Small-Signal Stability Enhancement of Power Systems with Renewable Distributed Energy Resources

Adirak Kanchanaharuthai * Vira Chankong * Kenneth A. Loparo *

* Department of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland OH 44106 USA

Abstract: This paper examines the application of energy storage to enhance the small-signal stability of an electrical power system with renewable power generation. In particular, a linearized model of the system and an LMI-based control design method are used to achieve frequency and voltage regulation subject to small perturbations to mechanical power inputs to synchronous generators in the system. The controller design is validated using the following simulation studies: (1) a single machine connected to an infinite bus (SMIB) (2) a two-machine (synchronous generator and doubly-fed induction generator) connected to an infinite bus. Simulation results show that the proposed controller can simultaneously achieve frequency and voltage regulation.

Keywords: Wind power systems; STATCOM; Battery energy storage systems; Small-signal stability; LMI.

1. INTRODUCTION

Due to the environmental impact of fossil fuels (Kyoto Protocol, European Directive on renewable energy, etc.), renewable energy is receiving increased interest as an alternative power generation option. There have been recent efforts that integrate wind power into the conventional power grid (Ackermann [2005] and Heier [2006]). As the diversity of the generation mix increases with larger amounts of power from renewable sources, differences in the dynamic behavior between conventional and alternative generators can play an increasingly important role in the stability and security of the grid.

In general, both conventional and alternative energy generation are faced with the difficult task of maintaining stability when small or large disturbances occur in the power system. Small disturbances, for example fluctuations in mechanical power input to a generator, are constantly perturbing the power system, especially when the influence of wind speed variability is considered in a wind energy conversion system.

Our objective is to investigate small-signal stability in power systems that include the integration of renewable energy sources (generation and storage) with conventional generation. Flexible AC Transmission System (FACTS) devices can be used to enhance power transfer capability and augment the stability of a power system (Song & Johns [1999] and Hingorani & Gyugyi [1999]), and of particular interest in this work is the Static Synchronous Compensator (STATCOM) that is capable of providing smooth and rapid reactive power compensation for voltage support and to improve damping and transient stability.

It is evident that energy storage technologies (Ribeiro et al. [2000]) are important for dealing with the intermittency of many alternative energy sources, and also provide the opportunity to improve power quality, especially frequency and power stability, as reported in Lu et al. [1995]. For stability enhancement of wind energy conversion systems, battery energy storages has been used to improve frequency stability through the regulation of active power levels. As reported in Spahic [2009] and Chahwan [2007], battery energy storage can be used to (1) reduce power fluctuations from large wind farms, (2) deliver large amounts of energy in a short time period when needed, and (3) regulate active power output to improve the economics of wind farms.

The STATCOM and (battery) energy storage have independently been used to improve power system operations, and integrating these devices, the STATCOM to improve voltage stability and the battery to regulate active power and improve frequency stability, provides an opportunity to improve overall small-signal stability of the power system. Relatively little prior work has been devoted to the integration of STATCOM and battery energy storage. Yang et al. [2001] have shown that the integration of STATCOM and battery can provide additional benefits beyond the STATCOM in conventional power systems and Baran et al. [2008] have shown the ability of the STATCOM to smooth intermittent wind farm power and compensate reactive power through simulation. In addition, Arulampalam et al. [2006] have shown that improvements in power quality can be obtained by a STATCOM and battery combination for a wind farm application.

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This paper continues this line of investigation and examines how a STATCOM and battery system can be used to improve the small-signal stability of a wind energy conversion system interconnected to the grid through a Doubly Fed Induction Generator (DFIG). An LMI-based controller is designed to achieve both frequency and voltage regulation. The wind power system considered incorporates a DFIG. Simulation results are provided for two operating scenarios: (1) single machine, a Synchronous Generator (SG) or DFIG, connected to an infinite bus, (2) two-machines, SG and DFIG, connected to an infinite bus.

The paper is organized as follows. The problem formulation is provided in Section 2. Simplified SG, DFIG, STATCOM and battery models are briefly described in Section 3. Power analysis for the STATCOM and battery system is provided in Section 4. Linearized models of the various operating configurations are derived in Section 5. The LMI controller is given in Section 6. Simulation results are given in Section 7. Conclusions are given in Section 8.

