Delay-based input shapers in feedback interconnections

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Abstract: A design method for a feedback controller with integrated delay-based command shaper is presented in this paper, based on the Smith predictor concept. PosiCast and related input command shapers (ZV, ZVD, EI etc.) are primarily intended as feedforward filters for reference (set point) signals. Nevertheless, due to their superior effectiveness in suppressing vibrations, there is a good motivation to incorporate them directly into positioning feedback loops for flexible structures. Compared to a purely feedforward setup, the feedback control system is commanded not to excite flexible vibrations by aggressive response of the controller, designed purely for the rigid body dynamics, to unmeasurable disturbances, not only to known reference command. If the shaper is tuned carefully to the flexible mode(-s) of interest, the overall feedback appears fast and responsive compared to the conventional approach of a simple high-frequency-roll-off compensator. Related experimental results (positioning servo with an attached flexible beam) are reported.

Keywords: Systems with time-delays; Disturbance rejection (linear case).

1. INTRODUCTION

Input command shaping is a feed-forward control technique for reducing vibrations in computer controlled flexible machines [2, 10]. Input shaping is implemented by convolving a sequence of time-delayed impulses, defining the input shaper, with the reference signal (step, or any other). The shaped command that results from the convolution is then used to drive the system. If the impulses defining the shaper are arranged in a smart way, the flexible system will respond without vibration to the reference command. The amplitudes and time locations of the impulses are obtained from the system’s natural frequencies and damping ratios.

Input command shaper as a feedforward controller can be regarded as a smart filter of the reference signal, an add-on to a functional reference-tracking feedback control system. If the controlled system is flexible, such a servomechanism typically features oscillatory behavior due to excitation of underlying flexible modes by setpoint changes (step commands), unless special attention - e.g. in the form of properly designed input command shaper, or, in a more complex case, an active damper [13] - is paid to these issues. Therefore, posicast control can be regarded as a complementary measure in a two-degree-of-freedom control scheme, when the feedback loop is closed first to guarantee stability, disturbance rejection and positioning, and then the input command prefilter shapes the reference signal such that the transient response is less oscillatory.

Standard Input shapers have proved most useful in many industrial projects related to controls for flexible devices reference tracking for flexible manipulators and cranes [11], orientation and pointing of solar panels of satellites [7] etc.

Obviously, this standard feedforward configuration does not have any impact on the control system’s response to unmeasurable disturbances. If, for instance, the position of a flexible large telescope [14] is affected by a wind gust, resulted control action pushing the system back to the desired position can easily further excite the flexible vibrations (next to the disturbance itself), unless special attention is paid. In principle, two approaches are possible. If one can measure and evaluate the amount of vibrations in the structure, by tensometers, accelerometers or similar sensors located at suitable points on/in the structure, and if also necessary related actuators are available (e.g. piezo elements), a complex active damping system can be designed with excellent performance [13, 14]. However, in many situations, such measurements and actuators are not available. In this case, the control goal is naturally reduced from active actuation of the flexible modes (in order to attenuate them by means of negative feedback) to a passive concept of, say, insensitiveness of the feedback loop in and around the flexible modes frequencies. Since the input command shapers have proved very efficient in targeting selected vibrational mode(-s), they are clear candidates for such an insensitive feedback key element. Note that for this setup still an adequate mathematical model of the flexible system covering the most prominent flexible modes is necessary, which is fortunately fairly common nowadays thanks to many available FEM-based modelling tools like Comsol Multiphysics (www.comsol.com) or Ansys (www.ansys.com).

The feedback concept for input shaping was first investigated in the papers [9] and [8]. In [9] the author suggests applying classical frequency methods to the augmented
system with a pre-tuned input shaper attached, while the sliding-mode approach to nonlinear control design is elaborated in [8]. In this paper, an alternative procedure is devised: a baseline controller is designed first with acceptable closed loop performance and robustness. Any suitable method for LTI systems can be used, including advanced tools like e.g. LQG theory or H-infinity optimization [15]. In contrast to [9], the shaper is added subsequently (and is therefore not a part of the design model). In order to assure stability and robustness of thus constructed loop, extra connections according to the Smith predictor theory are introduced to compensate for the effect of the shaper’s added dynamics. Analysis of robust stability of the overall feedback interconnection is presented leading to a strong result that the robustness is not deteriorated by addition of the delay-based shaper, compared to the baseline controller.

