Control of a Five-axle, Three-steering Coupled-vehicle System and its Experimental Verification

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Abstract: This paper presents a new type of coupled-vehicle system: a five-axle, three-steering coupled-vehicle system consisting of two vehicles which are car-like mobile robots, two carriers, and a steering system, and presents its path-following feedback control method. We first show that, by assuming virtual mechanical elements, it is possible to convert the kinematic equation of the coupled-vehicle system into a three-chain, single-generator chained form in a new coordinate system where an arbitrary path whose curvature is two times differentiable is one coordinate axis and a straight line perpendicularly intersecting the tangent of the path is the other coordinate axis. Based on the chained form, we secondly derive the path-following feedback control method which enables the orientations of the two carriers relative to the tangent of the path to be controllable. By the feedback control method, it is possible to form the two carriers into a straight-bed carrier configuration or a V-bed carrier configuration and to cause them to maintain such configurations while performing a path-following operation thus enabling the two carriers to be configured in accordance with shapes of objects to be transported. The validity of the mechanical design of the coupled-vehicle system and its path-following feedback control method has been verified experimentally.

Keywords: nonlinear systems, nonholonomic systems, differential geometry, chained form, nonlinear control, autonomous vehicles, coupled-vehicles, path-following

1. INTRODUCTION

Automation of transportation operation is an important research subject in control engineering. This paper presents a new type of coupled-vehicle system with five axles and three steering systems, and a path-following feedback control method for the system.

Coupled-vehicle systems generally consist of two or more vehicles each having one or more tractors, such as mobile robots with two independently driven wheels or car-like mobile robots, connected to one or more trailers by passive revolute joints. In coupled-vehicle systems, movement is usually obtained by delivery of driving force generated in a mobile robot at the front end of the system via its drive wheels, with the trailers towed behind on passive wheels. It is also possible to use two mobile robots with drive wheels, with one at the front end and the other at the back end of the system. In some cases, moreover, mobile robots are placed elsewhere in the system.

For coupled-vehicle systems which transport large structural objects, such object is generally loaded onto a single trailer, which is towed by a single car-like mobile robot (Sampei (1995), Samson (1995), Altafini (2001), Lizarra (2001), and Deligiannis et al. (2006)). The required length of the trailer increases with the length of the structural object, which increases the likelihood of interference with obstacles in the operating environment and thus complicates the transportation operation, and therefore there have been proposed various methods to increase the maneuverability of the coupled-vehicle system by the addition of a mechanism for actively varying the rolling direction of the trailer’s passive wheels relative to the orientation of the trailer (Bushnell et al. (1995), Tilbury et al. (1995), Tilbury and Sastry (1995), Nakamura (2001), and Lamiraux et al. (2005)). For the same purpose, the authors have previously proposed the connection of one trailer to two car-like mobile robots via two passive revolute joints together with cooperative steering (Yamaguchi et al. (2010)). In such transportation systems (Sampei (1995), Samson (1995), Altafini (2001), Lizarra (2001), Deligiannis et al. (2006), Bushnell et al. (1995), Tilbury et al. (1995), Tilbury and Sastry (1995), Nakamura (2001), Lamiraux et al. (2005), and Yamaguchi et al. (2010)), however, the load bed configuration cannot be changed to facilitate the transportation of large structural objects of differing configurations, and such objects therefore extend well beyond the trailer bed during the transpiration, increasing the risk of load and trailer overturning.

To resolve such problem, we propose a modification of our previously proposed cooperative transportation sys-
tem employing two car-like mobile robots, which as shown in Figure 1: (a) the single bed is replaced by two beds (two "carriers") connected by a passive revolute joint; and (b) a steering system is added to the forward carrier in the same position of the passive revolute joint coupling the two carriers. The modified cooperative transportation system thus employs five axles and three steering systems. Key advantages of this coupled-vehicle system over previously proposed systems include the capabilities for: (1) formation of a single straight-bed carrier configuration by the two carriers, with the combined length thus enabling the transportation of structural objects as large as those transported by a conventional carrier of that length; (2) formation of a single V-bed carrier configuration, thus enabling the transportation of structural objects for which stable transportation is not possible with conventional carriers; and (3) cooperative operation by the two car-like mobile robots with drive wheels, thus enabling generation and delivery of greater driving force. In summary, the proposed coupled-vehicle system facilitates the transportation of structural objects with a carrier whose configuration can be adjusted for the shapes of the objects and increases driving force.