2. PROBLEM FORMULATION

In this paper we are interested in studying the small-signal (local) stability of a nonlinear power system in the region of a steady-state operating point. The nonlinear system is linearized at an operating point to obtain the (stabilizable and detectable) linear state space model given by:

\[ \dot{x}(t) = Ax(t) + B_u w(t) + B_a u(t) \]
\[ y(t) = C_y x(t) \]

where \( x(t) \) is the state variables, \( u(t) \) is the control input, \( y(t) \) is the measured output, and \( w(t) \) is the disturbance (small mechanical power fluctuations, \( \Delta P_m \)) and the system matrices \( A, B_u, B_a \) and \( C_y \) have appropriate dimensions.

The problem of interest can be formulated as a dynamic output feedback controller design problem where \( x_K(t) \) is the state of the controller and:

\[ \dot{x}_K(t) = A_K x_K(t) + B_K y(t) \]
\[ u(t) = C_K x(t) + D_K y(t) \]

with \( A_K, B_K, C_K, \) and \( D_K \) matrices of appropriate dimensions to be determined so that the closed-loop system satisfies:

1. All closed-loop poles are located in the open left-half plane \( \text{Re}(A_{cl}) < 0 \).
2. Frequency and voltage regulation are simultaneously achieved.

The closed-loop system is as follows:

\[ \dot{x}_{cl}(t) = A_{cl} x_{cl}(t) + B_{cl} w(t) \]
\[ y(t) = C_{cl} x_{cl}(t) \]

where

\[ x_{cl}(t) = \begin{pmatrix} x(t) \\ x_K(t) \end{pmatrix}, A_{cl} = \begin{pmatrix} A + B_a D_K C_y & B_K C_y \\ B_K C_y & A_K \end{pmatrix}, B_{cl} = \begin{pmatrix} B_u \\ 0 \end{pmatrix}, C_{cl} = \begin{pmatrix} C_y & 0 \end{pmatrix}. \]

In the next section, we develop simplified nonlinear models of power system elements and use these to develop a linearized model for the design of a dynamic output feedback controller that meets the requirements (1) and (2) given above.

3. POWER SYSTEM MODELS

In this section, dynamic models of synchronous and doubly-fed induction generators, STATCOM and battery are provided.

3.1 Synchronous Generators: SG

A dynamic model of a synchronous generator (SG) can be obtained by representing the SG by a transient voltage source, \( E \), behind a transient reactance. The problem of interest can be formulated as a dynamic output feedback controller design problem where \( u_s \) is the measured output, \( E \) is the mechanical input power, \( P_m \) is the mechanical power, \( P_E = \frac{E V \sin(\delta - \theta)}{X_s} \) is the active power delivered by the generator to the terminal voltage \( V \) of SG, \( \omega_s \) is the synchronous machine speed, \( \omega_s = 2\pi f \) is the system frequency, and \( \theta \) is the angle of the SG terminal voltage.

3.2 Doubly-Fed Induction Generators: DFIG

The DFIG dynamic equations have two rotor inputs, \( V_{dr} \) and \( V_{dr} \), in the \( d-q \) reference frame. By transforming to polar coordinates \( E = \sqrt{E_r^2 + E_q^2} \) and \( \delta = \tan^{-1} E_q/E_r \), an equivalent DFIG dynamic model is:

\[ \dot{\delta} = \omega - \omega_s = \frac{X_r - X_s'}{X_s'} \frac{V \sin(\delta - \theta)}{E} + \omega_s V_r \cos(\delta - \theta) \]
\[ \dot{\omega} = \frac{\omega'}{2H} (P_m - P_E) \]
\[ \dot{E} = -\frac{X_r E}{X_s T_0} + \frac{X_r - X_s'}{X_s T_0} V \cos(\delta - \theta) + \omega_s V_r \sin(\delta - \theta) \]

where \( s = 1 - \omega/\omega_s \) is the speed of the DFIG terminal voltage \( V, \angle \theta = \angle V_r, V_{dr} = V_{dr} + j V_{qr} = V_r e^{j \theta}, \) and \( P_E = \frac{E V \sin(\delta - \theta)}{X_s} \) is the active electrical power delivered by the generator to the terminal voltage \( V \) (identical to the SG case).

3.3 STATCOM/Battery Energy Storage Model

STATCOMs and Batteries can be used to support electrical power networks that have poor voltage, frequency, and power stability (both small-signal and large-signal (transient)) (Song & John [1999], Hingorani & Gyugyi [1999], Ribeiro et al. [2000]), and references therein.

