Automotive Applications of Sliding Mode Control

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Abstract: This paper reviews recent automotive applications of sliding mode control. The design methods discussed include sliding mode optimization, integral sliding mode and higher order sliding mode. Examples of applications of conventional sliding mode as well as advanced techniques in vehicle suspension control, engine throttle position control, parameter estimation and fault diagnosis are summarized.

Keywords: Sliding mode control; Automotive applications; Observers

1. INTRODUCTION

Sliding mode control, due to its decoupling and disturbance rejection features, has found broad application in linear and nonlinear systems with uncertainties and disturbances. See, for example, Utkin and Chang (2002)’s review on sliding mode control in electro-mechanical systems. Automotive systems are typical among such applications. They possess complicated nonlinearities and are subject to significant model uncertainties and external disturbances. A variety of automotive control applications have benefited greatly from sliding mode techniques in terms of control accuracy and robustness.

This article is an effort to review developments in this field during the last decade. In an earlier work, Haskara et al. (2002) provided a tutorial style overview on automotive related sliding mode methods and applications. Their survey focused on sliding mode based extremum seeking, disturbance estimation and compensation, state observation and friction compensation. The reader is recommended to read their review to appreciate recent development.

This paper briefly summarizes the sliding mode techniques popular in automotive systems and presents a survey on recent applications. Controllers and observers utilizing conventional sliding mode, as well as advanced techniques including sliding mode based extremum seeking, integral sliding mode and higher order sliding mode are reviewed. Section 2 provides an overview of the principles and design methods of sliding mode optimization, integral sliding mode and higher order sliding mode. Section 3 to 7 reviews the modeling and control development for a number of applications spanning different levels and components in automotive control. Section 3 provides a summary of extremum seeking control for a time-delayed anti-lock brake system. Section 4 focuses on electronic throttle position control with a backstepping sliding mode design by Pan et al. (2008). Section 5 introduces the work in Choi et al. (2000) of sliding mode control in semi-active suspensions with electro-rheological dampers. Section 6 briefly presents the study of Kim et al. (2001) using integral sliding mode in engine actuator and sensor fault diagnosis and compensation. Section 7 summarizes the gasoline engine parameter estimator in Butt and Bhatti (2008) based on first and second order sliding mode observers. Section 8 provides a brief list of the use of sliding mode in other automotive applications and presents the conclusion.

2. SLIDING MODE DESIGN METHODS

2.1 Extremum seeking

Extremum seeking is a special type of control problems that requires optimization of dynamical systems with unknown performance functions. Sliding mode based approach has been developed in a number of papers (Korovin and Utkin, 1974; Drakunov and Özgüner, 1992; Haskara et al., 2000a; Yu and Özgüner, 2002b; Pan et al., 2003; Yu and Özgüner, 2003). It features a periodic switching function and a strictly monotonic reference function for real-time optimization of unknown performance functions.

To illustrate the approach, consider a first-order system:

\[ \dot{x} = u \]

where \( x \in \mathbb{R} \) is the state variable and \( u \in \mathbb{R} \) is the control input. An objective function

\[ y = F(x) \]

is to be maximized, with only its realtime measurement available.

Define a reference signal

\[ \dot{y} = \rho, \quad \rho > 0, \quad (1) \]

and let the sliding surface to be

\[ s = y - g. \]

The following control

\[ u = k \text{sign}(\pi s/\alpha), \quad \alpha > 0 \quad (2) \]
will enforce the performance $y$ to increase towards its maximum, when proper conditions hold. The sliding mode existence condition is given by

$$\left| \frac{\partial F}{\partial x} \right| k > |\rho|$$

It has been shown (Yu and Özgüner, 2002b; Pan et al., 2003) that in sliding mode, $s = n_0\alpha$, where $n_0$ is a constant depending on the initial condition of $s$. Consequently, $y$ follows $g$ to increase or decrease towards the optimal point.