We show that the kinematical equation of the proposed coupled-vehicle system can be converted into a chained form which is a canonical form in a new coordinate system where an arbitrary path whose curvature is two times differentiable is one coordinate axis and a straight line perpendicularly intersecting the tangent of the path is the other coordinate axis. Especially, to permit this conversion, we assume a "virtual vehicle" as a third car-like mobile robot in Figure 2 whose virtual rear wheels are equivalent to the passive wheels of the steering system at the passive revolute joint connecting the two carriers, and we incorporate the time derivatives of the state variables of the virtual vehicle into the kinematical equation of the coupled-vehicle system.

We then propose a path-following feedback control method for this coupled-vehicle system, based on a chained form, to control the orientations of the first and second carriers ("carrier-1" and "carrier-2") relative to the direction of the tangent of the path as the midpoint on the rear axle of the leading car-like mobile robot ("vehicle-1") moves along the path. With this feedback control method, the coupled-vehicle system follows desired paths as the relative orientations of carriers-1 and -2 converge to and maintain their desired values. Using this method, it is possible to maintain the constant relative orientation between the two carriers in either a straight-bed carrier configuration or a V-bed carrier configuration and perform the path-following operation with the chosen configuration in which the two carriers function as a single carrier. The validity of this control method is verified by an experimental apparatus of the coupled-vehicle system developed by the authors.

2. STRUCTURE OF COUPLED-VEHICLE SYSTEM

The basic structure of the five-axle, three-steering coupled-vehicle system is shown in Fig.1. The first car-like mobile robot ("vehicle-1") is coupled via a first passive revolute joint ("joint-1") at the midpoint of the vehicle-1 rear axle to the first carrier ("carrier-1"). Carrier-1 is coupled to the second carrier ("carrier-2") via a second passive revolute joint ("joint-2"), and a steering system whose steering axis is in the same position of the rotating axis of joint-2 is attached to carrier-1, thus enabling active control of the rolling direction of the passive wheels of the steering system relative to the orientation of carrier-1. Carrier-2 is coupled to the second car-like mobile robot ("vehicle-2") via a third passive revolute joint ("joint-3") at the midpoint of the vehicle-2 rear axle.

This paper presents a path-following feedback control method which causes the coupled-vehicle system to move along desired paths as the two carriers form into and maintain a single straight-bed carrier configuration or a single V-bed carrier configuration by the control inputs, the moving velocity of vehicle-1 and the angular velocity of its steering system, the angular velocity of the steering system between carriers-1 and -2, and the angular velocity of the steering system of vehicle-2.

![Fig. 1. A Five-axle, Three-steering Coupled-vehicle System](image-url)

3. KINEMATICAL EQUATION OF COUPLED-VEHICLE SYSTEM WITH VIRTUAL MECHANICAL ELEMENTS

We derive, in this section, the kinematical equation of the coupled-vehicle system, including the time derivatives of the state variables of the virtual mechanical elements which are added to the system as shown in Fig.2 to facilitate the conversion of the kinematical equation into a chained form which is a canonical form. As shown in Fig.2, we assume a virtual car-like mobile robot (thus, a "virtual vehicle") whose rear wheels are equivalent to the passive wheels of the steering system in the same position of joint-2. Then, the kinematical equation of the coupled-vehicle system is derived as:

