Type-2 Fuzzy Model Inverse Controller Design Based on BB-BC Optimization Method

Tufan Kumbasar, Ibrahim Eksin, Mujde Guzelkaya, Engin Yesil

Istanbul Technical University, Faculty of Electrical and Electronics Engineering, Control Engineering Department, Maslak, TR-34469, Istanbul, Turkey, (e-mail: {kumbasart, eksin, guzelkaya, yesileng}@itu.edu.tr)

Abstract: The use of inverse system model as a controller might be an efficient way in controlling nonlinear systems. It is also a known fact that type-2 fuzzy models can represent nonlinear systems better than type-1 fuzzy models. Therefore, an inverse type-2 fuzzy model can be used as a controller for controlling processes with nonlinearities and/or uncertainties. In the case of uncertainties and/or disturbances, an inverse type-2 fuzzy model based nonlinear internal model control structure is proposed to compensate these errors.

Keywords: Type-2 Fuzzy Models, Inverse Fuzzy Models, Big Bang-Big Crunch, Internal Model Control

1. INTRODUCTION

Inverting the system as a controller might be an effective way to control nonlinear systems. It is a known fact that type-2 fuzzy models can represent nonlinear processes and smoothly integrate a priori knowledge with information obtained from process data. Lately, it has been shown that, type-2 fuzzy systems are more suitable in circumstances where it is difficult to determine the accurate membership function for a fuzzy set; which is very useful for mapping uncertainties and nonlinearities (Karnik et al., 1999; Lin et al., 2005; Barkati et al., 1999).

Fuzzy logic systems that are described with at least one type-2 fuzzy set are called type-2 fuzzy logic systems. It has been shown that type-2 fuzzy sets are much more powerful tools to represent the inputs and/or outputs of FLC (Hagras et al., 2007). Nevertheless, the computations of type-2 fuzzy systems are more complex than type-1 fuzzy systems. Therefore, Liang et al. (2005) and Mendel et al. (2005) proposed a special type of type-2 fuzzy models called interval type-2 fuzzy sets. The benefits of interval type-2 fuzzy systems are demonstrated in several control applications: liquid-level process control (Wu et al., 2005); autonomous mobile robots (Martinez et al., 2009; Juang et al., 2009); plants control (Castillo et al., 2005); marine diesel engine control (Lynch et al., 2006); bioreactor control (Galluzzo et al., 2008).

In the control engineering literature, there have been a number of applications of nonlinear model inversion. There exists various type-1 fuzzy model inversion methods developed in literature. Different type-1 fuzzy model inversion techniques for certain fuzzy models have been suggested that can only be applied under certain limitations. Babuska, et al. (1995), stated that under some certain invertibility conditions the exact inverse of a fuzzy model can be obtained. Moreover, Boukezzoula et al., (2003) and Boukezzoula et al. (2007) proposed an inversion method in which the fuzzy model is decomposed into fuzzy meshes and the inverse of the global fuzzy system is obtained through inversion of each fuzzy mesh. In (Abonyi et al., 1999), the inverse model is obtained directly through mapping the output and input data of the process via fuzzy logic approach. Furthermore, the iterative fuzzy model inversion methods are also proposed by researchers (Varkonyi-Koczy et al., 1998; Varkonyi-Koczy et al., 1999; Kumbasar, et al. 2008a) which do not require any invertibility conditions to be satisfied.

Recently, a new evolutionary computation algorithm named as Big Bang Big Crunch (BB-BC) is presented in (Erol & Eksin, 2006). The leading advantage of BB-BC is the high convergence speed and the low computation time. The working principle of this method can be explained as the transformation of a convergent solution to a chaotic state and then back to a single tentative solution point. This evolutionary search algorithm was first used for on line adaptation of the fuzzy model and on line type-1 fuzzy model inversion (Kumbasar, et al. 2008b).

In this paper, a type-2 fuzzy model inverse controller structure is proposed. In this structure, the output of the inverse type-2 fuzzy model is generated as a solution of an optimization problem. Even though inverse model controllers may produce perfect control while operating in an open loop fashion, this open loop control would not be sufficient in the case of modelling mismatches or disturbances that might occur over the system. In order to compensate these errors a nonlinear internal model control (IMC) scheme is proposed, based on BBC based type-2 fuzzy model inverse controller. The effectiveness of the proposed method is shown on a simulation study. The control performance is compared with a type-1 fuzzy model inverse control structure.
2. TYPE-2 FUZZY MODEL STRUCTURE