2.2 Integral sliding mode

Integral sliding mode, as the name indicates, features an integral component in the definition of the sliding surface. The integral term is designed such that the system is in sliding mode from the start. As a result, the system trajectory is enforced to follow a nominal one robustly against disturbances. Also, an estimation of the disturbance is attainable using equivalent control method. Note that the order of the system with integral sliding mode is the same with the original system.

Utkin et al. (2009) provides a thorough introduction on integral sliding mode. Here we present a brief summary of the design method and analysis. Consider a linear system subject to matched disturbance:

$$\dot{x} = Ax + Bu + f(t), \quad |f(t)| < f_0 \quad (3)$$

The control objective is to enforce the trajectories of the system to follow those of a stabilized nominal system

$$\dot{x}_o = Ax_o + Bu, \quad u = Kx_o \quad (4)$$

with $(A + BK)$ Hurwitz.

Let $u = u_0(x) + u_1$ with $u_0 = Kx$. Define the sliding surface as $s = Cx + z$ with $CB$ positive definite, and

$$\dot{z} = -(CAx + CBu_0), \quad z(t_0) = -Cx(t_0)$$

Then

$$\dot{s} = C(Ax + Bu + Bf) + \dot{z} = CB(u_1 + f).$$

The second component of the control input is given by

$$u_1 = -Ms\text{sign}(s), \quad M > f_0.$$ 

to enforce sliding mode condition. In sliding mode, the trajectory of the perturbed system (3) tracks that of the nominal system (4). Also, the equivalent value of $u_1$ is equal to the opposite of the disturbance $f(t)$. Therefore, the disturbance can be estimated from $u_1$ through a low pass filter with $\dot{f} = -u_{av}$:

$$\tau u_{av} + u_{av} = u_1$$

2.3 Second-order sliding mode

The objective of second-order sliding mode is to enforce finite time convergence to a sliding surface, when the sliding surface function has relative degree two with respect to the discontinuous input. The discontinuous input could either be the actual control input or its derivative. In the latter case, the control input fed into the system is continuous, avoiding the chattering phenomenon that usually comes with sliding mode. Also, it is proved that in discrete-time implementations, the accuracy of tracking sliding surface is higher than that in first-order sliding mode (Levant, 2007). Note that second order sliding mode is not to be confused with a sliding function containing higher order derivatives of tracking error.

There are several different algorithms for second-order sliding mode design (Levant, 2007; Bartolini et al., 1998; Utkin et al., 2009, Ch.3). One is the twisting algorithm, where the sign of the derivative of the sliding function $s$ is needed:

$$\ddot{u} = v, \quad v = -Ms\text{sign}(s) - M_1\text{sign}(\dot{s}), \quad M_0 > M_1 > 0$$

A second algorithm is the super-twisting algorithm, where the derivative is not necessary.

$$\dot{u} = -a\sqrt{|s|}\text{sign}(s) + v, \quad M, a > 0$$

There is also a suboptimal algorithm

$$\dot{u} = -r_1\text{sign}(s - s_M/2) + r_2\text{sign}(s_M), \quad r_1 > r_2 > 0$$

where $s_M$ is the measurement of $s$ at the most recent time instant when $\dot{s} = 0$.

The proof of the mechanisms of these algorithms is beyond the scope of this review. Interested readers are referred to relevant references.

3. ANTI-LOCK BRAKE CONTROL

Examples of extremum seeking applications include vehicle Anti-lock Brake System (ABS) and traction control (Drakunov and Özgüner, 1992; Drakunov et al., 1995; Haskara et al., 2000b; Yu and Özgüner, 2002a). In ABS control, it is desired to maximize the tire/road friction and prevent skidding. A major challenge is that the analytical form of the performance function is unknown. The performance function is associated with the optimal tire slip ratio, and depends on the vehicle speed and road condition. Sliding mode based extremum seeking control has been utilized to directly address the uncertainties in the performance function as well as in the system dynamics. For example, in Drakunov et al. (1995), an extremum seeking algorithm was developed for a hydraulic brake system. The tire/road friction was maximized in real-time without gradient information.

