Some remarks on stabilizability of linear stochastic hybrid systems.∗

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Abstract: The problem of the stabilizability of stochastic linear and bilinear hybrid systems is considered. Construction methods of stabilizing control signals for a class of linear and bilinear systems are derived. Also, the possibility of stabilizing of unstable dynamic systems with the hybrid control signal given in the form of a signal switching is discussed. The obtained results are illustrated by examples and simulations.

Keywords: hybrid systems, bilinear systems, stability, stabilizability, switching rules.

1. INTRODUCTION

Hybrid systems are dynamic systems consisting of several structures described by deterministic or stochastic differential equations [3], [5], [8]. In the successive moments of time their structures may change according to the given switching rule thereupon creates the hybrid system. Since systems in the real world often need to run for a long period of time, problems of stability and control of hybrid systems have recently received a lot of attention (see for example [2], [10], [8], [16]). Hybrid systems are applicable in many fields for instance in nuclear, thermal, chemical processes, biology, socioeconomics, immunology and many other.

It is well known fact that even if all subsystems are stable the whole hybrid system with a special switching signal can be unstable [8]. Since the application of some switching signals leads to the instability of the system it is natural to ask about the existence of a class of switching signals that makes the system is stable. By the switching control theory we mean a constructing of switching signals that render the hybrid system is stable. In this case the switching signal is also the switching control. In this paper we give sufficient conditions for the stabilization of the considered class of hybrid systems, and present some methods of constructing stabilizing switching signals.

We find sufficient conditions for the stabilizability of a special class of linear hybrid systems (LHS) satisfying Lie-algebra conditions. These conditions are used in the determination of mathematical models of aperiodic processes appearing in chemical, biochemical reactions, electrical power systems, robotics and nonholonomic mechanical systems, see for instance [1], [14], [4], [15], [12].

The paper is organized as follows. In the second section we introduce the notation and basic definitions that we use in this paper. In next sections we find switching controls for LHS with any switching and we present the method for constructing stabilizing switching signals which also is a state depending controller for the hybrid system. In the fifth section we consider the possibilities of applying stabilizing switching signals to stabilize systems with one structure. In the sixth section we find open-loop controls for bilinear hybrid systems with any switching.

2. MATHEMATICAL PRELIMINARIES

Throughout this paper we use the following notation. Let |·| and <·> be the Euclidean norm and the inner product in ℜn, respectively. By λ(A) we denote the eigenvalue of the matrix A, λmin(A) and λmax(A) denotes the smallest and the biggest eigenvalue of the matrix A, respectively. We denote the indicator function of a set G by 1G. We mark ℜ+ = [0, ∞), T = [t0, ∞), t0 ≥ 0. Let Ξ = (Ω, ℱ, {ℱt}t≥0, ℙ) be a complete probability space with a filtration {ℱt}t≥0 satisfying usual conditions. Let w(t) = [w1(t),…,wm(t)]T, t ≥ 0 be the m - dimensional Wiener process defined on the probability space Ξ, where wκ are standard Wiener processes mutually independent and independent of initial values. The process σ(x(t), t) : ℜn × ℜ+ → S is a right - continuous switching signal (cadlag) and S = {1,…,N} is the set of states. We assume that processes wκ(t) and σ(t) are mutually independent and both are {ℱt}t≥0 adapted.

Let us consider a linear hybrid system described by the vector Itō differential equation

\[ d\mathbf{x}(t) = \mathbf{A}(\sigma(t))\mathbf{x}(t)dt + \sum_{k=1}^{m} \mathbf{B}_k(\sigma(t))\mathbf{x}(t)dw_k, \]

\[ \mathbf{x}(t_0) = \mathbf{x}_0, \quad \sigma(t_0) = \sigma_0, \]
where $x \in \mathbb{R}^n$, $t \in T$, $A(\cdot), B_k(\cdot) : S \to \mathbb{R}^{n \times n}$, $k = 1, \ldots, m$, $x_0 \in \mathbb{R}^n$ and $\sigma_0 \in S$ are initial values.

For any twice continuously differentiable function $\phi(\cdot, t)$ the $t$-th process has a generator $L_t$ (Itô operator for $t$-th subsystem of system (1)) given by

$$L_t \phi(x, l) = x^T A(l)^T \triangledown \phi(x, l) + \frac{1}{2} \sum_{k=1}^{m} \text{tr}(B_k(l)x^T B_k(l)^T \triangledown^2 \phi(x, l)), \quad l \in S,$$  

where $\triangledown$ and $\triangledown^2$ denote the gradient and Hessian, respectively.