Type-2 fuzzy sets are generalized forms of those of type-1 fuzzy sets. Mathematically, type-2 fuzzy sets are not easy to describe like type-1 fuzzy sets. A type-2 fuzzy set, denoted as $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_s \subseteq [0,1]$, i.e.:

$$\tilde{A} = \left\{ (x,u), \mu_{\tilde{A}}(x,u) \mid \forall x \in X, \forall u \in J_s \subseteq [0,1] \right\}$$

(1)

in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. For a continuous universe of discourse, $\tilde{A}$ can be expressed as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_s} \mu_{\tilde{A}}(x,u)/(x,u) \ J_s \subseteq [0,1]$$

(2)

where $\int\int$ denotes union over all admissible x and u. $J_s$ is referred to as the primary membership of x, and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set known as the secondary set. The uncertainty in the primary membership of a type-2 fuzzy set $\tilde{A}$ is defined by a region named footprint of uncertainty (FOU) which is also the union of all primary memberships. An example of a Gaussian type-2 fuzzy set is given in Fig. 1. It can be described in terms of an upper membership function $\bar{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

![Fig. 1. Illustration of type-2 fuzzy gaussian membership function.](image)

A fuzzy logic system described with at least one type-2 fuzzy set is called a type-2 fuzzy logic system (T2FLS). A block diagram of a T2FLS, that is a special fuzzy logic system, is given in Fig. 2. Similar to a type-1 FLS, a type-2 FLS includes type-2 fuzzifier, rule-base, inference engine, and substitutes the defuzzifier by the output processor.

A T2FLS is characterized by IF-THEN rules, but its antecedent and/or consequent fuzzy sets are defined by type-2 fuzzy sets. The type-2 fuzzy set outputs are then processed by the type-reducer which combines the output sets and then perform a centroid calculation, which leads to type-1 fuzzy sets called the type-reduced set. The defuzzifier can then defuzzify the type-reduced type-1 fuzzy outputs to produce crisp outputs (Mendel et al., 2000).

![Fig. 2. Block diagram of type-2 fuzzy logic system.](image)
3. BIG BANG–BIG CRUNCH OPTIMIZATION

The Big Bang-Big Crunch (BB-BC) optimization method proposed by Erol & Eksin (2006) consists on two main steps. The first step is the “Big Bang” phase where candidate solutions are randomly distributed over the search space and the next step is the “Big Crunch” phase where a contraction procedure calculates the centre of mass for the population. The initial Big Bang population is randomly generated over the entire search space just like the other evolutionary search algorithms. All subsequent “Big Bang” phases are randomly distributed about the centre of mass or the best fit individual in a similar fashion (Erol & Eksin, 2006).

After the “Big Bang” phase, a contraction procedure is applied as the “Big Crunch” phase to form a centre or a representative point for further “Big Bang” operations. In this phase, the contraction operator takes the current positions of each candidate solution in the population and its associated cost function value and computes a centre of mass. The centre of mass can be computed as:

\[ x_c = \frac{\sum_{i=1}^{N} \frac{1}{f_i} x_i}{\sum_{i=1}^{N} \frac{1}{f_i}} \]

where \( x_i \) is the position of the centre of mass, \( x_c \) is the position of the candidate, \( f_i \) is the cost function value of the ith candidate, and \( N \) is the population size. Instead of the centre of mass, the best fit individual can also be chosen as the starting point in the “Big Bang” phase. The new generation for the next iteration “Big Bang” phase is normally distributed around \( x_c \). The new candidates around the centre of mass are calculated by adding or subtracting a normal random number whose value decreases as the iterations elapse. This can be formalized as

\[ x_{new} = x_c + \frac{\alpha (x_{max} - x_{min})}{k} \]

where \( r \) is random number; \( \alpha \) is a parameter limiting the size of the search space, \( x_{max} \) and \( x_{min} \) are the upper and lower limits; and \( k \) is the iteration step. Erol & Eksin, (2006) the working principle of this evolutionary method is explained as to transform a convergent solution to a chaotic state which is a new set of solutions.

In order to improve the computational efficiency and performance of the general BB-BC algorithm, an elitist strategy introduced by Camp (2007) is applied in this BB-BC optimization. In this modification, positions of new candidate solutions at the beginning of each Big Bang are normally distributed around a new point located between the center of mass and the best solution.

\[ x_{new} = \beta x_c + (1-\beta)x_{best} + \frac{\alpha (x_{max} - x_{min})}{k} \]

where \( \beta \) is the parameter controlling the influence of the global best solution \( x_{best} \) on the location of new candidate solutions. This modification of generating the new solution can be viewed as to an elitist strategy, where the best solution influences the direction of the search, and consequently potentially improve the over all search performance.