Yu and Özgüner (2002a) investigated pneumatic braking systems which are subject to time delay. They established convergence conditions for systems with time delay. Also, the paper provided sufficient conditions regarding the choice of the controller parameters $\rho$, $k$ and $\alpha$ in (1) and (2), to ensure convergence of extremum seeking. In particular, denoting the time delay in the system as $\tau$, the convergence conditions are

$$\max_{t-t_0} \left( \frac{\partial F}{\partial x} \right) \dot{\hat{k}} + \rho \dot{\tau} \leq \alpha \quad (5)$$

$$k\tau < |x_{pk} - x_0| \quad (6)$$

where $x_0$ is the initial state value, and $x_{pk}$ is the value of $x$ when the performance is at the optimum.
The system model was based on the model of the pneumatic system in Acarman et al. (2000), and the parameters were determined from the real data of a truck. It features slow dynamics (with a time constant $\tau_1 = 0.6$ sec) and a significant time delay of $\tau_2 = 0.1$ sec. Extremum seeking control satisfying the conditions in (5) and (6) were employed to maximize the friction force.

4. ELECTRONIC THROTTLE CONTROL

A throttle valve regulates the air intake of the engine and is an important component of automotive systems. In an electronic throttle system, the unit measures the gas pedal level and, based on conditions of the powertrain system and the environment, generates an optimal valve opening angle. The control objective is to track this optimal trajectory. One of the main challenges here include non-smooth nonlinearities from friction, nonlinear spring and gear backlash, which cannot be suppressed by a continuous control. Also, the uncertainties and disturbances in the system do not satisfy matching condition. Sliding mode based throttle position control has been studied in a number of publications. Nakano et al. (2006) utilized a function-augmented sliding hyperplane to enforce the positioning error to converge to zero in finite time. Özgür et al. (2001) derived the model of the throttle valve and presented a discrete-time sliding mode controller for the electronic throttle valve. Dagci et al. (2002) utilized feedback linearization as an intermediate step before sliding mode control. Reichhartinger and Horn (2009) neglected the electrical dynamics to simplify the model and employed second-order sliding mode to eliminate steady state error. This section summarizes the work in Pan et al. (2008) for robust electronic throttle position control. The sliding mode controller with a backstepping design in the paper addresses the mismatched disturbances and model uncertainties, and achieves robust tracking of the reference throttle angle in the presence of parameter uncertainties.

The mathematical model of an electronic throttle is formulated as follows:

$$\dot{\theta} = K_g \omega$$

$$J_{tot} \dot{\omega} = -B_{tot} \omega - T_f(\omega) - T_{sp}(\theta) + K_i i$$

$$L_1 \dot{i} = -K_c \omega - Ri + u$$

where $\theta$ is the throttle angle, $\omega$ is the motor angular velocity, $i$ is the armature current, and $u$ is the motor input voltage. The constant $K_g$ is the total gear ratio from the motor to the throttle valve, $J_{tot}$ lumps the overall inertial on the motor, $B_{tot}$ represents damping ratio, $K_t$ is the torque constant, $K_e$ is the back EMF constant, and $L$ and $R$ are the inductance and resistance of the armature circuit respectively. The friction force is given by $T_f = F_\text{r} \text{sign}(\omega)$, while the spring force is $T_{sp}(\theta) = m_1 (\theta - \theta_0) + D \text{sign}(\theta - \theta_0)$. Here $F_\text{r}$, $m_1$ and $D$ are constants.