\[
\begin{align*}
\dot{x}_1 &= u_1 \cos \theta_1 \\
\dot{y}_1 &= u_1 \sin \theta_1 \\
\dot{\phi}_1 &= u_2 \\
\dot{\theta}_1 &= \tan \phi_1 \\
\dot{\phi}_2 &= u_3 \\
\dot{\theta}_2 &= \sin(\theta_1 - \theta_2) u_1 \\
\dot{\phi}_3 &= u_4 \\
\dot{\theta}_3 &= \tan \phi_3 \cos(\theta_1 - \theta_2) u_1 \\
\dot{\phi}_4 &= l_3 \cos(\theta_2 - \theta_3) \sin(\theta_3 - \theta_5) u_1 \\
\dot{\theta}_4 &= l_3 \cos(\theta_2 - \theta_3) \cos(\theta_3 - \theta_5) u_1 \\
\dot{\phi}_5 &= \frac{u_3 \cos(\theta_1 - \theta_2) \cos(\theta_3 - \theta_4) u_1}{l_5 \cos(\theta_2 - \theta_3) \cos(\theta_3 - \theta_5)}.
\end{align*}
\]
As understood from Fig.2, a vector \((x_1,y_1)^T\) represents the position of the midpoint of the rear axle of vehicle-1; angles \(\phi_1\) and \(\theta_1\) represent the steering angle and orientation of vehicle-1, respectively; an angle \(\theta_2\) the orientation of carrier-1; angles \(\phi_2\) and \(\theta_3\) the steering angle and orientation of the virtual vehicle, respectively; an angle \(\theta_4\) the orientation of carrier-2; and angles \(\phi_3\) and \(\theta_5\) the steering angle and orientation of vehicle-2, respectively.

As defined in Fig.2, the point \(P_s\) is taken as positive in sign when the midpoint of the rear axle of vehicle-1 is off the path, the position of that midpoint relative to the path as follows. The point \(P_s\) is the point on the path which satisfies the condition of the perpendicular intersection between the tangent of the path at that point and the straight line through that point to the midpoint of the rear axle of vehicle-1. If two or more such points exist on the path, the one closest to the midpoint of the rear axle of vehicle-1 is chosen. As indicated in Fig.2, the position of the point \(P_s\), the coordinate axis defined along the path is designated by \(s\). The position of the midpoint of the rear axle of vehicle-1 relative to the point \(P_s\) on the other coordinate axis defined along the straight line through the point \(P_s\) to the midpoint is designated by \(d\). The variable \(d\) is taken as positive in sign when the midpoint of the rear axle of vehicle-1 is to the left of the point \(P_s\) when looking in the positive direction of the coordinate axis \(s\), negative in sign when it is to the right of the point \(P_s\), and zero when it is on the path. The direction of the tangent of the path at the point \(P_s\) relative to the x-axis is designated by \(\theta_i\). The orientations of vehicle-1, carrier-1, the virtual vehicle, carrier-2, and vehicle-2 relative to the tangent of the path at the point \(P_s\) are designated by \(\theta_{pi}\), \(i = 1, 2, \ldots, 5\), and these are given as:

\[
\theta_{pi} = \theta_i - \theta_1, \quad i = 1, 2, \ldots, 5.
\]  

(2)

The derivative of the angle \(\theta_i\) with respect to \(s\) is given as:

\[
c(s) = \frac{d\theta_i}{ds}.
\]  

(3)

The derivative \(c(s)\) is the curvature of the path at the point \(P_s\) and the inverse of its absolute value is the radius of curvature. The time derivative of the angle \(\theta_i\) is therefore represented in terms of \(\dot{s}\), the moving velocity of the point \(P_s\), as:

\[
\dot{\theta}_i = \frac{d\theta_i}{ds} \frac{ds}{dt} = c(s)\dot{s}.
\]  

(4)

4.2 Control Strategy

We propose, in this subsection, a new control strategy to achieve the path-following operation of the coupled-vehicle system. As the coupled-vehicle system is moved and thus as \(s\) is increased or decreased, vehicle-1, vehicle-2 and the virtual vehicle are steered so that: the position \(d\) of the midpoint of the rear axle of vehicle-1 relative to the point \(P_s\) and the orientation \(\theta_{p1}\) of vehicle-1 relative to the direction of the tangent of the path at the point \(P_s\) both converge to zero; the orientation \(\theta_{p2}\) of carrier-1 relative to the direction of the tangent of the path at the point \(P_s\) converges to the desired value \(\theta_{p2d}\); the orientation \(\theta_{p3}\) of carrier-2 relative to the direction of the tangent of the path at the point \(P_s\) converges to the desired value \(\theta_{p3d}\). Thereafter, the steering angles of vehicle-1, vehicle-2, and the virtual vehicle are determined in accordance with the path curvature \(c(s)\) and its first-order derivative \(c'(s)\), to maintain \(d\) and \(\theta_{p1}\) at zero and \(\theta_{p2}\) and \(\theta_{p3}\) at \(\theta_{p2d}\) and \(\theta_{p3d}\), respectively. In this control strategy, the vector \(x\) below is taken as the state variable vector of the coupled-vehicle system.