In \( d-q \) coordinates, the dynamic model of a STATCOM and Battery are as follows:
\[
\begin{align*}
\dot{I}_d &= -\frac{R_s \omega_s I_d + \omega_I q + h k \cos(\gamma + \beta)}{L_s} - \frac{\omega_v V_s \cos \beta}{L_s} \\
\dot{I}_q &= -\frac{\omega_I I_q - R_s \omega_s I_q + h k \sin(\gamma + \beta)}{L_s} - \frac{\omega_v V_s \sin \beta}{L_s} \\
\dot{V}_{dc} &= -\frac{1}{C} \left( \frac{R_b R_{dc}}{R_b + R_{dc}} \right) V_{dc} + \frac{\omega_v V_s}{R_b C} k \cos(\gamma + \beta) - \frac{\omega_s I_d}{C} k \sin(\gamma + \beta)
\end{align*}
\]

where \( I_d \) and \( I_q \) are the injected or absorbed per unit d-q STATCOM currents, \( V_{dc} \) is the per unit voltage across the dc capacitor \( C \). \( R_s \) and \( L_s \) are used to model the STATCOM/Battery transformer losses. \( V_s \) denotes the per unit battery voltage. \( V_{dc} \) represents the per unit system side (AC) bus voltage and \( h = \frac{\omega_v V_s}{L_s} \). \( R_b \) and \( R_{dc} \) model the battery and the switching losses, respectively. \( k \) and \( \gamma \) are control variables representing the PWM modulation gain and firing angle, respectively. \( V_{dc} \) is treated as a constant. See Yang et al. [2001] for more details.

The STATCOM/Battery dynamic model is given as follows:

\[
\begin{align*}
\dot{I}_d &= -\frac{R_s \omega_s}{L_s} I_d + \omega_I q - V_{td} + (h \cos \beta) \cdot u_M - (h \sin \beta) \cdot u_\gamma \\
\dot{I}_q &= -\omega_I I_q - \frac{R_s \omega_s}{L_s} I_q - V_{tq} + (h \sin \beta) \cdot u_M + (h \cos \beta) \cdot u_\gamma \\
\end{align*}
\]

where \( V_{td} = \frac{\omega_v V_s \cos \delta}{L_s} \) and \( V_{tq} = \frac{\omega_v V_s \sin \delta}{L_s} \). \( u_M = k \cos \gamma, u_\gamma = k \sin \gamma, k = \sqrt{u_M^2 + u_\gamma^2} \), and \( \gamma = \arctan u_\gamma / u_M \). \( V_i, \cos \beta, \) and \( \sin \beta \) are given in the next Section.

4. ANALYSIS OF TRANSMITTED POWER WITH STATCOM/BATTERY

In this section, we study the transmitted power characteristics of the STATCOM/Battery system using two simplified models consisting of conventional (SG) and wind power generators (DFIG). We assume that any losses in the STATCOM/Battery are negligible, and we model the STATCOM/Battery system as a parallel current source that can inject or absorb both active and reactive power simultaneously. We focus on the transmitted power in single-machine (SMIB) and two-machine infinite bus systems with STATCOM/Battery.

4.1 Single Machine Infinite Bus

![Fig. 1. Network](image1)

Consider the network and a phasor diagram shown in Figs. 1 and 2, respectively, where \( X_1 \) denotes a transformer and transient reactance of the SG or DFIG. \( X_2 \) denotes the transmission line reactance between the bus terminal voltage \( V_t \) and the infinite bus voltage \( V_\infty \). \( I_d \) and \( I_q \) are the active and reactive currents, respectively. \( E \) is the transient voltage of either the SG or DFIG.

Using Kirchoff’s voltage and current laws and extending the results from Song & Johns [1999], the terminal STATCOM/Battery voltage \( V_t \) is given by:

\[
\begin{align*}
V_t &= \left( \frac{E \cos \delta - V_{\infty} \cos \theta}{X_1 + X_2} \right) - j \frac{I_{SB} M}{|V_t|} \\
&= V_t + \alpha - j \left( I_d + j I_q \right) M \left( \frac{V_t \angle \alpha}{|V_t|} \right)
\end{align*}
\]

where \( M = X_1 X_2 / (X_1 + X_2) \), \( (X_1 + X_2) V_t \angle \alpha = X_2 E \cos \delta + X_1 V_\infty \angle \theta \). \( |V_t| \) represents the unit vector of \( V_t \). Using the phasor diagram shown in Fig. 2, the terminal STATCOM/Battery voltage \( V_t \) can be written as follows:

\[
V_t = V_t = \left( 1 + \frac{M}{|V_t|} \right) I_d \left( \frac{M}{|V_t|} \right), \quad \beta = \alpha - \theta
\]