The coordinate transformation $x_1 = \theta$, $x_2 = K_g \omega$ and $z = i$ results in

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a_{21}(x_1 - x_{10}) + a_{22} x_2 + a_{23} z - \mu \text{sign}(x_2)$$

$$\dot{z} = a_{32} x_2 + a_{33} z + b_3 u$$

The Backstepping based control design involves two steps. The first step is to treat the state variable $z$ as a fictitious control, and determine its desired trajectory. For the purpose, define the sliding surface as

$$s_1 = cc + \dot{c}, \quad c(t) = \theta(t) - \theta_0(t)$$

where $\theta_0(t)$ is the reference trajectory. The desired trajectory for $z$ to enforce sliding mode is

$$z_d = \frac{1}{a_{23}} (\dot{\theta}_0 - c(x_2 - \dot{\theta}_0) - a_{21}(x_1 - x_{10}) - a_{22} x_2 + \mu \text{sign}(x_2) + k_1 \text{sign}(s_1)),$$

where $k_1 > 0$ is large enough. For practical implementation, it is necessary to replace the $\text{sign}(\cdot)$ function with a smooth approximation such that $z_d$ is smooth.

The second step is to determine the actual control input $u$. Define the sliding surface as the tracking error of $z$ with respect to $z_d$:

$$s_2 = z - z_d$$

The control input that enforces sliding mode is:

$$u = -k_2 \text{sign}(s_2)$$

where $k_2 > |a_{32} x_2 + a_{33} z - z_d|/b_3 + \epsilon_2$ with $\epsilon_2 > 0$.

The unmeasured angular speed and armature current of the motor are estimated by a sliding mode observer:

$$\dot{x}_1 = x_2 + l_1 \text{sign}(\epsilon_1)$$

$$\dot{x}_2 = a_{21}(\dot{x}_1 - x_{10}) + a_{22} \dot{x}_2 + a_{23} z + l_2 \text{sign}(\epsilon_1) - \mu \text{sign}(x_2) - k_2 \text{sign}(x_1 - x_{10})$$

$$\dot{z} = a_{32} \dot{x}_2 + a_{33} z + b_3 u + l_4 \text{sign}(\epsilon_1)$$

where $\epsilon_1 = x_1 - \dot{x}_1$.

5. ELECTRO-RHEOLOGICAL SUSPENSION CONTROL

Vehicle suspension control systems aim to improve ride comfort by minimizing suspension movement, including vertical displacement, roll and pitch. The control is subject to model uncertainties of the suspension systems and disturbances from the road surface. There are three different types of suspension systems in terms of vibration reduction: passive, active and semiactive. Passive suspension systems use oil dampers and are cost effective with limited performance. Active suspensions are equipped with additional sensors and actuators to provide high degree of freedom in control but require high power sources. Semiactive suspensions utilize electric or magnetic field sensitive fluids to achieve controllable damping forces. Regardless of the suspension type, sliding mode control have been widely applied for disturbance rejection and robust control. Sam et al. (2004) studied proportional-integral sliding mode control for active suspension system. Yokoyama et al. (2001) investigated semiactive suspension control of a quarter-car system with magnetic-rheological dampers, utilizing sliding mode to robustly follow a desired reference model. Choi et al. (2000) presented sliding mode control of a full car system with electro-rheological (ER) dampers.

In Choi et al. (2000), sliding mode control is designed for the ER dampers to robustly stabilize the suspension displacement and angles at the origin, in the presence of
model uncertainties and road disturbances. The damping force in an ER damper is modeled as
\[ F_e = k_e x_p + c_e \dot{x}_p + F_{er} \text{sign}(\dot{x}_p) \]
where \( k_e \) and \( c_e \) are constants, \( x_p \) is the displacement of the damper, and \( F_{er} \) changes with applied electric field \( E \):
\[ F_{er} = (A_p - A_r) \frac{2E_0}{h} E^3. \]
Here \( A_p, A_r, L \) and \( h \) are constants associated with the structure of the damper, and the value of constants \( \alpha \) and \( \beta \) can be determined experimentally.