We use the following definition of the stochastic stability:

**Definition 1.** The null solution $x \equiv 0$ of the stochastic differential equation (1) is said to be $p$-th mean exponential stable if there exists a pair of positive scalars $\alpha, c$ such that $\forall (x_0, t_0) \in \mathbb{R}^n \times \mathbb{R}$,

$$E[|x(t, x_0, t_0)|^p] \leq cE[|x_0|^p] \exp\{-\alpha(t - t_0)\}, \quad t \geq t_0$$

or if

$$\limsup_{t \to \infty} \frac{1}{t} \log(E[|x(t, x_0, t_0)|^p]) \leq -\alpha, \quad t \geq t_0.$$

The left hand side of (4) is called the $p$-th mean Lyapunov exponent of the solution of equation (1). In the case of $p = 2$ the null solution $x \equiv 0$ of the stochastic differential equation (1) is called exponential mean-square stable.

At the end of this section we remind some basic facts concerning a Lie algebra.

**Definition 2.** A Lie algebra $L$ over a field $\mathbb{R}$ is a triple $(V, +, [, \cdot])$, where $(V, +)$ is a vector space over a field $\mathbb{R}$ and $[,] : V \times V \to V$ is a bilinear mapping such that

1. $[v_1, v_2] = -[v_2, v_1]$
2. $[v_1, [v_2, v_3]] + [v_2, [v_3, v_1]] + [v_3, [v_1, v_2]] = 0$.

For a Lie algebra of matrices we have $[A, B] = AB - BA \forall A, B \in L$, $L(A_1, \ldots, A_r)$ denotes a Lie algebra generated by matrices $A_1, \ldots, A_r$.

**Definition 3.** For every Lie algebra $L$ we find

1. $L^0 = L$
2. $\ldots$
3. $L^{(n+1)} = [L^n, L^n]$.

We say that a Lie algebra $L$ is solvable if $L^n = \{0\}$ for some $n$.

**Lemma 1.** A Lie algebra of matrices $L(A_1, \ldots, A_r)$ is solvable iff there exists a nonsingular matrix $M$ such that matrices of a form $MA_iM^{-1}$ are upper-triangular for every matrix $A_i \in L$, $1 \leq i \leq r$.

3. **STABILIZATION OF LHS FOR ANY SWITCHING**

We consider the stabilizability of a hybrid system consisting of linear subsystems [10]

$$dx(t) = [A(\sigma(t))x(t) + C(\sigma(t))u(\sigma(t))]dt + \sum_{k=1}^{m} [B_k(\sigma(t))x(t) + D_k(\sigma(t))u(\sigma(t))]dw_k(t),$$

$$x(t_0) = x_0, \quad \sigma(t_0) = \sigma_0,$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the control vector, $t \in T$, $A(\cdot), B_k(\cdot) : S \to \mathbb{R}^{n \times n}$, $C(\cdot), D_k(\cdot) : S \to \mathbb{R}^{p \times n}$, $k = 1, \ldots, m$ are piecewise constant matrices, $x_0 \in \mathbb{R}^n$ and $\sigma_0 \in S$ are initial values.

We are looking for a control signal of the form $u(x(t), \sigma(t)) : \mathbb{R}^n \times S \to \mathbb{R}^p$ that stabilizes system (5).

We will find sufficient conditions which must be satisfied by each structures of the hybrid system to ensure the stability of the whole hybrid system with any switching. We assume that all subsystems of the hybrid system are stable. If not and for example the $j$-th system is unstable, then the hybrid system will be unstable, because we may set $\sigma(t) \equiv j$.

First we recall some facts about the stability of hybrid systems. It is well known that the stability of all subsystems of the hybrid system is not sufficient for the stability of the whole hybrid system [8]. Hence it follows that to establish the sufficient conditions of the stability of hybrid systems we had to find an additional condition. To do it we use the common Lyapunov approach.

**Definition 4.** If there is a common definite constant matrix $P$ satisfying

$$A(l)^T P + PA(l) + \sum_{k=1}^{m} B_k(l)^T PB_k(l) < 0, \quad \forall l \in S,$$

then $V(x) = x^T Px$ is called a common quadratic Lyapunov function for all subsystems of system (1).