\[
x = (s, d, x_1, \phi_1, \theta_{p1}, \phi_2, \theta_{p2}, \phi_3, \theta_{p3}, \phi_4, \theta_{p4})^T
\]  

(5)

The time derivatives of \(s\) and \(d\), which are the components of the vector \(x\), are given as:

\[
\begin{aligned}
\dot{s} &= u_1 \cos \theta_{p1} + d\dot{\theta}_1 \\
d &= u_1 \sin \theta_{p1}.
\end{aligned}
\]  

(6)

The time derivatives of \(s\) and \(\theta_i\) are derived from Eqs.(4) and (6) as:

\[
\begin{aligned}
\dot{s} &= \frac{\cos \theta_i}{1 - d\frac{dc(s)}{ds}} u_1 \\
\dot{\theta}_i &= \frac{c(s)\cos \theta_{p1}}{1 - d\frac{dc(s)}{ds}} u_1.
\end{aligned}
\]  

(7)

The time derivative of the state variable vector \(x\) of Eq.(5) is derived from Eqs.(1), (2), (6), and (7) as:

\[
\dot{x} = g_1(x_1)u_1 + g_2(x_2)u_2 + g_3(x_3)u_3 + g_4(x_4)u_4,
\]  

(8)
The time derivative of Eq. (12) thus becomes the three-chain, single-generator chained form shown in Eq. (14).

\[
\begin{align*}
\dot{z}_{11} &= w_1, & \dot{z}_{14} &= w_4, \\
\dot{z}_{23} &= z_{11} w_1, & \dot{z}_{33} &= z_{12} w_1, \\
& & i &= 2, 3, 4.
\end{align*}
\]

\[w_1 = \ddot{u}_1, \quad w_4 = \sum_{i=1}^{3} L_{ij} L_{ji}^2 h_j \dot{u}_i, \quad j = 2, 3, 4.\]

In Eq. (14), each of the terms \(L_{ij} L_{ji}^2 h_j \dot{u}_i\), \(L_{ij}^3 h_j \dot{u}_i\), \(L_{ij}^2 h_j \dot{u}_i\), and \(L_{ij}^3 h_j \dot{u}_i\), which are the first terms on the right hand sides of the equations of \(w_2, w_3,\) and \(w_4\) respectively, contains the second-order derivative \(c''(s)\) of the curvature of the path. For calculation of the coupled-vehicle system control inputs, the curvature of the path to be followed must be therefore two times differentiable.

This conversion is effective in the following open subset \(U\) as:

\[
U = \{ x : \phi_1 \neq \pm \pi/2, \phi_2 \neq \pm \pi/2, \phi_3 \neq \pm \pi/2, \theta_{p4} - \theta_{p1} \neq \pm \pi/2, \theta_{p4} - \theta_{p3} \neq \pm \pi/2; dc(s) < 1 \}.
\]

The state variable vector \(x\) is limited to the open subset \(U\) in order to avoid singular attitudes in which the kinematical equation of the coupled-vehicle system performing the path-following operation cannot be converted into the three-chain, single-generator chained form.

### 4.4 Desired Values of Converted Variables in Path-following Operation

We show, in this subsection, the desired values for the convergence of the converted variables shown in Eq. (12) and thus for the path-following operation of the coupled-vehicle system.

The variable \(z_{23}\) is \(d\), the variable \(z_{43}\) is \(\theta_{p2}\), and the variable \(z_{41}\) is \(\theta_{p4}\); and, as understood from Eqs. (12) and (14), the following relationships about \(d, \theta_{p2}\), and \(\theta_{p4}\) are satisfied:

\[
\begin{align*}
\ddot{d} &= z_{23}, & \ddot{d} &= z_{22} \ddot{u}_1, \quad \theta_{p2} = z_{33}, \\
\ddot{\theta}_{p2} &= z_{32} \ddot{u}_1, & \ddot{\theta}_{p4} &= z_{43}, \\
\ddot{\theta}_{p4} &= z_{42} \ddot{u}_1, \quad \theta_{p4} = z_{41} \dot{u}_1 + z_{42} \ddot{u}_1.
\end{align*}
\]