4. BB-BC BASED INVERSE TYPE-2 FUZZY CONTROLLER

In this proposed control structure, the inverse fuzzy model control signal generation is handled as an optimization problem. The optimization objective is defined as minimization of the error between the type-2 fuzzy model output and the reference signal. If the type-2 fuzzy model matches the process dynamics perfectly, then one can assume that the process output will converge to the set point. The inverse control signal generation will calculated via the nature inspired global BB-BC Algorithm. An online implementation of this evolutionary algorithm has been feasible at each sampling time due to the simplicity and high convergence speed of the algorithm (Erol & Eksin, 2006).

![Fig. 3. Illustration of BBBC type-2 fuzzy model inverse controller (BBBC-T2FMIC)](image)

The simplest way to design a model based controller for processes is to set up the inverse model of the system as the controller. Let us consider a nonlinear system without time delay, characterized with a state vector \( x(k) \). The model can be expressed as

\[ x(k+1) = f(x(k),u(k)) \]

where \( x(k) \) is the state vector, \( u(k) \) is the current input, \( x(k+1) \) is the predicted state and \( f \) represents the linear/non-linear relationship of the mapping. The optimal control problem can be stated to force the system output from any initial state \( x(0)=x_0 \) to a desired state \( x(k)=x_d(k) \) in an optimal way. The cost function \( J \), which is minimized at every sampling time is chosen as:

\[ J = \sum_{k=0}^{K-1} e^{T}(k+1)Qe(k+1) + u^{T}(k)Ru(k) \]

where \( e(k+1) = x(k+1) - x_d(k) \) is the error at time \( k+1 \), \( Q \) and \( R \) are positive definite matrices with appropriate dimensions. The matrices \( Q \) and \( R \) determine the speed and aggressiveness of the controller. This optimization problem of \( J \) gives the optimal inverse model signal or the best approximation. In Fig. 3, the proposed BBBC type-2 fuzzy model inverse controller (BBBC-T2FMIC) is presented.

In the case of existence of disturbances and/or modeling errors, the proposed inverse controller can be used in Internal Model Control (IMC) scheme which is illustrated in Fig.4.
5. SIMULATION STUDY

In the simulation study, in order to show the effectiveness of the proposed T2FMIC the level control of a spherical tank is considered (Agrawal and Lakshminarayan, 2003). The inverse controller closed loop performance will be compared with the T1FMIC proposed by Kumbasar et al., (2008a). In order to make a fair comparison of the T2FMIC scheme with the T1FMIC scheme two performance measures are considered to be:

i) Integral Absolute Error (IAE), which is defined as:

\[
IAE = \int |r(t) - y(t)| dt
\]

ii) Total Variation (TV) of the control input (Skogestad, 2003), which is defined as:

\[
TV = \sum_{i=1}^{\infty} |u_{i+1} - u_i|
\]

A schematic diagram of the tank level system is illustrated in Fig.5. The differential equation related to the tank level system is given as:

\[
Q_i(t - d) - Q_o = (\pi - y(R - y)^2) \frac{dy}{dt}
\]

where R is the radius of the spherical tank, \(Q_i\) is the inlet volumetric flow rate, and \(Q_o\) is the outlet flow rate. The delay from the manipulated input \(Q_i\) to the controlled output \(y\) is indicated by \(d\). The outlet flow rate \(Q_o\) is related to the level \(y\) via the Bernoulli equation:

\[
Q_o = \sqrt{2g(y - y_o)}
\]

where \(g\) represents the gravitation constant and \(y_o\) represents the height of the outlet pipe as measured from the base of the column. The parameters of the process are given in Table 1. The nominal operating value of \(y\) is taken as 0.5 m. At this nominal operating value, the output \(y\) will respond faster than the case when \(y\) is close to 1. Moreover it is assumed the system output inherits noise.

