A Nonlinear Observer for Estimating Transverse Stability Parameters of Marine Surface Vessels

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Abstract: This paper presents a nonlinear observer for estimating parameters associated with the restoring term of a roll motion model of a marine vessel in longitudinal waves. Changes in restoring, also referred to as transverse stability, can be the result of changes in the vessel’s centre of gravity due to, for example, water on deck and also in changes in the buoyancy triggered by variations in the water-plane area produced by longitudinal waves—propagating along the fore-aft direction along the hull. These variations in the restoring can change dramatically the dynamics of the roll motion leading to dangerous resonance. Therefore, it is of interest to estimate and detect such changes.

Keywords: Nonlinear observer, Parameter estimation, Parametric resonance, Marine systems

1. MOTIVATION

A marine surface vessel can be conceptually described as the analogue of a multi-degree of freedom mass-spring-damper system [Perez, 2005]. Hydrodynamic pressure forces have a component proportional to the accelerations and a component proportional to the velocities. The pressure forces due to accelerations give rise to the so-called added-mass terms whereas the pressure forces due to velocity give rise to damping terms related to the energy transferred to the fluid as a consequence of wave making, skin friction and vortex shedding. Hydrostatic forces tend to bring the vessel to its up-right equilibrium position. These forces are due to weight and buoyancy.

If a vessel takes water on deck or if there are fluid tanks (fuel and water) with a free surface (not completely full), the motion of the fluid creates a change in the restoring forces due to the motion of the centre of gravity [Biran, 2003]. Also, when the vessel sails in longitudinal waves—propagating along the fore-aft direction along the hull—the passage of the wave along the hull can produce changes in the water-plane area, which produces changes in the buoyancy forces [Biran, 2003]. This effect depends on the shape of hull, and for modern hull designs of container ships and also some fishing vessels, these changes in buoyancy have been known to trigger a kind of parametric resonance resulting in dangerous roll motions [Holden et al., 2007a, France et al., 2001, Neves et al., 1999].

From an operational perspective, it is important to monitor parameters associated with the roll restoring forces and detect changes, which can lead to catastrophic losses.

Few publications seem to be devoted to the design of estimation/detection schemes to detect the onset of instability conditions induced by the inception of parametric roll resonance. Holden et al. [2007b] introduced an observer based predictor, which provides an estimate of the eigenvalues of a linear second-order oscillatory system describing the roll motion by using different estimation schemes. The predictor sets an alarm when those eigenvalues move into the right-half plane. McCue and Bulian [2007] investigated the feasibility of using finite time Lyapunov exponents to detect the onset of parametric roll for ships operating in irregular seas condition. Statistical approaches have been pursued by Galeazzi et al. [2009a,b] where an energy flow index and a generalized likelihood ratio test for non-Gaussian signals have been proposed.

In this paper, we present an alternative to the above methods based on a nonlinear observer. The proposed observer is simple to implement and tune, which provides an advantage with respect to the above proposed methods.

2. MATHEMATICAL MODELING

A two degree-of-freedom model in roll and pitch is considered. Namely,

$$\dot{\Theta}_{nb} = T_\Theta \omega_{b/n}^b$$  \hspace{1cm} (1)

$$M_\omega \omega_{b/n} + D(\omega_{b/n}^b)\omega_{b/n}^b + G(\Theta_{nb})\Theta_{nb} = \tau_w$$  \hspace{1cm} (2)

The vector $\Theta_{nb} = [\phi, \theta]^T$ has as components the roll angle $\phi$ and the pitch angle $\theta$. The angular-velocity vector in body-fixed coordinates is $\omega_{b/n}^b = [p, q]^T$, where $p$ is the roll rate and $q$ is the pitch rate. The wave-induced excitation is given by $\tau_w = [K, M]^T$ where $K$ is the roll moment and $M$ is the pitch moment. The parameters of the model are
where $M$ is the inertia matrix, which includes the rigid-body moments of inertia along the longitudinal and transverse axis ($I_x$, $I_y$) and the added inertia components due to hydrodynamic forces ($K_p$, $M_t$); $D(\omega_{b/n})$ is the hydrodynamic damping matrix given by the sum of potential damping ($K_p$, $M_t$) and nonlinear effects in roll ($K_{ppp}$); $G(\Theta_{nb})$ is the restoring moment matrix. Finally, $T_\Theta$ is a kinematic transformation that relates the component of the angular velocity to the time-derivatives of roll and pitch:

$$T_\Theta = \begin{bmatrix} 1 & \sin \phi \tan \theta \\ 0 & \cos \phi \end{bmatrix}.$$