![Fig. 2. Phasor Diagram](image2)

Then, the power, \( P_{E \infty} \), transmitted from bus 1 to the infinite bus is:

\[
P_{E \infty} = \frac{E V_\infty \sin \delta}{(X_1 + X_2)} \left( 1 + \frac{I_d X_1 X_2}{\Delta} \right) + \frac{E X_2 I_d}{(X_1 + X_2)} V_\infty X_1 \cos \delta + E X_2 \frac{\Delta}{\Delta}
\]

where \( \Delta = \sqrt{(E X_2)^2 + (V_\infty X_1)^2 + 2 X_1 X_2 E V_\infty \cos \delta} \).

4.2 Two Machine Infinite Bus

![Fig. 3. Network](image3)
Consider the network and phasor diagrams shown in Figs. 3 and 4, respectively, incorporating a DFIG wind power generator. Here, $X_3$ denotes the transient reactance of the DFIG and transformer and using Kirchoff’s voltage and current laws, the STATCOM/Battery terminal voltage is given as follows:

$$V_L = V_L - jI_f L_1$$

$$= V_L - jI_f M'$$

where $M' = \frac{\alpha}{\sqrt{X_1 X_2}}$, $K = 1 + \frac{\alpha}{\sqrt{X_1 X_2}}$, $AV \omega \alpha = X_2 X_1 E_1 \omega \alpha + X_1 X_3 V_{\omega \alpha} + X_1 X_2 E_3 \omega \alpha$, $\lambda = X_1 X_3 + X_3 X_3 + X_1 X_3$, $\omega I_q$ and $\omega I_q$ denote the active and reactive currents, respectively, and $|V_L|$ is the unit vector of $V_L$.

Fig. 4. Phasor diagram

The STATCOM/Battery terminal voltage can be rewritten as given in (8) and by using phasor diagrams shown in Fig. 4, the transmitted power, $P_{L_{\text{E}}} \text{E}_{\text{E}}$, from bus 1 to the infinite bus is:

$$P_{L_{\text{E}}} = (E_1 V_{\omega \alpha} |Y_{12}| \sin \delta_1 + E_1 E_3 |Y_{13}| \sin(\delta_1 - \delta_3))$$

$$\cdot (1 + M) + \left(\frac{X_2 X_3 E_3^2}{X_3 L} + E_1 V_{1 \omega \alpha} \cos \delta_1ight)$$

$$+ E_3 E_1 |Y_{13}| \cos(\delta_1 - \delta_3) \cdot N$$

where $M = \frac{I_1 X_1 X_3 X_3}{\sqrt{X_1 X_2}}$, $N = \frac{I_1 X_1 X_3 X_3}{\sqrt{X_1 X_2}}$, $A = \sqrt{A^2 + B^2}$, $B = X_2 X_3 E_1 \cos \delta_1 + X_1 X_3 V_{\omega \alpha} + X_1 X_2 E_3 \cos \delta_3$, $\delta = X_2 X_3 E_1 \sin \delta_1 + X_2 X_3 E_3 \sin \delta_1$, $|Y_{12}| = X_3 / L$, and $|Y_{13}| = X_3 / L$. Likewise, the transmitted power, $P_{L_{\text{E}}} \text{E}_{\text{E}}$, from bus 3 to the infinite bus is:

$$P_{L_{\text{E}}} = (E_3 V_{\omega \alpha} |Y_{32}| \sin \delta_3 + E_1 E_3 |Y_{13}| \sin(\delta_3 - \delta_1))$$

$$\cdot (1 + M) + \left(\frac{X_2 X_3 E_3^2}{X_3 L} + E_1 V_{1 \omega \alpha} \cos \delta_1ight)$$

$$+ E_3 E_1 |Y_{13}| \cos(\delta_1 - \delta_3) \cdot N$$

where $|Y_{32}| = X_1 / L$.

5. LINEARIZED POWER SYSTEM MODELS INCLUDING STATCOM/BATTERY

Power system models are highly nonlinear, to study small-signal stability we linearize the models and include the dynamics of the terminal $V_L$ for the single machine infinite bus case, to address the problem of voltage regulation.

