SHAPING DESIGN PROCEDURE

From the discussion in Section 3, the uncertainty in the design model ((1) and (2)) stems from the prediction of the beam position on the target from the FSM azimuth and elevation. Specifically, the basic output model from (2)

$$y = \begin{bmatrix} 2d & 0 \end{bmatrix} x$$

(3)

that is used in the controller design does not contain information about the relationship between the vibration of the platform and the motion of the beam on the target. Therefore, robust stabilization is a key design criterion because the controller obtained from the design model ((1) and (3)) must stabilize the true system represented by (1) and (2). The \(H_\infty\) Loop Shaping Design Procedure (LSDP) provides a mechanism to balance robust stability...
requirements with disturbance rejection specifications [McFarlane 1992].

The $H_\infty$ robust stabilization problem forms the basis for the LSDP. The robust stabilization problem is stated as follows

$$\begin{bmatrix} -K \\ 1 \end{bmatrix} (1+GK)^{-1} \begin{bmatrix} 1 \\ K \end{bmatrix} < \gamma$$ \hspace{1cm} (4)

where $G(s)$ is the transfer function of the system to be controlled (e.g., the laser system) and $K(s)$ is the controller transfer function to be computed. This problem arises when trying to maximize the robustness of the closed-loop system to uncertainty in the normalized left coprime factorization of the plant $G(s)$. For a SISO plant

$$G(s) = \frac{N(s)}{D(s)}$$

the uncertainty is represented as stable perturbations to the numerator and denominator of the transfer function yielding an uncertain model of the form

$$G(s) = \frac{N(s)+\Delta_n(s)}{D(s)+\Delta_n(s)}$$

The optimal solution of the (4) maximizes the value $\varepsilon$ such that

$$\|\Delta_n - \Delta_n\|_\infty < \varepsilon$$

and the closed-loop remains stable. The optimal (minimum) value $\gamma_{\min} = \frac{1}{\varepsilon_{\max}}$ of (4) is computed from $G(s)$ and a suboptimal problem ($\gamma > \gamma_{\min}$) is solved to obtain $K(s)$.

The controller design process shapes the loop gain of the system $L(s) = G(s)K(s)$. For a disturbance rejection problem such as described in Section 2, the goal is to shape the sensitivity transfer function between the output disturbance and the error

$$S(s) = \frac{1}{1+L(s)}$$

If $|L(j\omega)|$ is large at frequencies where a disturbance is expected, the sensitivity at those frequencies will be small and the disturbance will be rejected. In the SISO case of the LSDP, the engineer chooses a weighting transfer function $W(s)$ so that the weighted plant $G_w(s) = W(s)G(s)$ achieves the desired loop shape. The construction of the weight $W(s)$ as the product of a series of fundamental weighting functions is described in Section 5.

Given the weighted plant $G_w(s)$, the robust stabilization problem in (4) is solved yielding a controller $K_w(s)$. The solution of the robust stabilization problem involves the solution of the Generalized Control and Filtering Algebraic Riccati Equations (GCARE and GFARE) based on the given plant data [McFarlane 1992].

If the chosen weight is compatible with the robust stability requirements from (4), a tolerance in the range $1 < \gamma < 10$ is obtained and the loop gain $G_w(s)K_w(s)G_w(s)K_w(s)$ approximates the desired loop shape, $G_w(s) = W(s)G(s)$, closely. Therefore, the tolerance $\gamma$ provides valuable information about the closed-loop performance before simulation or experimentation. Since the controller will be implemented on the given plant $G(s)$ instead of the weighted plant, $G_w(s) = W(s)G(s)$, the weight $W(s)$ must be appended to the computed controller $K_w(s)$ yielding the implemented controller $K(s) = K_w(s)W(s)$.

5. JITTER REJECTION CONTROLLER DESIGN

Section 4 presented the general $H_\infty$ design process and the selection of the weighting transfer function, or weight, represents the primary design input from the engineer. From the problem statement in Section 2, the disturbance contains a set of tones with known frequencies. The weight is a cascade of the transfer functions chosen to attenuate each of the tonal frequencies. Additional weights are added to address the response to broadband noise.