The roll restoring moment is given by the sum of three contributions

$$\tau_\phi = K_\phi \phi + K_{\phi \phi} \dot{\phi} \theta + K_{\phi \phi \phi} \dot{\phi}^3,$$

which are a linear term $K_\phi \phi$ that is dominant for relatively small roll angles, a cubic term $K_{\phi \phi \phi} \dot{\phi}^3$, which shows its effect for large roll oscillations, and a cross-coupling term $K_{\phi \theta} \dot{\theta}$, which is responsible for the onset of parametric resonance if certain conditions are fulfilled:

- Sailing in longitudinal seas,
- Wave length similar to the length of the vessel,
- Wave amplitude above certain threshold (vessel dependent),
- Encounter frequency $\omega_c$ close to twice the roll natural frequency $\omega_\phi$,
- Hull with small roll damping.

Despite the apparent restriction that the above conditions seems to impose, vessels satisfying these conditions are not uncommon and several cargo losses have been reported due to large induced roll angles and accelerations [Holden et al., 2007a, Carmel, 2006, France et al., 2001, Neves et al., 1999].

The linear term in roll in (3) is often expressed as

$$K_\phi = \rho g \nabla GM_0,$$

where $\rho$ is the water density, $g$ is the acceleration of gravity, $\nabla$ is the displaced volume, and $GM_0$ is the transverse metacentric height in calm water. The latter is a parameter commonly used in the marine environment to characterise the changes in the roll restoring term – this also refers to the transverse stability – and the onset of parametric resonance has been described in terms of periodic variations of the transverse metacentric height [Shin et al., 2004, France et al., 2001].

3. OBSERVER FOR PARAMETER ESTIMATION IN NONLINEAR SYSTEMS

In this section, we revisit a particular observer for parameter estimation in nonlinear systems proposed by Friedland [1997].

Given a nonlinear system

$$\dot{x} = f(x, u, \beta),$$

where $x$ is the state vector, $u$ is the control and $\beta$ is a vector of parameters, one can form the following estimator

$$\dot{\beta} = \gamma(x) + z$$

$$\dot{z} = -\Gamma(x)f(x, u, \beta),$$

where $\gamma$ and $\Gamma$ are chosen such that

$$\Gamma(x) = \left[\frac{\partial f(x, u, \beta)}{\partial z}\right],$$

where the right-hand side is the Jacobian of $\gamma$. The matrix $\Gamma$ plays the role of observer gain. If the parameters $\beta_j$ are slowly-varying, namely, $\dot{\beta} \approx 0$, then the dynamics of the parameter estimation error $e = \beta - \hat{\beta}$ is given by

$$\dot{e} \approx -\beta$$

$$= -\Gamma(x)\ddot{x} - \ddot{z}$$

$$= -\Gamma(x)[f(x, u, \beta) - f(x, u, \beta - e)].$$

If the model (5) is affine in the parameter vector $\beta$, then it can be expressed as

$$f(x, u, \beta) = F(x, u) \beta + g(x, u),$$

where $F$ is a Jacobian matrix given by

$$F(x, u) = \left[\frac{\partial f(x, u, \beta)}{\partial \beta_j}\right].$$

Substituting (12) into (11), we obtain

$$\dot{e} \approx -L(t)e,$$

where

$$L(t) = \Gamma(x)F(x, u).$$

In order to analyze the stability of the estimation error, we can use Lyapunov’s direct method. Let a candidate Lyapunov function be

$$V(t) = e(t)^T e(t),$$

whose derivative along the trajectories is

$$\dot{V}(t) = -2e(t)^T L(t)e(t).$$

If $\Gamma(x)$ is chosen such that $L(t)$ is symmetric and positive semi-definite – see (15) – then (17) shows that $e(t) \in L_2 \cap L_\infty$, but it does not show asymptotic stability of the error. If we have the additional requirement that the error is uniformly continuous, or equivalently its time derivative is bounded, then Barbalat’s Lemma (see Lemma 1.2.1 and Corollary 1.2.1 in Sastry and Bodson [1989]) establishes the asymptotic stability of the error. An alternative necessary and sufficient condition for asymptotic stability is that

$$\int_0^t L(t) dt \to \infty \text{ as } t \to \infty,$$

which follows from Theorem 1 of Morgan and Narendra [1977], and relates to the standard condition for persistence of excitation as discussed by Friedland [1997].