**Theorem 1.** [13] If there is a common quadratic Lyapunov function for all subsystems, then the hybrid system (1) is exponentially mean-square stable for any switching.

Theorem 1 with use of a common quadratic Lyapunov function establishes sufficient conditions for the exponential mean-square stability for linear hybrid systems with any switching.

Other sufficient conditions of the exponential mean-square stability for hybrid systems can be proposed for LHS with a special structure defined by the Lie algebra generated by the matrices $A(l), B_k(l), l \in S$.

**Theorem 2.** [13] If the Lie algebra $L(A(l), B_k(l), l \in S = k = 1, \ldots, m)$ is solvable and furthermore for all $l \in S$

$$2Re(\lambda_j(A(l))) + \sum_{k=1}^{m} |\lambda_j(B_k(l))|^2 < 0, \quad j = 1, \ldots, n,$$

then the hybrid system (1) is exponentially mean-square stable for any switching.

**Remark 1.** If all eigenvalues of matrices $B_k(l), k = 1, \ldots, m$ are real, then condition (7) is equivalent to the Hurwitz character of matrices $2A(l) + \sum_{k=1}^{m} B_k^2(l), l \in S$.

We note that Theorem 2 is very useful in many switching control problems. Suppose that we have the hybrid system (5) and suppose that we can design a set of feedback controllers $u(x(t), \sigma(t)) = K(\sigma(t))x(t)$, $K(\cdot) : S \to \mathbb{R}^{p \times n}$ such that the Lie algebra

$$L(A(l) + C(l)K(l), B_k(l) + D_k(l)K(l),$$

$$k = 1, \ldots, m, l \in S)$$

is solvable and condition

5725
is satisfied. Then according to Theorem 2 the control signal \( u(x(t), \sigma(t)) = K(\sigma(t))x(t) \) stabilizes system (5).

**Example 1.** Let us consider the hybrid system (5) with \( n = 2, N = 2, m = 1, p = 1 \) defined by matrices

\[
A(1) = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}, \quad C(1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix},
\]

\[
B(1) = \begin{bmatrix} 0.1 & 0.5 \\ 0.5 & 0.2 \end{bmatrix}, \quad D(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},
\]

\[
A(2) = \begin{bmatrix} -2 & -1 \\ 0.5 & -1 \end{bmatrix}, \quad C(2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]

\[
B(2) = \begin{bmatrix} 0.5 & 0.01 \\ 0.1 & 0.04 \end{bmatrix}, \quad D(2) = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix},
\]

We find that the hybrid controller \( u \) given by

\[
u(x(t), 1) = -7x_1, \quad u(x(t), 2) = 0.5x_2 \]

stabilizes the hybrid system (5) with matrices (10)-(11) because conditions (8) and (9) with \( u \) given by (12) are satisfied.

![Fig. 1. A path of system (5) with \( u \equiv 0 \).](image1)

![Fig. 2. A path of system (5) with \( u \) given by (12).](image2)

### 4. Stabilization of LHS by Switching Signals

We consider a hybrid system described by the vector Itô differential equation in every structure

\[
dx(t) = A(x(t))x(t)dt + \sum_{k=1}^{m} B_k(\sigma(t))x(t)dw_k(t),
\]

where \( x \in \mathbb{R}^n \) is the state vector, \( t \in \mathbb{T}, A(\cdot), B_k(\cdot) : \mathbb{S} \rightarrow \mathbb{R}^{n \times n}, k = 1, \ldots, m, x_0 \in \mathbb{R}^n \) and \( \sigma_0 \in \mathbb{S} \) are initial values.

We are looking for a stabilizing control signal of the form \( u(x(t), t) \equiv \sigma(x(t), t) \).

We find sufficient conditions for existence of the switching rule that renders the whole system is stable.

We assume in this case that none of the subsystems is stable. If at least one of the individual structures is stable (for example the \( j \)-th subsystem), then the problem is trivial because we may set \( \sigma(t) \equiv j \).

Since the application of some switching signals leads to the instability of the hybrid system we may ask if it is possible to find a switching signal that renders the stochastic hybrid system is stable. Such stabilizing signals may exist even if all the individual subsystems are unstable [8]. It follows from a simple example.