2. DELAY-BASED INPUT COMMAND SHAPERS

![Fig. 1. Feed-forward input shaping configuration](image)

In the 1990’s Singer and Seering [2] developed alternative methodology and time-domain formulas for the Smith’s posicast [1], giving rise to a new modification with improved robustness, the zero-vibration-derivative shaper. Singer and Seering’s approach is based on analysis of the Smith predictor theory and elaborates the vibration ratio concept (2) defined by (6).

\[ V(\xi, \omega) = e^{-\xi \omega_1} \sqrt{C(\xi, \omega)^2} + |S(\xi, \omega)|^2 \]  
\[ C(\xi, \omega) = \sum_{i=1}^{n} A_i e^{\xi \omega_1} \cos(\omega \sqrt{1 - \xi^2 t_i}) \]  
\[ S(\xi, \omega) = \sum_{i=1}^{n} A_i e^{\xi \omega_1} \sin(\omega \sqrt{1 - \xi^2 t_i}) \]

\( \xi \) is the damping and \( \omega \) is the natural frequency of the system. \( A_i \) is the amplitude, \( t_i \) is trigger time of one of the impulses defining the shaper, \( n \) is the number of these impulses. The vibration equation (2) yields the trigger times and amplitudes (3) of each impulse for zero vibration response of the second order system in the shortest possible time,

\[ \begin{bmatrix} A_1 \\ t_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 + K \\ 1 + K \\ 0 \end{bmatrix} ++ \begin{bmatrix} 1 \\ 1 + K \\ 1 + K \\ 0 \end{bmatrix} , A_i > 0, \sum_{i=1}^{n} A_i = 1 \]  

where \( K \) is given by (4).

\[ K = e^{\frac{-\xi \omega}{\sqrt{1 - \xi^2}}} \]  

So far, the results correspond to classical posicast. Singer and Seering’s formulation however allows adding an extra equation (5) which gives rise to improved robustness w.r.t. model uncertainty. Thus obtained ZVD shaper features weaker dependency of performance on uncertain or drifting frequency of the underlying flexible mode. Resulting shaper is defined by three impulses (compared to just two for posicast or ZV shaper) defined by (6).

\[ \frac{\partial V(\xi, \omega)}{\partial \omega} = 0 \]  
\[ \begin{bmatrix} A_1 \\ t_1 \end{bmatrix} = \begin{bmatrix} 1 \\ (1 + K)^2 \\ 0 \end{bmatrix} \]  
\[ 0 (1 + K)^2 \]  
\[ K^2 \]  
\[ \frac{1}{2} \]

Further development of the vibration ratio concept (2) led the authors to yet another robust variant - extra insensitive shaper (EI, [7]), representing a trade-off between robustness and nominal performance. If one admits some tolerable low level of vibration (7) for the nominal system model, equation (2) can be modified accordingly. Resulting EI shaper is slightly detuned for the targeted mode with parameters given by (8) (and hence does not perfectly suppress related vibrations), though, for a wider considered frequency band, its performance is superior in average compared to ZV or ZVD.

\[ V_i(\xi, \omega) = 5\% \]  
\[ \begin{bmatrix} A_1 \\ t_1 \end{bmatrix} = \begin{bmatrix} 1 + V_i \\ 1 - V_i \\ 1 + V_i \end{bmatrix} \]  
\[ \begin{bmatrix} 4 \\ 0 \end{bmatrix} \]  
\[ \frac{9 \beta d}{2} \]  
\[ \frac{4 \beta d}{2} \]

This fact, and performance and robustness of all introduced shapers in general, can be visualized by the sensitivity curves [2], see Fig.2, which shows dependency of the vibration ratio on (normalized) frequency. That is, the value of vibration ratio at the time of the last impulse when reference command reaches a desired value for systems with uncertainty at natural frequency.