### Table 1. Spherical tank parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of tank [m]</td>
<td>R = 1m</td>
</tr>
<tr>
<td>Delay from (Q_i) to (y) [s]</td>
<td>(d = 0)s</td>
</tr>
<tr>
<td>Gravity acceleration [m/s^2]</td>
<td>(g = 9.81) m/s^2</td>
</tr>
<tr>
<td>Height of the output pipe [m]</td>
<td>(y_o = 0.1)m</td>
</tr>
<tr>
<td>Inlet volumetric flow rate [m^3/s]</td>
<td>(Q_i(t) : 0-6) m^3/s</td>
</tr>
<tr>
<td>Outlet volumetric flow rate [m^3/s]</td>
<td>(Q_o(t))</td>
</tr>
<tr>
<td>Height of liquid level [m]</td>
<td>(y)</td>
</tr>
</tbody>
</table>

Through empirical studies, the parameters for the evolutionary algorithm \(\alpha\) and \(\beta\) have chosen as 1 and 0.25, respectively, such that the optimal solution can be provided for T1FMIC and T2FMIC design problems. The population size and the number of iterations for the BB-BC optimization method have been chosen as 20.

At first, the effectiveness of type-2 fuzzy modeling will be demonstrated. Therefore, the type-1 and type-2 fuzzy models of the process have been obtained and compared.

The state vector for the type-1 and type-2 fuzzy model has been chosen as \((T_s=0.05s)\):

\[
x(k + 1) = [y(k), Q_o(k)]
\]

In order to represent the nonlinearities of spherical tank better, the membership functions of the type-1 fuzzy model is defined with 4 type-1 gauss functions while the membership functions of the type-2 fuzzy model are defined with 4 type-2 gauss functions.
To check the obtained type-1 and type-2 fuzzy models, a uniform random test signal which is illustrated in Fig. 6a is applied. The fuzzy models are simulated against the model output which is illustrated in Fig. 6b. In order to make a fair comparison between the fuzzy models, a performance index is defined as follows:

\[
IAE = \int_0^{200} |y_p(t) - y_{FM}(t)| dt
\]  

(20)

where \(y_p(t)\) is the process output and \(y_{FM}(t)\) is the fuzzy model output. The performance values obtained from (20) for the T1FM and T2FM are found as 6.915 and 5.824, respectively. Moreover, it has been demonstrated from Fig. 6 that the type-2 fuzzy model represents the defined nonlinear process better than the type-1 fuzzy model.

First of all, since the nonlinearity is related to the level of the tank, a servo performance of the T1FMIC and T2FMIC structures have been compared. It has been illustrated in Fig. 7 that the T2FMIC scheme provides better transient state performances than the T1FMIC under varying reference signals. Moreover, the IAE value of T2FMIC structure is less than T1FMIC controller structure, as it can be seen in Table 2. Additionally, the T2FMIC scheme has a low value of TV that shows that it has the smoothest control signal.

Table 2. Performance comparison of the simulation example

<table>
<thead>
<tr>
<th></th>
<th>Servo Performance</th>
<th>Disturbance Rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Simulation</td>
</tr>
<tr>
<td>IAE</td>
<td>TV</td>
<td>IAE</td>
</tr>
<tr>
<td>T1FMIC</td>
<td>0.3893</td>
<td>2.9776</td>
</tr>
<tr>
<td>T2FMIC</td>
<td>0.3134</td>
<td>2.4816</td>
</tr>
</tbody>
</table>

Fig. 7. Illustration of (a) the system outputs for varying reference values (b) the control signals for varying reference values.

Secondly, the case of a disturbance rejection performance of the T2FMIC and T1FMIC scheme has been examined. First of all, a reference signal of “1m” has been applied. After the process converged to the desired tank level, a step output disturbance with the magnitude of “0.3” and a step input disturbance with the magnitude of “2” have been applied in 1st and 3rd seconds, respectively. As it can be seen from the Fig. 8 the proposed control structure compensated very effectively the input/output disturbances in a short period of time compared to the T1FMIC structure. Moreover, the control signal of the T2FMIC scheme is smoother and therefore has a lower TV value. Also, the IAE value is the better.

Fig. 8. Illustration of (a) the system output (b) the control signal for input and output disturbances.

6. CONCLUSIONS

In this study, a new iterative type-2 fuzzy model inversion technique based on the BB-BC optimization is developed to improve the system performance and a simulation example is presented to show the superiority of the approach. In this new approach, the inverse type-2 fuzzy model control signal generation is handled as an optimization problem. Since the BB-BC optimization algorithm has a high convergence speed and low computational time, the optimal inverse type-2 fuzzy model control signal can be generated within each sampling period.
time in an online fashion. In the simulation studies, the proposed T2FMIC IMC structure has been compared with the T1FMIC IMC structure. Since the type-2 fuzzy models map the nonlinearities and uncertainties better than the type-1 fuzzy models, the T2FMIC ameliorate the overall performance of the system with respect to disturbance rejection and set-point following much better.

REFERENCES