**Example 2.** A stabilizing switching signal for a hybrid system with all unstable structures.

Let us consider a special case of the hybrid system (13) with two unstable structures, \( n = 2, N = 2, m = 1 \), defined by the following matrices

\[
A(1) = \begin{bmatrix} 0.2 & -0.4 \\ 3.0 & 0.2 \end{bmatrix}, \quad B(1) = \begin{bmatrix} 0.1 & 0.06 \\ 0.06 & 0.3 \end{bmatrix}
\]

and

\[
A(2) = \begin{bmatrix} 0.3 & -3.0 \\ 0.6 & 0.3 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0.02 & 0.05 \\ 0.06 & 0.03 \end{bmatrix}.
\]

Despite the fact that both structures are unstable, the hybrid system (13) can be stabilized by the special switching \( \sigma^*(x(t), t) \) defined as

\[
\sigma^*(x(t), t) = \begin{cases} 1 & x_1(t)x_2(t) < 0 \\ 2 & x_1(t)x_2(t) > 0. \end{cases}
\]

Exemplary simulations are shown in Figure 3-4.

Now, we consider a convex combination of the structure matrices \( A(l) \), namely

\[
A = \sum_{l=1}^{N} \alpha_l A(l), \quad \alpha_l \in (0, 1), \quad \sum_{l=1}^{N} \alpha_l = 1.
\]

We extend results of Liberzian and Morse [7] to stochastic case.

**Theorem 3.** If there exists \( \alpha_l \in (0, 1) \) such that a system with one structure described by

\[
dx(t) = A(x(t))x(t)dt + \sum_{l=1}^{N} \sum_{k=1}^{m} \sqrt{\alpha_l} B_k(l)x(t)dw_k(t)
\]

is satisfied.
Then \( \mathcal{L}_t V(x) < 0 \) for function \( V(x) = x^T P x \) in the region \( \Omega_t \), where \( \mathcal{L}_t \) is the Itô operator for the \( t \)-th subsystem of system (13). Using this fact it is possible to construct a switching signal such that \( \sum_{l=1}^{N} \mathcal{L}_l V(x) \|_{\Omega_l} < 0 \) along solution of (13) and

\[
x^T \left( A(\sigma(x,t))^T P + PA(\sigma(x,t)) + \sum_{k=1}^{m} B_k(\sigma(x,t))^T PB_k(\sigma(x,t)) \right) x < 0 \quad \forall x \neq 0 \tag{23}
\]

which implies the exponential mean–square stability of system (13) [6].

We note that in this case a stabilizing switching signal takes the state feedback form, i.e. the value of \( \sigma \) at any given time \( t \geq 0 \) depends on \( x(t) \) and \( \sigma = \sigma(x,t) \). The proof of Theorem 3 gives us the method for constructing stabilizing switching signals for hybrid systems. The problem of the stability of hybrid systems reduces to the problem of the stability of systems with one structure.

### Example 3. Hybrid control signal.

Let us consider a special case of system (13) with \( n = 2, N = 2, m = 1 \) defined by the following matrices

\[
A(1) = \begin{bmatrix} 1 & 2 \\ -10 & 4 \end{bmatrix}, \quad B(1) = \begin{bmatrix} 0 & 0 \\ 0.01 & 0 \end{bmatrix} \tag{24}
\]

and

\[
A(2) = \begin{bmatrix} -2 & 5 \\ 2 & 0 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0 & 0 \\ 0.02 & 0 \end{bmatrix}. \tag{25}
\]

Matrices \( A(l), l = 1, 2 \) are not stable but the matrix \( A = \alpha A(1) + (1 - \alpha) A(2) \) is stable, for example for \( \alpha = 0.25 \) and the following system

\[
dx(t) = Ax(t) dt + \sum_{l=1}^{2} \sqrt{\alpha_l B(l)} x(t) dw(t) \tag{26}
\]

with \( \alpha_1 = \alpha, \alpha_2 = 1 - \alpha \) is exponentially mean–square stable. Using Theorem 3 we can see that the hybrid system (13) with matrices (24)-(25) can be stabilized by a hybrid control signal which can be constructed as it is shown in the proof of Theorem 3.

Also an opposite way is possible, i.e. we may stabilize a class of (non-hybrid) linear systems by a special switching control corresponding to a linear hybrid system.