In the path-following operation, while moving the coupled-vehicle system, i.e., while maintaining at non-zero the moving velocity \(\ddot{u}_1\) of the point \(P_s\) which represents the position of the midpoint of the rear axle of vehicle-1 on the path, it is necessary to achieve the convergence of the position \(d\) of the midpoint of the rear axle of vehicle-1 relative to the path to the desired value of zero, the orientation \(\theta_{p2}\) of carrier-1 relative to the direction of the tangent of the path at the point \(P_s\) to the desired value \(\theta_{p2}(=\text{const})\), and the orientation \(\theta_{p4}\) of carrier-2 relative...
to the direction of the tangent of the path at the point \( P_a \) to the desired value \( \theta_{p4d} (= \text{const}) \). As understood from Eqs.(16), (17), and (18), it is therefore necessary to achieve the convergence of the variables \( z_{21}, z_{22} \) and \( z_{23} \) to zero, the variables \( z_{31} \) and \( z_{32} \) to zero and the variable \( z_{33} \) to \( \theta_{p2d} \), and also the convergence of the variables \( z_{41} \) and \( z_{42} \) to zero and the variable \( z_{43} \) to \( \theta_{p4d} \). In other words, the desired values of the variables \( z_{21}, z_{22}, \) and \( z_{23} \) are respectively zero, zero, and zero, those of the variables \( z_{31}, z_{32}, \) and \( z_{33} \) are respectively zero, zero, and \( \theta_{p2d} \), and those of the variables \( z_{41}, z_{42}, \) and \( z_{43} \) are respectively zero, zero, and \( \theta_{p4d} \).

5. PATH-FOLLOWING FEEDBACK CONTROL METHOD

We propose, in this section, a method of feedback control for the path-following operation of the coupled-vehicle system based on the chained form. With this method, we achieve the path-following operation while moving the coupled-vehicle system, i.e., while increasing or decreasing the variable \( z_{11} \) (which is the position \( s \) of the point \( P_a \) on the path), by obtaining the convergence of the variable \( z_{33} \) (which is the orientation \( \theta_{p2} \) of carrier-1 relative to the direction of the tangent of the path at the point \( P_a \)) to its desired value \( \theta_{p2d} \), the convergence of the variable \( z_{43} \) (which is the orientation \( \theta_{p4} \) of carrier-2 relative to the direction of the tangent of the path at the point \( P_a \)) to its desired value \( \theta_{p4d} \), and the convergence of all of the other variables (which are \( z_{21}, z_{22}, z_{23}, z_{31}, z_{32}, z_{41}, \) and \( z_{42} \)) to zero. We thus propose the following feedback control method as:

\[
\begin{align*}
\begin{cases}
w_1 &= a_0 \\
w_2 &= p_{21} z_{21} + p_{22} \frac{z_{22}}{a_0} + p_{23} \frac{z_{23}}{a_0} \\
w_3 &= p_{31} z_{31} + p_{32} \frac{z_{32}}{a_0} + p_{33} \frac{z_{33} - \theta_{p2d}}{a_0} \\
w_4 &= p_{41} z_{41} + p_{42} \frac{z_{42}}{a_0} + p_{43} \frac{z_{43} - \theta_{p4d}}{a_0} 
\end{cases}
\end{align*}
\]  

(19)

In Eq.(19), \( a_0 \) is a non-zero constant value, and physically means the moving velocity \( s \) of the point \( P_a \) on the path. Here, when we define the three-dimensional vectors \( \zeta_i \), \( i = 2, 3, 4 \) as shown in Eq.(20), their time derivatives \( \dot{\zeta}_i \), \( i = 2, 3, 4 \) are given as shown in Eq.(21).

\[
\begin{align*}
\zeta_2 &= \begin{pmatrix} z_{21} \\ z_{22} \\ z_{23} \end{pmatrix}^T \\
\zeta_3 &= \begin{pmatrix} z_{31} \\ z_{32} \\ z_{33} - \theta_{p2d} \end{pmatrix}^T \\
\zeta_4 &= \begin{pmatrix} z_{41} \\ z_{42} \\ z_{43} - \theta_{p4d} \end{pmatrix}^T 
\end{align*}
\]  

(20)

\[
\dot{\zeta}_i = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \zeta_i = A_i \zeta_i, \quad i = 2, 3, 4.
\]  

(21)

When \( p_{11}, p_{12}, p_{13}, i = 2, 3, 4 \) are given so as to obtain negative values for the real parts of the eigenvalues of the matrices \( A_i \), \( i = 2, 3, 4 \) of Eq.(21), the vectors \( \zeta_i, i = 2, 3, 4 \) exponentially converge to zero, resulting in achieving the path-following operation.

