Adaptive Interconnected Observer-Based Backstepping Control Design For Sensorless PMSM

Marwa Ezzat¹, Jesus de Leon², Alain Glumineau¹

¹ LUNAM Université, Ecole Centrale de Nantes, IRCCyN UMR CNRS 6597, Nantes, France,
² FIME, Universidad Autonoma de Nuevo Leon, Mexico.
Avenida Universidad s/n, San Nicolas de los Garza, Nuevo Leon, Mexico, C.P. 66451.
marwa.ezzat@irccy.ec-nantes.fr, alain.glumineau@irccy.ec-nantes.fr, drjleon@gmail.com.

Abstract: In this paper, a robust sensorless speed observer-controller scheme for a surface permanent magnet synchronous motor (SPMSM) is proposed. First-of-all, an adaptive high gain interconnected observer is designed. This observer estimates the rotor speed, the stator resistance and the load torque as well. A non linear backstepping controller is developed. The above observer is associated to this controller and the proof of the complete scheme is given. The overall system is tested by simulation in the framework of an industrial benchmark.

Keywords: Permanent magnet synchronous motor, sensorless control, adaptive observer.

1. INTRODUCTION

The sensorless techniques become essential and attract the attention of the industrial applications and the researches as well. Therefore, many approaches for speed/position estimation have been investigated in the literature. Different methods as electromotive force (Chen et al. (2003)), adaptive observers (Cascella et al. (2003)), and extended Kalman filter (Boussak (2005)) have been used to estimate both the speed and the position of the PMSM. When analyzing all these results, it is clear that previous results have rarely evaluated robustness of the closed-loop system with respect to parameters variations while this is a key-point to get robust performances for sensorless control. Moreover these studies never investigate the robustness of the observer-controller scheme when the observability is lost. Nearly all the published observers have been tested with classical vector control using a proportional-integral controller. From this point-of-view, adapted nonlinear robust controls as high order sliding mode and backstepping (Ouassaid et al. (2004); Plestan et al. (2007)) could be more efficient. But all of them use the speed measurement. In (Ke and Lin (2005)) an angular velocity observer is proposed for estimating the speed and measures the position.

2. MATHEMATICAL MODEL

The SPMSM model in the synchronous (d − q)-reference frame reads as

\[
\begin{align*}
\frac{d\theta}{dt} &= \Omega \\
\frac{d\Omega}{dt} &= \frac{p}{J} \psi_f i_q - \frac{f_v}{J} \Omega - T_l \\
\frac{di_d}{dt} &= -\frac{R_s}{L_s} i_d + p\Omega i_q + \frac{1}{L_s} v_d \\
\frac{di_q}{dt} &= -p\frac{\psi_f}{L_s} \Omega - p\Omega i_d - \frac{R_s}{L_s} i_q + \frac{1}{L_s} v_q.
\end{align*}
\]

Note that, \(i_{d,q}\) and \(v_{d,q}\) are the measured states. On the other hand, \(\Omega, \theta\) and \(T_l\) are the unmeasured states that are estimated in the sequel; where
- \(p\) Pole pairs
- \(T_l\) Load torque
- \(\psi_f\) Magnet flux
- \(\Omega\) Rotor mechanical speed
- \(\theta\) Rotor angular position
- \(i_{d,q}\) Stator currents in \((d-q)\) reference frame
- \(v_{d,q}\) Stator voltages in \((d-q)\) reference frame
- \(R_s\) Stator-winding resistance
- \(L_s\) Stator-winding inductance
- \(f_v\) Viscous damping coefficient
- \(J\) Rotor moment of inertia.

3. ADAPTIVE INTERCONNECTED OBSERVERS DESIGN

In the sequel an adaptive interconnected observer will be designed for the sensorless PMSM. It is assumed that load torque and stator resistance are slowly varying with respect to electric and mechanic variables. Then the dynamic behavior of these two variables can be read as
Remark 1. Equation (2) means that the load torque and stator resistance values are assumed to be approximated by piecewise constant function. Only the bound of the load torque is assumed to be known. Furthermore, it is clear that the stator resistance slowly changes with the temperature. However, using step constant functions this variation can be approximated and the proposed approach works. Other approaches can be used, for instance singular perturbation methodology, however the dynamics of the PMSM is fast with respect to the variations of the stator resistance that it could be considered constant.