![Bode plot of weighting transfer function designed to attenuate vibration at 10 Hz.](image-url)
filter weight of the form \( W(s) = \frac{b}{a + bs} \) where \( a < b \) to the tonal weights in (5) used in the \( H_{\alpha} \) controller design. However, the resulting controller produces very limited attenuation (<1 dB) at high frequency. From the Bode Integral Theorem, excessive attenuation at low frequency may be a cause of the increased spillover. To reduce the attenuation at 5 Hz, the low-pass filter weight

\[
W(s) = \frac{(0.2)(s+10)}{s+2}
\]

is added.

6. SIMULATION RESULTS

The closed-loop system consisting of the mirror model described in Section 3 and the jitter control system described in Sections 4 and 5 is created using MATLAB and SIMULINK and shown in Fig. 4. The disturbance represents the effect of the platform motion on the beam position at the target and is modelled as an external position signal added to the predicted target position from the mirror model as in (2). The tones at 10, 20, and 30 Hz in the disturbance are represented by a sinusoid at each frequency with amplitudes of 1 mm. The broadband noise is implemented using the Band-limited white noise block in SIMULINK with a sampling period of 10 ms and a gain of 0.1 mm.

The addition of the broadband noise to the disturbance produces amplification at high frequencies (for this problem, above 25 Hz). This effect is known as spillover and is a consequence of the Bode Integral Theorem that implies the sensitivity cannot be less than unity (0dB) at all frequencies using output feedback with finite-bandwidth controllers and that, combined with the open-loop roll-off requirements for stability, the primary cost of feedback is in increased sensitivity at high frequencies [Mohktadi 1990]. A possible solution to reduce high frequency noise is to add a high-pass...
Fig. 5: Power spectral density of the disturbance and the controlled x-axis beam position at the target.

Fig. 6: Power spectral density of the disturbance and the controlled x-axis beam position at the target.

Fig. 7: Power spectral density of the disturbance (10, 20, 30 Hz tones plus broadband noise) and the controlled beam position in the x-direction at the target.

7. EXPERIMENTAL VERIFICATION

While the proposed controller has not been implemented on the apparatus in Section 3, an indirect experimental verification is accomplished using comparisons to a linear quadratic regular (LQR) controller designed and successfully implemented as part of the system identification project [Shreffler 2010].

Fig. 8 shows the comparison of the simulated power spectral densities of the multi-tonal disturbance (tones at 10, 12, 20, 27, and 30 Hz plus broad band noise) and the controlled beam position at the target for the $H_\infty$ and LQR controllers. Due to the proximity of the power spectral densities are displayed over three plots (Figs. 8 (a), (b), and (c)) The $H_\infty$ controller achieves at least 26 dB reduction at the tones included in the design (10, 20, and 30 Hz) and at least 23 dB at the other tonal frequencies. The LQR controller achieves about 8 dB reduction at each frequency.

This comparison is important because the LQR controller has been implemented on apparatus discussed in Section 3.1 and achieved similar reduction to that shown in Fig. 8. Therefore, one expects the proposed $H_\infty$ controller to achieve the reduction shown in Fig. 8 when implemented on the physical apparatus in Section 3.1.
In this paper, a controller was designed for a platform-mounted laser to reduce the transmission of platform jitter, modeled as a multi-tonal signal plus broadband noise, to the laser beam and to ensure minimal deviation of the center of the beam from its intended position. The $H_\infty$ loop shaping controller design procedure ensured a proper balance between robust stability and disturbance rejection criteria. Numerical simulation demonstrated at least a 20 dB (or, equivalently, > 90%) reduction at the tonal frequencies. Comparison to a previously implemented controller provided verification of a successful implementation of the proposed controller.

In future work, the proposed controller will be implemented on the laser test bed to formally verify the design. Furthermore, the $H_\infty$ loop shaping design procedure will be coupled with an adaptive control scheme to permit rejection of unknown tonal frequencies.

8. CONCLUSIONS

In this paper, a controller was designed for a platform-mounted laser to reduce the transmission of platform jitter, modeled as a multi-tonal signal plus broadband noise, to the laser beam and to ensure minimal deviation of the center of the beam from its intended position. The $H_\infty$ loop shaping controller design procedure ensured a proper balance between robust stability and disturbance rejection criteria. Numerical simulation demonstrated at least a 20 dB (or, equivalently, > 90%) reduction at the tonal frequencies. Comparison to a previously implemented controller provided verification of a successful implementation of the proposed controller.

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REFERENCES


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