### 5. STABILIZATION OF SYSTEMS WITH ONE STRUCTURE BY THE SWITCHING CONTROLLER

Suppose that we consider a linear control system with one structure (\( N = 1 \))

\[
dx(t) = (Ax(t) + Cu) dt + \sum_{k=1}^{m} B_k x(t) dw_k(t), \tag{27}
\]

where \( x \in \mathbb{R}^n \) is a state vector, \( u \in \mathbb{R}^p \) is the control vector, \( t \in \mathbb{T}, A, B \) are \( n \times n \)-dimensional matrices, \( C \) is...
a $n \times p$ dimensional matrix, $x_0 \in \mathbb{R}^n$ is an initial value. It is possible to stabilize (27) by using a hybrid control signal with a finite number of discrete states [7].

Suppose that we have a collection of constant matrices $K_1, \ldots, K_N$, $K_i \in \mathbb{R}^{n \times n}$, $l = 1, \ldots, N$. Setting $u = K_i x$, $l \in S$ we obtain the family of systems with the index $l \in S$.

$$dx(t) = (A + CK_i)x(t)dt + \sum_{k=1}^{m} B_k x(t)dw_k(t) \quad (28)$$

Thus we can stabilize the origin system (27) if we can find a switching law which renders the hybrid system (28) is stable. To find a stabilizing switching rule we can use Theorem 3. Let us denote $A(l) = A + CK_l$, $l \in S$, $A = \sum_{l=1}^{N} \alpha_l K_l(l)$ and assume that none of subsystems is stable which makes the problem is trivial. Next we find $\alpha_l$ such that assumptions from Theorem 3 are satisfied that is

$$A^T P + PA + \sum_{k=1}^{m} B_k^T P B_k < 0 \quad (29)$$

and system (28) can be stabilized by a linear feedback control signal $u = K x(t)$ with $K = \sum_{l=1}^{N} \alpha_l K_l$, because

$$A = \sum_{l=1}^{N} \alpha_l A_l = \sum_{l=1}^{N} \alpha_l (A + CK_l) = A + C \sum_{l=1}^{N} \alpha_l K_l = A + CK. \quad (30)$$

In some cases even if the thesis of Theorem 3 is not satisfied we still can find a stabilizing switching rule which follows from the following example.

**Example 4. Stabilizing switching signals.**

Let us consider a special case of system (27) with $n = 2, \ m = 1, p = 1$

$$d\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3.75 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} dt + \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} ud, \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} dw. \quad (31)$$

This system cannot be stabilized by a static control signal, but it still can be stabilized by a hybrid control signal. One of the possible stabilizing strategy is letting $u = -0.25x_1$ and $u = 3.5x_1$ for the first and the second structure, respectively. Then the corresponding equations for the closed loop structure (with feedback controls) have the form

$$d\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} dt + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} dw \quad (32)$$

$$d\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} dt + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} dw \quad (33)$$

Let us denote $A = \alpha A_1 + (1 - \alpha) A_2, \alpha \in (0, 1)$. We note that there is not possible to find an $\alpha \in (0, 1)$ such that $A$ is a stable matrix and therefore we cannot use Theorem 3 but we still can stabilize system (31) with the following switching signal $\sigma = \sigma^{**}(x(t), t)$ given by

$$\sigma^{**}(x(t), t) = \begin{cases} 1 & \text{for } x_1(t)x_2(t) > 0 \\ 2 & \text{for } x_1(t)x_2(t) < 0. \end{cases} \quad (34)$$

6. STABILIZATION OF BILINEAR HYBRID SYSTEMS BY OPEN-LOOP CONTROLLERS

Let us consider now a bilinear hybrid system described by the vector Itô differential equation

$$dx(t) = [A(\sigma(t))x(t) + u(\sigma(t))C(\sigma(t))x(t)]dt + \sum_{k=1}^{m} B_k(\sigma(t))x(t)dw_k(t), \ x(t_0) = x_0, \ (35)$$

where $x \in \mathbb{R}^n, \ u \in \mathbb{R}^p, \ t \in T, A(\cdot), C(\cdot), B_k(\cdot) : S \to \mathbb{R}^{n \times n}$, $i = 1, \ldots, p, \ k = 1, \ldots, m, x_0 \in \mathbb{R}^n$ and $\sigma_0 \in \mathbb{S}$ are initial values. We assume that the Lie algebra

$$L(A(l), B_k(l), C(l), l \in S, \ k = 1, \ldots, m) \quad (36)$$

is solvable [9]. To stabilize system (35) we seek a piecewise constant hybrid controller i.e.