![Fig. 2. Robustness by sensitivity curves of input shapers](image)

3. INPUT SHAPERS IN FEEDBACK LOOPS

Hung proposed in [9] a straightforward augmentation of the standard feedback loop with pre-tuned input command shaper, see Fig. 3. The IS shaper is incorporated in order to prevent the controller to excite flexible modes by control action. Obviously, the extra term IS affects the closed loop performance and robustness. Any suitable method for LTI systems can be used, including advanced tools like e.g. LQG theory or H-infinity optimization [15]. In contrast to [9], the shaper is added subsequently (and is therefore not a part of the design model). In order to assure stability and robustness of thus constructed loop, extra connections according to the Smith predictor theory are introduced to compensate for the effect of the shaper’s added dynamics. Analysis of robust stability of the overall feedback interconnection is presented leading to a strong result that the robustness is not deteriorated by addition of the delay-based shaper, compared to the baseline controller.

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\[ G(s) = \frac{\omega^2}{s^2 + 2\xi \omega s + \omega^2} \]  
\[ V(\xi, \omega) = e^{-\xi \omega_1} \sqrt{C(\xi, \omega)^2} + |S(\xi, \omega)|^2 \]  
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\( \xi \) is the damping and \( \omega \) is the natural frequency of the system. \( A_i \) is the amplitude, \( t_i \) is trigger time of one of the impulses defining the shaper, \( n \) is the number of these impulses. The vibration equation (2) yields the trigger times and amplitudes (3) of each impulse for zero vibration response of the second order system in the shortest possible time,

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behaviour and must be therefore taken as a part of the design model for the controller \( C(s) \). If IS were omitted and just added a-posteriori, one would most probably lose performance or even stability. The trouble is that the input command shaper is an infinite-dimensional system (it does not have a finite state-space representation, its transfer function is not rational) and most of the advanced tools for LTI controllers (LQ, LQG, H2 or Hinfinity optimization) cannot be employed directly. Hung avoids these problems by calling classical frequency methods based on “manual” shaping of the open-loop frequency response [16], by adding zeros and poles at suitable locations. The frequency methods, however popular and elaborated they are, suffer nevertheless from well known drawbacks and limitations (correspondence between time- and frequency-domain is not always transparent and reliable for higher order flexible systems; no optimization criteria can be applied; difficult generalization for true MIMO systems; etc.) and modern control tools, as those mentioned above, could be more appropriate in many applications, if a way is found how to tract them within the introduced framework.

Fig. 3. Hung feedback input shaping configuration

A closed loop transfer function from reference to output, for instance with a ZV shaper,

\[
IS(s) = A_2 + A_1 e^{-st_2}
\]

linear controller \( C(s) \) and linear system \( G(s) \) can be written as

\[
T(s) = \frac{L(s)IS(s)}{1 + L(s)IS(s)} = \frac{C(s)G(s)(A_2 + A_1 e^{-st_2})}{1 + C(s)G(s)(A_2 + A_1 e^{-st_2})}
\]  

Characteristic quasi-polynomial of this configuration is then equal to

\[
p(s) = a(s)p(s) + b(s)q(s)(A_2 + A_1 e^{-st_2})
\]

with infinitely many poles due to the delay element, where

\[
C(s) = \frac{q(s)}{p(s)} \quad G(s) = \frac{b(s)}{a(s)}
\]

To handle delay terms in feedback loops, a powerful concept is known and has been widely applied for decades: the Smith predictor [5]. Originally developed for systems with time delays in process industry, this concept can be readily applied in our case. Resulting closed loop configuration is then given in Fig.4. Note that, analogously to the original Smith predictor idea, \( C(s) \) is designed for the plant \( G(s) \) only (without IS, with its non-rational transfer function), and the IS shaper is included additionally, with a few extra linkages as in Fig.4. Compare to [9] where IS became a necessary part of the design model.