6. TRANSPORTATION BY STRAIGHT-BED AND V-BED CARRIERS

We propose, in this section, methods of operation of the coupled-vehicle system for transportation with carriers-1 and -2 for causing the two carriers to be (a) in a straight-line-shaped configuration as a straight-bed carrier; and (b) in a V-shaped configuration as a V-bed carrier. Figs.3 and 4 illustrate transportation by the straight-bed carrier formed by causing the relative orientation between carriers-1 and -2 to be an angle of zero degrees and by the V-bed carrier formed by causing the relative orientation between them to be an obtuse angle. Of course, it is possible to make it an acute angle.

![Fig. 3. A Straight-bed Carrier](image)

![Fig. 4. A V-bed Carrier](image)
Procedure 1-5: Load an object onto the straight-bed carrier, and perform the path-following operation.

6.2 Transportation by V-bed Carrier

For formation of carriers-1 and -2 into a V-bed carrier configuration and transportation of an object in that configuration, as illustrated in Fig.4, we propose the following procedures.

Procedure 2-1: Set a circular path of a radius \( R \) with a center at \((0, R)^T\) such that when the midpoints of the rear axles of vehicles-1 and -2 lie on the path and when the straight line between these two midpoints passes through the center of the circular path the relative orientation between carriers-1 and -2 becomes the desired value \( \theta_{pc} \). Place the starting point of the circular path at the origin \((0, 0)^T\) of the Cartesian coordinate system, and take the counterclockwise direction on the path as the positive direction of the coordinate axis \( s \).

Procedure 2-2: With the midpoints of the rear axles of vehicles-1 and -2 on the circular path and with the line between these midpoints passing through the center of the circular path, set the desired orientations \( \theta_{p2d} \) and \( \theta_{p4d} \) of carriers-1 and -2 relative to the direction of the tangent of the path at the point \( P_s \), so as to cause the relative orientation between the two carriers to be \( \theta_{pc} \), i.e., so as to satisfy the geometrical condition \( \theta_{pc} = \theta_{p2d} - \theta_{p4d} \).

Procedure 2-3: With the origin \((0, 0)^T\) of the Cartesian coordinate system as a starting point, set a path to be followed as a free-form curve (e.g., a Bezier curve) such that the direction of the tangent of the path at the origin corresponds to the direction of the x-axis and the path curvature as well as its first- and second-order derivatives at the origin are all equal to zero.

Procedure 2-4: Obtain the convergence of \( d \) to zero, \( \theta_{p2} \) to \( \theta_{p2d} \), and \( \theta_{p4} \) to \( \theta_{p4d} \) while moving the coupled-vehicle system along the circular path by the path-following operation. More specifically, move the point \( P_s \) which represents the position of the midpoint of the rear axle of vehicle-1 on the path from the origin \((0, 0)^T\) of the Cartesian coordinate system through the distance \( 2\pi R \) thus through one round on the circular path in either the clockwise or the counterclockwise direction. When the point \( P_c \) is not initially at the origin, move it to the origin before beginning the single round on the circular path.

Procedure 2-5: Repeat Procedures 2-1 to 2-4 sufficiently to ensure the convergence of \( d \) to zero, \( \theta_{p2} \) to \( \theta_{p2d} \), and \( \theta_{p4} \) to \( \theta_{p4d} \). The orientations of vehicle-1 and vehicle-2 then correspond to the directions of the tangents of the path at the midpoints of their rear axles, and they are parallel to each other.

Procedure 2-6: To cause the coupled-vehicle system to follow the path which is a free-form curve while maintaining the relationships \( d = 0, \theta_{p2} = \theta_{p2d}, \) and \( \theta_{p4} = \theta_{p4d} \), set all the steering angles so that the steering directions of vehicle-1, the steering system between carriers-1 and -2, and vehicle-2 all correspond to the direction of the x-axis, since the direction of the tangent of the path at its starting point corresponds to the direction of the x-axis, the path curvature and its first-order and second-order derivatives are all equal to zero.