4. NONLINEAR OBSERVER DESIGN

4.1 Observer Model

Given the system (1)-(2), we will consider a reduced-order model that takes into account only the roll motion component. The roll restoring moment (3) can be rewritten as
\[ \tau_\phi = (K_\phi + K_\phi \theta(t))\phi + K_{\phi \phi \phi} \phi^3 \]
\[ = \rho g \nabla [GM_t^0 + \delta GM_t(t)]\phi + K_{\phi \phi \phi} \phi^3 \]
\[ = \rho g \nabla GM_t(t)\phi + K_{\phi \phi \phi} \phi^3, \] (19)
where \( \delta GM_t(t) \) is the time-varying change of the metacentric height due to the passage of longitudinal waves along the hull and the coupling with pitch. Furthermore, since the observer is to be used when the vessel is sailing in longitudinal waves, the external moments acting in roll are assumed to be negligible. Hence, the oscillatory changes in \( GM_t \) can be ascribed to the onset and development of parametric resonance. This type of model has been used to investigate parametric resonance using the Mathieu’s equation [Shin et al., 2004, France et al., 2001].

Based on the above restoring model, we can design a reduced-order nonlinear observer for estimating parameter related to \( GM_t \). Indeed, the reduced state vector considered is \( x_r = [\phi, p]^T \), then the roll subsystem can be expressed as
\[ \dot{x}_r = f (x_r, \beta) \]
\[ = \begin{bmatrix} p + q_m \sin \phi \tan \theta_m \\ -\mu_1 p - \mu_2 p^3 - \beta \phi - \kappa \phi^3 \end{bmatrix} \] (20)
where \( \theta_m \) and \( q_m \) are the measured pitch angle and pitch rate, respectively, \( \mu_1 = -K_r/(I_x - K_p) \), \( \mu_2 = -K_{ppp}/(I_x - K_p) \), \( \beta = -\rho g \nabla GM_t/(I_x - K_p) \) is the time-varying parameter to be estimated, and \( \kappa = -K_{\phi \phi \phi}/(I_x - K_p) \).

The model (20) can be expressed as
\[ \dot{x}_r = F(x_r) \beta + g(x_r, u), \]
\[ = \begin{bmatrix} 0 \\ \beta + p \end{bmatrix} + \begin{bmatrix} p + q_m \sin \phi \tan \theta_m \\ -\mu_1 p - \mu_2 p^3 - \kappa \phi^3 \end{bmatrix}. \] (21)

Given the structure of this model, we can choose
\[ \gamma (x_r) = -\lambda \phi \]
where \( \lambda > 0 \) is a constant gain. Then,
\[ \Gamma (x_r) = -\lambda |p| \]
and thus
\[ L(t) = \Gamma (x_r) F(x_r) = \lambda \phi^2 \geq 0, \] (24)
from which we can see that \( \lambda \) determines the rate of convergence of the estimation error.

With the definitions (22) and (23), the conditions for convergence of the estimation error of the parameter \( \beta \) described in the previous section are satisfied. Note also from (24) that the condition for persistence of excitation is that there is roll motion.

In the analysis made thus far, it has been assumed that \( \beta \) is slowly varying, namely, \( \dot{\beta} \approx 0 \) since \( \dot{\beta} \) is a forcing term in the estimation error equation. This forcing term will produce a bias in the estimate, but it will not make the estimation error to diverge. As we show in the following section, the observer provides very good estimates to the time varying parameter in the application being considered.

5. CASE STUDIES

To assess the performance of the proposed observer, we use a high-fidelity container ship model developed for the study of parametric roll resonance by Holden et al. [2007a]. The full model is a three degree-of-freedom heave-pitch-roll fully coupled model, but for the study at hand the heave dynamics has been suppressed. Three scenarios are considered, all in the presence of conditions for parametric resonance.