$$u(\sigma(t)) = \beta(\sigma(t)), \ \beta(\cdot) : \mathbb{S} \to \mathbb{R} \quad (37)$$

such that the hybrid system

$$dx(t) = [A(\sigma(t)) + \beta(\sigma(t))C(\sigma(t))]x(t)dt + \sum_{k=1}^{m} B_k(\sigma(t))x(t)dw_k(t), \ x(t_0) = x_0, \ (38)$$

is exponentially mean-square stable. Because of the solvability of the Lie algebra $L$ given by (36) there exists a matrix $M$ (Lemma 1) that simultaneously triangularizes matrices $A(l), C(l), B_k(l)$, i.e., $A(l) = MA(l)M^{-1}$, $B_k(l) = MB_k(l)M^{-1}$, $k = 1, \ldots, m, C(l) = MC(l)M^{-1}$ are upper triangular. In this case the set of eigenvalues of system matrices is given by

$$\lambda(A(l) + \beta(l)C(l)) = \{\bar{\alpha}_{jj}(l) + \beta(l)\bar{\epsilon}_{jj}(l), \ j = 1, \ldots, n\}, \quad (39)$$

$$\lambda(B_k(l)) = \{\bar{b}_{jj}(l)^k, \ j = 1, \ldots, n\}, \ k = 1, \ldots, m. \quad (40)$$

In order to stabilize system (38) we can find a set of stabilizing controllers using Theorem 2

$$\Gamma_l = \{\beta(l) \in \mathbb{R} : Re(\bar{a}_{jj}(l) + \beta(l)\bar{\epsilon}_{jj}(l))$$

$$+ \frac{1}{2} \sum_{k=1}^{m} \bar{b}_{jj}(l)^k < 0, \ j = 1, \ldots, n\}, \ l \in S, \quad (41)$$

which is given by $n$ linear restrictions for every $l \in S$.

**Example 5.** Let us consider the hybrid system (35) for $n = 2, N = 2, \ m = 1, p = 1$ defined by the following matrices

$$A(1) = \begin{bmatrix} -0.5 & 0.1 \\ 0.1 & -0.5 \end{bmatrix}, \ C(1) = \begin{bmatrix} -0.4 & 0.3 \\ 0.3 & -0.4 \end{bmatrix}, \quad (42)$$

$$B(1) = \begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix},$$

$$A(2) = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}, \ C(2) = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, \quad (43)$$

$$B(2) = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}.$$

5728
The Lie algebra $L(A(l), B(l), C(l), l = 1, 2)$ is solvable with the matrix $M$ given by
\[
M = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}
\] (44)
and we can stabilize system (42) - (43) by a piecewise constant hybrid controller (37). To determine a set of stabilizing controllers (41) we must solve the following linear inequalities
\[
\begin{cases}
-0.6 - 0.7\beta(1) + 2 < 0 \\
-0.4 - 0.1\beta(1) + 2 < 0 \\
0.6 - 0.1\beta(2) + 2 < 0 \\
0.4 - 0.3\beta(2) + 2 < 0
\end{cases} \Rightarrow \beta(1) > 16 \quad (45)
\]
and a set of stabilizing controllers is given by
\[
\Gamma_1 = \{\beta(1) \in \mathbb{R} : \beta(1) > 16\} \\
\Gamma_2 = \{\beta(2) \in \mathbb{R} : \beta(2) > 26\} 
\] (47)
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0.6 - 0.1\beta(2) + 2 < 0 \\
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\] (47)

7. CONCLUSIONS

In this paper linear and bilinear hybrid systems parametrically excited by a white noise consisted of both stable and unstable structures described by Itô stochastic differential equations have been analyzed. We have used the Lie algebra approach and a common Lyapunov function technique to find switching controls for linear hybrid systems with any switching rules and also to find open-loop controls for bilinear hybrid systems with any switching.

We have showed the method for constructing stabilizing switching signals for linear hybrid systems and the possibility for applying hybrid controllers for stabilizing systems with one structure. Example 4 presents the non-hybrid system which cannot be stabilized by classical methods but it still can be stabilized by a special switching rule. Obtained results were illustrated by examples and simulations.

REFERENCES