Fig. 4. Feedback input shaping configuration with smith predictor

The overall controller with Smith predictor has the transfer function [5]

\[
C_{smith}(s) = \frac{C(s)}{1 + C(s)G_m(s)(1 - IS(s))}
\]

Smith predictor effectively cancels the effect of shaper in the characteristic quasipolynomial (11) when the model \( G_m \) and time delay are precisely known:

\[
T(s) = \frac{C_{smith}(s)IS(s)G(s)}{1 + C_{smith}(s)IS(s)G(s)}
\]

The characteristic polynomial with the wired Smith predictor is therefore given solely by the system \( G_m(s) \) and linear controller \( C(s) \):

\[
p(s) = a(s)p(s) + b(s)q(s)
\]

4. ROBUST STABILITY

Although the Smith predictor is known to be sensitive to uncertainties in the time delays, it is not an issue in the proposed setup as the delay is actually a part of the controller and is therefore given precisely. Robustness with respect to uncertainties in the model \( G_m(s) \) is another story however and is studied in this section. We can rely on recent results presented in [5] where the multiplicative uncertainty for \( G(s) \) is considered in the time-delay context.

Let

\[
G(s) = (1 + \Delta W(s))G_m(s)
\]

where \( W(s) \) is a weighting frequency-dependent filter, \( \Delta \) is a bounded arbitrary complex number, \( ||\Delta||_{\infty} \leq 1 \), and \( G_m(s) \) is the nominal model. Equation (14),

\[
T(s) = \frac{C(s)IS(s)G(s)}{1 + C(s)G_m(s) - C(s)G_m(s)IS(s) + C(s)G(s)IS(s)}
\]

for the closed loop complementary sensitivity function with properly incorporated Smith predictor as Fig. 4 and precisely given IS term can then be re-written according to [5] as

\[
T(s) = \frac{T_{yr}(s)(1 + \Delta W(s))}{1 + T_{yr}(s)IS(s)\Delta W(s)}IS(s)
\]
where
\[ T_{yr}(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \]
According to the small gain theorem [15],
\[ ||T_{yr}||_\infty < \frac{1}{W(s)} \]  
(17)
Note that robust stability of Fig. 4 is equivalent to robust stability of the feedback connection composed of \( G(s) \) and \( C(s) \) only as the IS term itself is stable. This is actually the best finding one could have imagined: as long as the baseline controller \( C(s) \) is designed as robust w.r.t. uncertainties defined by (14), e.g. by dedicated H-infinity methods like mixed-sensitivity, DK-iterations or robust loopshaping [15], addition of the IS shaper, provided it is interconnected according to Fig.4, will still fully preserve the robustness properties of the baseline interconnection. As stated above, this strong conclusion is due to the fact that the IS part with its delays is known precisely (as a part of the overall controller) and the Smith predictor therefore works perfectly.

Note that the considerations above apply to stable systems only. For unstable systems, extra measures must be taken in the form of MSP-based control (Modified Smith Predictor, [5]).

5. EXPERIMENTAL RESULTS

Experiments with the quanser flexible-link laboratory positioning mechanism [3] are presented in this section, see Fig.5. Position - angle - is the measured signal used for feedback, and the angular position is actuated by a DC motor. Vibrations of the attached flexible beam are indicated by a piezoelement; note that this signal serves just as an indicator of flexible vibrations and it is not used for feedback, according to the set-up considered in this paper.

6. CONCLUSIONS

This paper is devoted to integration of delay-based command shapers into feedback controls for flexible structures. A design approach is proposed based on the Smith predictor concept. Such a way, any design method for linear finite dimensional (read: state-space) system can be readily applied and the delay-based input shaper can be incorporated subsequently so that the properties of the design controllers like tracking performance and robust stability are not degraded. Related experimental results (positioning servo with an attached flexible beam) are reported.

REFERENCES

Fig. 6. Lead compensation

Fig. 7. IS included in feedback

Fig. 8. IS with Smith predictor
Fig. 9. Disturbance compensation