In the first scenario, we test ability of the observer to estimate a constant change in \( GM_t^0 \). At \( t = 0 \) the value of \( GM_t^0 \) is reduced of 30%, which simulates the event of green water on deck. In order to evaluate the convergence of the observer’s estimate the time-varying part \( \delta GM_t \) is known by the observer. Figure 1 shows the time series of roll and pitch: for \( t < 200 \) s the roll motion enters in parametric resonance driven by the pitch motion, which oscillates at a frequency approximately twice the roll natural frequency (\( \omega_r/\omega_\phi = 1.9633 \)); at \( t = 200 \) s a sudden drop of \( GM_t^0 \) takes place, and it breaks the frequency condition (\( \omega_r/\omega_\phi = 2.3465 \)), which eliminates the parametric resonance. The convergence of the observer’s estimate is shown in Figs. 2-3. The value of the observer gain has been taken as \( \lambda = 5 \). The observer converges very quickly to the new value of \( GM_t^0 \) (Fig. 2), in particular Fig. 3 shows that the estimation error \( e \) is already small after only 30 seconds and it keeps decreasing confirming the asymptotic stability of the error dynamics.

In the second scenario, we test the ability of the observer to track time-varying changes in the parameter, i.e. to estimate the contribution \( \delta GM_t(t) \). This produces periodic variations of the metacentric height \( GM_t(t) \) about its nominal value. Figure 4 shows the roll and pitch time series during a fully developed parametric roll event in regular waves. Figures 5-6 show the performance of the observer. Figure 5 shows a good performance in tracking a time-varying parameter: when the observer has reached the steady state the estimation error settles to maximum 3% (as shown in Fig. 6). This is due to the periodic nature of the derivative of the true parameter, which is forcing term in the estimation error equation.
In the third scenario, we test the tracking performance for multiple changes of the metacenter height. First, the pitch motion is excited at a frequency, which does not fulfill the requirements for the inception of parametric resonance. As consequence the roll motion is very close to zero \((\phi < 0.2^\circ)\) for the first 500 seconds. Then a new wave train with encounter frequency approximately twice the roll natural frequency hits the vessel and parametric roll is induced into the system. Suddenly, the wave amplitude is halved and the roll oscillations start decaying again, until a new resonance phenomenon takes place, as shown in Fig. 7. Figure 8 shows the outcome of the nonlinear observer in tracking the different changes in \(GM_t\) \((t)\). The observer is not capable of tracking the periodic variation of \(GM_t\) for the first 500 seconds. This may be ascribed to the very small roll oscillations, which determines the quasi absence of external excitation in the roll subsystem. In fact (3) points out that although the pitch is different from zero the roll restoring moment is zero when the roll motion is zero, i.e. if the vessels does not roll then it is not possible to estimate the metacenter height. However, as soon as the roll oscillations increase in magnitude the observer quickly converges to the true metacentric height, and it keeps tracking it even if further changes occurs. The ability of tracking the changes of metacenter height in these conditions is the sought property of the observer since these conditions are the likely conditions to be encountered in irregular seas [Holden et al., 2007a].

6. CONCLUSIONS

In this paper, we discussed the application of a nonlinear observer for the estimation of the time-varying transverse metacentric height \(GM_t\) of vessels sailing in longitudinal waves. The ability to monitor this parameter is important since its changes can lead to dangerous situations triggered by an auto-parametric resonance phenomenon. This is the case for modern container ships and fishing vessels. The observer design relies on a method previously proposed in the literature. A reduced-order model that considers
only the roll motion is re-parameterised and used for estimation. This model is in agreement with previous studies on parametric resonance. The stability of the estimation error is discussed, and the performance of the observer is tested based on three different scenarios using a high-fidelity container ship model. The obtained performance and rate of convergence of the estimation error seems to be acceptable for the purpose of detecting changes in transverse stability of vessels sailing in longitudinal waves.

REFERENCES


Fig. 6. The observer tracks the time-varying parameter very well committing an error, which is maximum about 3%.

Fig. 7. Roll and pitch time series for multiple variations of GM$_t$(t).

Fig. 8. The observer tracks multiple variations of GM$_t$(t) keeping the estimation error almost always below 10%, after the initial transient.