Define $\Delta V_L = V_L - V_{\text{ref}}$ as a voltage deviation where $V_{\text{ref}}$ represents the reference voltage and

$$V_L = \frac{\Delta}{(X_1 + X_2)} \sqrt{\left(1 + \frac{L_1 X_1 X_2}{\Delta}\right)^2 + \left(\frac{L_1 X_1 X_2}{\Delta}\right)^2}$$

The objective is to drive $\Delta V_L$ to zero at steady state, so we incorporate an additional state variable $\xi$ (integral action) as follows

$$\dot{\xi}(t) = \Delta V_L = V_L - V_{\text{ref}}$$

The voltage dynamics for the single machine infinite bus system are given as follows:

$$\Delta V_L = F_1(x) \dot{\delta} + F_2(x) E + F_3(x) \dot{I}_d + F_4(x) I_q$$

where $x = (\delta, \omega, E_1, I_d, I_q, \Delta V_L, \xi)$ are the state variables, $F_i$ are given in Kanchanaharuthai et al. [2011], and $M^T$ is the transpose of matrix $M$.

In order to illustrate the benefits of the STATCOM/Battery to both conventional and wind power systems, consider the SMIB case as shown in Fig. 1. There are two SMIB cases of interest, SG and DFIG.

Case 1: SG Dynamic models of the SG, STATCOM, Battery, the integral action (13), and voltage dynamics (14) are described as follows:

$$\dot{\delta} = \omega - \omega_s$$

$$\dot{\omega} = \frac{\omega_s}{2H}(P_m - P_{\text{E}})$$

$$\dot{E} = -aE + b \cos \delta + \frac{u_f}{T_0}$$

$$\dot{I}_d = -\frac{R_{\text{E}} \omega_s}{L_s} I_d + \omega I_q - V_{\omega \alpha} + (h \cos \beta) u_M - (h \sin \beta) u_y$$

$$\dot{I}_q = -\omega I_d - \frac{R_{\text{E}} \omega_s}{L_s} I_q - V_{\omega \alpha} + (h \sin \beta) u_M + (h \cos \beta) u_y$$

$$\Delta V_L = F_1(x) \dot{\delta} + F_2(x) E + F_3(x) \dot{I}_d + F_4(x) I_q$$

where $P_{\text{E}}$ represents the power transmitted from bus 1 to the STATCOM/Battery terminal in (9). $a = \frac{(X_1 + X_3 + X_2)}{(X_1 + X_2) \omega_s}$ and $b = \frac{(X_1 + X_3)}{(X_1 + X_2)}$.

Linearizing the model given above we obtain:

$$\Delta \dot{\delta} = A_S \Delta x + B_{S_u} \Delta P_m + B_{S_u} \Delta u$$

where $A_S, B_{S_u}, B_{S_u}$, and $\Delta u$ are given in Kanchanaharuthai et al. [2011]. $\Delta P_m$ denotes a small mechanical power perturbation as the disturbance.

Case 2: DFIG Dynamic models of the DFIG, STATCOM /Battery, integral action (13), and voltage dynamics (14) are described as follows:

$$\dot{\delta} = \omega - \omega_s$$

$$\dot{\omega} = \frac{\omega_s}{2H}(P_m - P_{\text{E}})$$

$$\dot{E} = -aE + b \cos \delta + \omega_s V_r \sin(\delta - \theta_r)$$

$$\dot{I}_d = \frac{R_{\text{E}}}{}$$

$$\dot{I}_q = -\omega I_d + \omega I_q - V_{\omega \alpha} + (h \cos \beta) u_M - (h \sin \beta) u_y$$

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\[ \dot{I}_q = -\omega I_d - \frac{R_s \omega}{L_s} I_q - V_{i_q} + (h \sin \beta) u_M + (h \cos \beta) u_{\gamma}, \]
\[ \Delta \dot{V}_t = F_1(x) \dot{\delta} + F_2(x) \dot{E}_1 + F_3(x) \dot{I}_d + F_4(x) \dot{I}_q, \]
where \( a, b, P, \dot{E}_1, V_t, I_d, I_q, \Delta P_m \) are identical to SG case. Linearizing the overall dynamic models we get as follows:
\[ \Delta \dot{V}_t = A_{D} \Delta x + B_{D} \Delta P_m + B_{D} \Delta u \]
where \( A_{D}, B_{D}, B_{Du}, B_{Dm} \), and \( \Delta u \) are given in Kanchanaharathai et al. [2011].