The full-car suspension model is given by
\[ M \ddot{z}_2 = -f_{s1} - f_{s2} - f_{s3} - f_{s4} + F_{er1} + F_{er2} + F_{er3} + F_{er4} \]
\[ J_0 \dot{\theta} = a f_{s1} + a f_{s2} - b f_{s3} - b f_{s4} - a F_{er1} - a F_{er2} + b F_{er3} + b F_{er4} \]
\[ J_s \dot{\phi} = -c f_{s1} + c f_{s2} - c f_{s3} + c f_{s4} + c F_{er1} - d F_{er2} + c F_{er3} - d F_{er4} \]
\[ m_i \ddot{z}_{usi} = f_{si} - F_{eri}, \quad i = 1, \ldots, 4. \]
where \( z_2 \) is the vertical displacement of the suspension, \( \theta \) and \( \phi \) are suspension roll and pitch respectively, and \( z_{usi} \) is the vertical displacement of the unsprung mass. The constant \( M \) is the sprung mass at the i-th wheel, \( J_0 \) and \( J_s \) are the roll and pitch inertia respectively, and \( m_i \) is the mass of the i-th wheel. The constants \( a \) and \( b \) are the distances of the left and right wheels to the roll axis, while \( c \) and \( d \) are the distances of the front and rear wheels to the pitch axis. The resistance forces \( f_{si} \) and \( f_{ti} \) are given by \( f_{si} = k_s (z_{si} - z_{usi}) + c_s (\dot{z}_{si} - \dot{z}_{usi}) \) and \( f_{ti} = k_t (z_{usi} - z_i) \), where \( k_s, c_s \) and \( k_t \) are constants, \( z_{si} - z_{usi} \) is the vertical displacement of the sprung mass at the i-th wheel, and \( z_i \) is the height of the road surface at the i-th wheel.

The control objective is to stabilize the system at the origin in the presence of model uncertainties and road disturbances. The system model can be represented as
\[ \dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u + Dw \]
with \( x = [z_2, \dot{z}_2, \theta, \dot{\theta}, \phi, \dot{\phi}, z_{usi1}, \ldots, z_{usi4}, \dot{z}_{usi4}]^T \), control input \( u = [F_{eri1}, \ldots, F_{eri4}]^T \), and disturbance \( w = [z_2, \ldots, z_4]^T \). There are usually uncertainties in the vehicle mass and inertia \( M_0 + \Delta M \), \( J_0 = J_{00} + \Delta J_0 \), \( J_s = J_{s0} + \Delta J_s \). The parameter uncertainties \( \Delta M, \Delta J_0 \) and \( \Delta J_s \) are characterized by \( \Delta A \) and \( \Delta B \) in the model. It is assumed that
\[ \| \Delta B \| \leq \sigma \| B_0 \|. \]

The sliding surface is designed as \( s = Gx \), where \( G \in \mathbb{R}^{4 \times 4} \) is selected according to Ackermann’s formula, such that the 10-th order dynamics on \( s = 0 \) has desired eigenvalues. The control input to enforce sliding mode is
\[ u = -\left( (GB_0) + \frac{1}{\sigma} \right) (\|G_0x\| + \|G_\Lambda x\|) \text{sign}(s) + K \text{sign}(s) \]
In actual implementation, \( u \) needs to be non-negative. Set \( u_i = 0 \) if \( u_i(z_{usi} - z_{si}) \leq 0 \), and the electric fields to apply at each wheel are calculated as
\[ E_i = \left( \frac{h u_i}{2La(A_p - A_r)} \right)^{1/3} \]