Procedure 2-7: Load an object onto the V-bed carrier, and perform the path-following operation.

7. EXPERIMENTAL SYSTEM

We describe, in this section, the experimental system developed to verify the validity of the path-following feedback control method for the coupled-vehicle system. Figs.5 and 6 show the experimental coupled-vehicle system and the layout of the visual feedback control system developed for this purpose, respectively.

Vehicles-1 and -2 are each operated by two-wheel rear drive and two-wheel front steering. The two front wheels of each vehicle are independently steered and the extensions of the axes of its two front wheels and that of its rear wheels intersect at a single point which is the center of its revolution. The two rear wheels are driven through a differential gear so that in rounding a curve the difference in rotational speed between the inner and outer wheels can be absorbed mechanically. These mechanical features enable operation of each of the vehicles to be in strict accordance with its kinematical equation. In each vehicle, each of the two front wheels is steered by a DC servomotor with a zero-backlash gear head, and the two rear wheels are driven by a DC servomotor with a gear head. The rotational angle of each motor is measured by a rotary encoder.

Rotary encoders are also used for measurement of the rotational angles of the three passive revolute joints: joint-1 between vehicle-1 and carrier-1, joint-2 between carriers-1 and -2, and joint-3 between carrier-2 and vehicle-2. In the steering system between the two carriers, the rolling direction of its passive wheels can be actively changed relative to the orientation of carrier-1. The passive wheels are steered by a DC servomotor with a zero-backlash gear head. The rotational angle of this servomotor is also measured by a rotary encoder.

The weight of the experimental coupled-vehicle system is 9.1[kg]. As shown in Fig.6, the positions of the two black circular marks on the top of vehicle-1 are measured by a CCD camera installed on the ceiling 3.0[m] above the experimental field 3.0[m] in diameter, and together with the measurement of the rotational angles of joints-1, -2, and -3, which makes it possible to determine the position and orientation of the coupled-vehicle system. The two identical marks are 120[mm] in diameter and located 219[mm] above the experimental field. The measurement error for the position of the center of mass of each circular mark is 3.0[mm] or less. The control period for the path-following feedback control system in this experiment is 33[ms].

8. EXPERIMENTAL VERIFICATION

We describe, in this section, the verification of the validity of the path-following feedback control method for the coupled-vehicle system, by transportation of an object by the experimental coupled-vehicle system in the V-bed carrier configuration. The design parameters determining the size of the experimental coupled-vehicle system are as:

\[
\begin{align*}
&l_1 = 0.27[m], \quad l_2 = 0.405[m], \quad l_3 = 0.27[m], \\
&l_4 = 0.405[m], \quad l_5 = 0.27[m].
\end{align*}
\]

In this experiment, as shown in Fig.7, a circle of radius \( R = 0.2025[m] \) with a center at \((0, R)^T\) is taken as the path for formation of the V-bed carrier by carriers-1 and -2, and a 10th-order Bezier curve with a starting point at
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For the Bezier-curve path-following, to obtain the movement of the point \( P_s \) on the path through the distance of \( L_b=3.513[m] \) from its starting point \( P_0 \) to its ending point \( P_{10} \) in the time of \( T_2=45[s] \), the moving velocity \( \dot{s} \) of the point \( P_s \) is given as:

\[
\dot{s} = a_0 = L_b/T_2 = 0.05225[m/s].
\]

For this Bezier-curve path-following, the coefficients in the control inputs of Eq.(19) are given as:

\[
\begin{align*}
p_{21} &= -0.42, & p_{22} &= -0.0588, & p_{23} &= -0.002744, \\
p_{31} &= -0.42, & p_{32} &= -0.0588, & p_{33} &= -0.002744, \\
p_{41} &= -0.42, & p_{42} &= -0.0588, & p_{43} &= -0.002744.
\end{align*}
\]