For the two machine infinite bus case (SG-DFIG) shown in Fig. 3, let \( \Delta \dot{V}_t = \dot{V}_t - \dot{V}_{ref} \) where \( \dot{V}_t = \frac{\dot{V}_m}{\sqrt{(1+M)^2 + N^2}} \).

The integral action \( \xi \) in (13) is intended to drive \( \Delta \dot{V}_t \) to zero in steady state, and the voltage dynamics for the two machine infinite bus case is as follows:
\[ \Delta \dot{V}_t = \frac{F_1(x) \dot{\delta} + F_2(x) \dot{E}_1 + F_3(x) \dot{I}_d + F_4(x) \dot{I}_q}{\dot{V}_t} \]
where \( x = (\delta_1, \omega_1, E_1, \delta_3, \omega_3, E_3, I_d, I_q, \dot{V}_t, \xi)^T \) are state variables and \( F_i \) are given in Kanchanaharathai et al. [2011].

Case 3: SG-DFIG Dynamic models of the two machine infinite bus system consisting of SG, DFIG, STATCOM/Battery, integral action (13), and voltage dynamics (14) are described as follows:
\[ \dot{\delta}_1 = \dot{\omega}_1 - \omega_1, \quad \dot{\omega}_1 = \frac{\omega_s}{2\Omega T} (P_m - P_{1E}), \]
\[ \dot{E}_1 = -a_1 E_1 + a_2 \cos \delta_1 + a_3 E_3 \cos (\delta_3 - \delta_1) + \frac{u_{\gamma}}{T_{i0}}, \]
\[ \dot{\delta}_3 = \omega_3 - \omega_s - b_1 E_1 \sin (\delta_3 - \delta_1) + b_2 \sin \delta_3 \]
\[ + \frac{\omega_s V_{c} \cos (\delta - \theta_1)}{E_3}, \quad \dot{\omega}_3 = \frac{\omega_s}{2\Omega T} (P_m - P_{3E}), \]
\[ \dot{E}_3 = -c_1 E_3 + c_2 \cos \delta_3 + c_3 E_1 \cos (\delta_3 - \delta_1) \]
\[ + \omega_s V_{c} \sin (\delta - \theta_1), \]
\[ \dot{I}_d = -\frac{R_s \omega}{L_s} I_d + \dot{I}_d - V_{i_d} + (h \cos \beta) u_{M} + (h \sin \beta) u_{\gamma}, \]
\[ \dot{I}_q = -\frac{R_s \omega}{L_s} I_q - V_{i_q} + (h \sin \beta) u_{M} + (h \cos \beta) u_{\gamma}, \]
\[ \Delta \dot{V}_t = F_1(x) \dot{\delta} + F_2(x) \dot{E}_1 + F_3(x) \dot{I}_d + F_4(x) \dot{I}_q \]
\[ + F_5(x) \dot{I}_d + F_6(x) \dot{I}_q, \quad \dot{\xi}(t) = \Delta \dot{V}_t \]
where \( a_1 = \frac{X_{m1} Y_{11}}{Y_{11}}, \quad a_2 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad a_3 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad b_1 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad b_2 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad c_1 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad c_2 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad c_3 = \frac{X_{m1} Y_{11} V_{c} \omega_1}{Y_{11}}, \quad X_{m1} = X_{s1} - X_{s1}, \quad X_{m3} = X_{s3} - X_{s3}, \quad P_{1E} \text{ and } P_{3E} \text{ represent the power transmitted from bus 1 and bus 3, respectively, to the STATCOM/Battery terminal in (11) and (12).} \]

Linearizing the overall dynamic model we obtain:
\[ \Delta \dot{v} = A_{DS} \Delta x + B_{DS} \Delta P_m + B_{DS} \Delta u \]
where \( A_{DS}, B_{DS}, B_{Du}, B_{Dm}, \Delta P_m, \) and \( \Delta u \) are given in Kanchanaharathai et al. [2011].

The linearized power system models for the three cases of interest are given in (15), (16), and (18). We assume that full state information is not available, and the following output variables are directly measurable:
\[ y = (\Delta \omega, \Delta V_t) = C_y \Delta x. \]

The linearized models of the conventional and wind power systems including a STATCOM/Battery are used in the next section to design an output feedback controller to achieve frequency and voltage regulation under small mechanical power perturbations to the system.