6. ENGINE FAULT TOLERANT CONTROL

Fault tolerant control aims to maintain satisfactory vehicle performance in the event of system faults, which is important for vehicle safety, energy efficiency and emission control. In fault tolerant control, it is essential to detect and isolate the possible faults. Integral sliding mode control is known to be capable of enforcing sliding mode without a reaching phase. Therefore, it guarantees robust system control, which is important in automotive systems. Also, faults or disturbances can be estimated from the switching control, and then be compensated through the controller. In Kim et al. (2001), a fault tolerant controller for a spark ignition engine was developed. Integral sliding mode and observer with hypothesis testing were designed for fault detection, isolation and fault tolerant control. Both actuator fault and sensor fault were considered. The actuator fault can be modeled by
\[ \dot{x} = f(x) + g(x)(u + \Delta u) \]
On the other hand, the model for sensor fault is
\[ \dot{y} = Cx + \Delta y \]
The system model for the spark ignition engine is represented in the domain of crank angle \( \theta \), with the assumption of zero exhaust gas recirculation (EGR):
\[ z_1(\theta) = u_1 x_1(\theta) + b_1 u_1(\theta) \]
\[ z_2(\theta) = u_2 x_2(\theta) + b_2 u_2(\theta) \]
\[ z_3(\theta) = u_3 x_3(\theta) - \frac{a_3 a_4}{z(\theta - \theta_d)} x_2(\theta - \theta_d) \]
\[ z(\theta) = \frac{a_1}{b_1} x_1 \]
with output \( y = [y_1, y_2]^T = [x_1, x_3 + \Delta y_2]^T \). The state variables and inputs are listed below. The term \( \theta_d \) represents system delay. The constants \( \theta_d, a_i \) and \( b_i \) (\( i = 1, \ldots, 4 \)) are known.
\[ x_1 = p_m \] intake manifold pressure
\[ x_2 = m_{ff} \] mass of fuel in fuel film
\[ x_3 = \phi_m \] exhaust equivalence ratio
\[ z = m_{ac} \] air mass rate into cylinder
\[ u_1 = m_{ath} \] air mass rate at throttle
\[ u_2 = m_{fj} \] fuel flow rate from the injector.
The unmeasured state variable \( z = m_{ac} \) is estimated through a sliding mode observer. The baseline air-fuel-ratio (AFR) control is determined by
\[ u_{20} = \frac{v}{1 - \frac{z}{\phi_s}} - \frac{m_{ff}}{1 - \frac{z}{\phi_s}} + K_p(\phi_s - y_2) \]
The authors considered only fuel injector (\( u_2 \)) fault and UEGO sensor (\( y_2 \)). To design the integral sliding mode controller, let
\[ u_2 = u_{20} + v \]
Denote \( s_0 = y_2 - x_{3d} \) and define the sliding surface as
\[ S = s_0 + \Psi \]
with
\[ \Psi = -a_3 y_2(\theta) + \frac{a_3 a_4}{z(\theta - \theta_d)} x_2(\theta - \theta_d) + a_4 b_4(\theta - \theta_d) \]
\[ + \frac{a_3 b_4}{z(\theta - \theta_d)} u_2(\theta - \theta_d) + \dot{x}_{3d} \]
and \( \Psi(0) = -y_2(0) + x_{3d}(0) \). The switching component of the control is
\[
\nu(\theta) = M \text{sign}(S)
\]
with \( M > 0 \) large enough to enforce sliding mode. An estimate of the fault or its derivative is then obtained based on equivalent control method. If it is a fuel injector fault, \( v(\theta) = M \text{sign}(S) \).

The actual system output is compared with the outputs of observers to determine the type of the fault. If the fault comes from the fuel injector, the actuator displacement can be estimated through (7). In the case of a UEGO sensor fault, the fault can be estimated from (8) along with the equivalent control in the second observer:
\[
\Delta \hat{y}_2 = \frac{1}{a_3} \left( \Delta \hat{y}_2 - [L_3 \text{sign}(y_2(\theta) - \hat{y}_2(\theta))]_{eq} \right)
\]
The estimate can be incorporated in the controller to compensate the fault and achieve fault tolerant control.