The initial state variables of the coupled-vehicle system are given as:

\[
\begin{align*}
x_1|_{t=0} &= 0.0[m], & y_1|_{t=0} &= 0.0[m], \\
\phi_1|_{t=0} &= 0.0[rad], & \theta_1|_{t=0} &= 0.0[rad], \\
\phi_2|_{t=0} &= 0.0[rad], & \theta_2|_{t=0} &= 0.0[rad], \\
\phi_3|_{t=0} &= 0.0[rad], & \theta_3|_{t=0} &= 0.0[rad].
\end{align*}
\]
initial position $(s, d)^T|_{t=0}$ of the midpoint of the rear axle of vehicle-1 and the initial orientations $\theta_{p1}|_{t=0}$ of vehicle-1, $\theta_{p2}|_{t=0}$ of carrier-1, $\theta_{p4}|_{t=0}$ of the virtual vehicle, $\theta_{p5}|_{t=0}$ of carrier-2, and $\theta_{p4}|_{t=0}$ of vehicle-2 relative to the direction of the tangent of the path at the point $P_0$ are given as:

$$
\begin{cases}
  s|_{t=0} = 0.0[\text{m}], \\
  d|_{t=0} = 0.0[\text{m}], \\
  \theta_{p1}|_{t=0} = 0.0[\text{rad}], \\
  \theta_{p2}|_{t=0} = 0.0[\text{rad}], \\
  \theta_{p4}|_{t=0} = 0.0[\text{rad}], \\
  \theta_{p5}|_{t=0} = 0.0[\text{rad}].
\end{cases}
$$

Fig. 8 shows the formation of the V-bed carrier configuration as the coupled-vehicle system follows the circular path. As vehicle-1 performs the two counterclockwise rounds of the circular path while maintaining $d$ at zero, the orientations $\theta_{p2}$ of carrier-1 and $\theta_{p4}$ of carrier-2 converge to $\theta_{p2d}$ and $\theta_{p4d}$, respectively, thus forming the V-bed carrier configuration with the relative orientation between carriers-1 and -2 converging to the desired value $\theta_{pc} = (\theta_{p2d} - \theta_{p4d})$. After the two counterclockwise rounds of the circular path, the orientations of vehicles-1 and -2 correspond to the direction of the $x$-axis of the Cartesian coordinate system, and the direction of the steering system between carriers-1 and -2 corresponds to the direction of the $y$-axis.

Following the formation of the V-bed carrier configuration by carriers-1 and -2, the coupled-vehicle system performs the transportation of an object by following the 10th-order Bezier-curve path with the origin $(0,0)^T$ of the Cartesian coordinate system as the starting point of the path. In preparation for the transportation operation, the directions of the steering systems of vehicle-1, -2, and between carriers-1 and -2 are all initially caused to correspond to the direction of the $x$-axis of the Cartesian coordinate system. This is because the 10th-order Bezier-curve path is planned so that the direction of the tangent of the path at the origin $(0,0)^T$ of the Cartesian coordinate system which is the starting point of the path corresponds to the direction of the $x$-axis and so that the path curvature and its first- and second-order derivatives at the origin are all equal to zero. An object weighing 3.0[kg] is next loaded onto the V-bed carrier, and the experimental coupled-vehicle system is then ready to begin the transportation operation. As shown in Fig.9, the experimental coupled-vehicle system performs the transportation of the V-bed carrier configuration and following the 10th-order Bezier-curve path, thus successfully executing entry into a garage with very tight clearances and verifying the validity of the path-following feedback control method for the coupled-vehicle system.

9. CONCLUSIONS

We have proposed, in this paper, a five-axle, three-steering coupled-vehicle system and a path-following feedback control method for the system. We have shown that the kinematical equation of the coupled-vehicle system can be converted into a chained form which is a canonical form by assuming the existence of virtual mechanical elements in a new coordinate system where an arbitrary path whose curvature is two times differentiable is one coordinate axis and a straight line perpendicularly intersecting the tangent of the path is the other coordinate axis. We have proposed a path-following feedback control method based on the chained form. With this method it is possible to form the two carriers into either a straight-bed carrier configuration or a V-bed carrier configuration, and to perform the path-following operation with either configuration as though the two carriers constitute a single carrier, thus enabling the carriers to be configured in accordance with shapes of objects to be transported. The validity of the path-following feedback control method is verified by the experimental coupled-vehicle system performing transportation of an object in a garage entry operation with very tight clearances.

REFERENCES


Fig. 8. Formation of a V-bed Carrier Configuration

Fig. 9. Transportation of an Object by a V-bed Carrier