6. OUTPUT FEEDBACK CONTROLLER DESIGN

The linearized power system models derived in the previous section are used to design to a full order output feedback controller to achieve the performance requirements given in the Problem Statement, (3)-(4). We use an LMI-based controller design methodology to achieve the following.

**Theorem 1.** For the linearized system, the closed-loop system can meet the expected performance requirements, if and only if there exist symmetric positive definite matrices \( X, Y \) and matrices \( A, B, C, \) and \( D \) such that
\[ \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} < 0 \quad \text{and} \quad \begin{bmatrix} X & I \\ Y & 0 \end{bmatrix} > 0 \]
where \( \Omega_{11} = A X + X A^T + B_{Cu} C + (B_{Du} C)^T, \Omega_{21} = A + \lambda B_{Du} C \), and \( \Omega_{22} = A^T Y + Y A + B_{Cu} C + (B_{Du} C)^T \).

A dynamic output feedback controller can be constructed as follows:
\[ A_K := (N^T)^{-1} (A - B_{Cu} C Y) X - Y B_{Du} C M^T \]
\[ -Y (A + B_{Du} D_{Cu} Y) (M^T)^{-1}, \]
\[ B_K := (N^T)^{-1} (B - Y B_{Du} D), \quad C_K := (C - D_{Cu} C Y) (M^T)^{-1}, D_K := D \] where \( X \) and \( Y \) are arbitrary nonsingular matrices satisfying \( M N^T = I - X Y \).

**Proof:** See Scherer et al. [1997].

7. SIMULATION RESULTS

Simulation results for SMIB and two machine infinite bus power systems are used to illustrate the effectiveness of the STATCOM/Battery in improving small-signal stability.

7.1 Single Machine Infinite Bus

Consider the single line diagram given in Fig. 1 with either SG or DFIG connected through a transmission line to an infinite-bus. The SG delivers 1.0 pu. power and the DFIG delivers 1.03 pu. power, with reference terminal voltages 0.9897 pu. for the SG and 0.9877 pu. for the DFIG, and the infinite-bus voltages are 1.0 pu. for both cases. However, when there is a small step increase of mechanical power (\( \Delta P_m = 0.05 P_m \)), this leads to rotor acceleration and voltage sag. We consider the two cases mentioned in the previous section and investigate the effectiveness of the STATCOM/Battery to improve small-signal stability.
applied to conventional and wind power systems to achieve voltage and frequency regulation.

**Case 1: SG** Fig. 5 shows rotor speed (frequency) and voltage responses with a SG. It is obvious that both frequency and voltage deviations ($\Delta\omega$ and $\Delta V_I$) are zero at steady state, the rotor speed returns to synchronous speed ($\omega \rightarrow \omega_s$) and the voltage response goes to the reference voltage ($V_I \rightarrow 0.9878 \text{ pu.}$).

**Case 2: DFIG** Fig. 6 shows rotor speed (frequency) and voltage responses of the wind power system. It is clear that voltage deviation goes to zero ($V_I \rightarrow 0.9878 \text{ pu.}$). The STATCOM is able to compensate the reactive power to track the reference voltage following a small mechanical change. Similarly, the Battery is able to help manage frequency deviations at steady state equal to zero, the DFIG rotor speed (frequency) eventually recovers $\omega = 1.03\omega_s$ (DFIG slip = 0.03) at steady state.

**Case 3: SG-DFIG** Fig. 7 shows the rotor speed (frequency) and voltage responses of the SG and DFIG. Voltage and frequency regulation are identical to the SMIB cases, namely voltage and frequency deviations converge to zero at steady state ($\omega_1 \rightarrow \omega_s$, $\omega_3 \rightarrow 1.03\omega_s$, and $V_I \rightarrow 0.9780 \text{ pu.}$) as expected.

**Remark 1.** In order to further improve the transient performance of the rotor speed and voltage responses given above, we can use a LMI-based controller design with regional pole placement proposed in Chilali and Gahinet [1996]. From Figs. 5-7, time responses of the proposed controllers with pole placement have better transient response characteristics.

8. CONCLUSIONS

In this paper, we have shown that the combination of a STATCOM and Battery energy storage can be used to enhance the small-signal stability of SG and DFIG generators. An analysis of transmitted power with the STATCOM/Battery configuration is investigated. Simulation results have demonstrated that voltage and frequency regulation are achievable for small disturbances using a linearized system model and a LMI-based control design methodology.

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