7. GASOLINE ENGINE PARAMETER ESTIMATION

For high performance control and fault diagnosis of the gasoline engine, accurate knowledge of the system parameters is highly desirable. Sliding mode observers can be applied to nonlinear systems for robust estimation of system state variables or unknown parameters. They are known to have a simple implementation structure as well as robustness against model uncertainties and perturbations. Hasan and Miano (1999) discussed the second-order sliding mode based estimation of two engine parameters, cylinder pressure and indicated torque, that are difficult to obtain otherwise. Butt and Bhatti (2008) considered the estimation of three parameters. First order and second order sliding mode observers were designed to estimate discharge coefficient of throttle body, load torque and indicated torque. The second order observer was able to estimate two unknown parameters at the same time.

The throttle-discharge coefficient represents the ratio of the actual to the ideal flow rates of air mass:
\[
C_d = \frac{m_{\text{actual}}}{m_{\text{ideal}}}
\]
The indicated torque \( T_i \) is assumed to be a function of the intake manifold pressure \( P_m \) in the form of \( T_i = a_1 P_m \) with the coefficient \( a_1 \) to be estimated.

The system model for the engine is given by a mean-value engine model
\[
P_m = -C_1 \eta_\nu P_m \omega + A_k (C_d - \Delta C_d) f(P_m) \alpha
\]
where \( \omega \) is engine speed, \( C_d \) is a nominal value for the throttle discharge coefficient, \( \alpha \) is throttle opening angle, and \( f(P_m) \) are known constants and function. The parameters \( \Delta C_d \), \( a_1 \) and \( L_T \) are to be estimated.

Let \( q_1 = P_m, q_2 = \omega, q_3 = \hat{\omega} \). The observer is
\[
\begin{align*}
\dot{q}_1 &= -C_1 \eta_\nu q_1 q_2 + A_k (C_d - \Delta C_d) f(q_1) \alpha - k_s \text{sign}(s_1) \\
\dot{q}_2 &= \hat{q}_3 \\
\dot{q}_3 &= \chi(s_2)
\end{align*}
\]
where the sliding surfaces
\[
s_1 = q_1 - q_1 \\
s_2 = q_2 - q_2
\]
and
\[
\chi(s_2) = \begin{cases} -k_m \text{sign}(s_2), & s_2 \delta_s > 0 \\ -k_m \text{sign}(s_2), & s_2 \delta_s \leq 0 \end{cases}
\]
are known and \( \delta_s = s_2(k) - s_2(k - 1) \) is the difference of \( s_2 \) at sampling interval \( T \), and \( k_M > k_m > 0 \).

Then the estimate for throttle discharge coefficient correction is
\[
\Delta C_d = \frac{k_s \text{sign}(s_1) \omega}{A_k f(q_1) \alpha}
\]
For indicated torque coefficient and load torque
\[
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}^{-1} \begin{bmatrix} q_1 + a_2 q_2 + a_3 q_3 \\ q_3 + a_2 q_2 + a_3 q_3 \\ q_4 + a_2 q_2 + a_3 q_3 \end{bmatrix}
\]
where \( G_1 = A_k (C_d - \Delta C_d) f(q_1) \alpha, G_2 = -(C_1 \eta_\nu q_1 q_2 + a_2 q_1 + a_3 q_3 q_1), \) and \( G_3 = a_2 + 2a_3 q_2 \).

8. CONCLUSION

There have been many other applications of sliding mode in the automotive field. For example, Fu and Özgüner (2009) proposed a sliding mode based approach for vehicle source tracking in a constrained field. Haskara et al. (2004) developed a sliding mode observer based disturbance estimator for robust camless engine control. Gokasan et al. (2006) proposed sliding mode engine speed and torque control for efficiency improvement in series hybrid-electric vehicles. Canale et al. (2008) applied second-order sliding mode in rear active differential control for better steering characteristics. M’Sirdi et al. (2008) proposed to use second-order sliding mode observer to estimate vehicle dynamic parameters.

Sliding mode control has brought great performance improvement to automotive systems, thanks to its simple implementation structure and disturbance rejection features. In a great number of applications, it has successfully
addressed the main issues in automotive control, including system nonlinearity, disturbances, model uncertainties, fault detection, as well as real-time optimization.

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