Thus, the extended SPMSM model (1)-(2) may be seen as the interconnection between two subsystems

\[
\Sigma_1 : \begin{cases} 
\dot{X}_1 = A_1(y)X_1 + F_1(X_2) + \Phi_1(u) \\
y_1 = C_1 X_1
\end{cases} 
\]

\[
\Sigma_2 : \begin{cases} 
\dot{X}_2 = A_2(y)X_2 + F_2(X_1, X_2) + \Phi_2(u) + \Phi T_l \\
y_2 = C_2 X_2
\end{cases} 
\]

with

\[
A_1(\cdot) = \begin{bmatrix} -1 & \frac{-i_q}{L_s} \\ 0 & 0 \end{bmatrix}, \quad A_2(\cdot) = \begin{bmatrix} 0 \frac{p_i y}{J} \\ 0 \frac{-R_s}{J} \end{bmatrix}, \\
F_1(\cdot) = \begin{bmatrix} 0 \frac{-\psi f}{J} \\ 0 \frac{-R_s}{J} \end{bmatrix}, \quad F_2(\cdot) = \begin{bmatrix} 0 \frac{p_i}{J} \end{bmatrix}, \\
\Phi_1 = \begin{bmatrix} \frac{1}{L_s} \nu d \\ \frac{1}{L_s} \nu d \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} \frac{1}{L_s} \nu d \\ 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \\
C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}
\]

\[
X_1 = [i_q \; R_s]^T \quad \text{and} \quad X_2 = [i_d \; \Omega]^T \quad \text{are respectively the state vectors of systems (3) and (4),} \quad u = [u_d \; u_q]^T \quad \text{is the input,} \\
\text{and} \quad y = [i_d \; i_q]^T \quad \text{is the output. Furthermore, the SPMSM physical operation domain} \quad \mathcal{D} \quad \text{is defined by the set of values}
\]

\[
\mathcal{D} = \{ X \in \mathbb{R}^5 \mid |i_d| \leq I_{d}^{\text{max}}, |i_q| \leq I_{q}^{\text{max}}, \quad |\Omega| \leq \Omega^{\text{max}}, \quad |T_l| \leq T_l^{\text{max}}, \quad |R_s| \leq R_s^{\text{max}} \}
\]

with \( X = [i_d \; i_q \; \Omega \; T_l \; R_s]^T \) and \( I_{d}^{\text{max}}, I_{q}^{\text{max}}, \Omega^{\text{max}}, T_l^{\text{max}}, R_s^{\text{max}} \) the actual maximum values for currents, speed, load torque and stator resistance, respectively.

The adaptive interconnected observer, developed in the sequel for the sensorless SPMSM, is based on the interconnection between several observers satisfying some required properties, in particular the property of input persistence. The input persistence is related to the observability properties of system (3)-(4). In order to design an observer for system (3)-(4), a separate synthesis of the observer for each subsystem is required.

Remark 2.

When the PMSM remains in the unobservable area, \( X_2 \) and \( X_1 \) do not satisfy the regularly persistence condition. Then, asymptotic stability of the observer is not guaranteed. This problem is solved, by using the practical stability introduced in Section 4.

Remark 3. From (3) and (4), it is clear that \( A_1(y) \) is globally Lipschitz w.r.t. \( X_2 \), \( A_2(y) \) is globally Lipschitz w.r.t. \( X_1 \), \( F_1(X_2) \) is globally Lipschitz w.r.t. \( X_2 \) and uniformly w.r.t. \( (u,y) \) and that \( F_2(X_2, X_1) \) is globally Lipschitz w.r.t. \( X_2, X_1 \) and uniformly w.r.t. \( (u,y) \).

Then, adaptive interconnected observers for subsystems (3) and (4) are given by

\[
\dot{\hat{Z}}_1 = A_1(y)Z_1 + F_1(Z_2) + \Phi_1(u) + S_1^{-1}C_1^T(y_1 - \hat{y}_1) \\
\dot{\hat{y}}_1 = C_1 Z_1
\]

\[
\dot{\hat{Z}}_2 = A_2(y)Z_2 + F_2(Z_1, Z_2) + \Phi_2(u) + \Phi T_l \\
+ (\Xi \Lambda S_p^{-1} \Lambda^T C_2^T + \Gamma S_x^{-1} C_2^T)(y_2 - \hat{y}_2) + K C_2^T(\hat{y}_1 - y_1)
\]

\[
\dot{\hat{Y}}_2 = \Xi S_p^{-1} \Lambda^T C_2^T (y_2 - \hat{y}_2) + B(y_1 - \hat{y}_1) \\
\hat{S}_x = -\rho_s S_2 - A_2(y)S_x - S_2 A_2(y) + C_2^T C_2
\]

\[
\hat{S}_p = -\rho_p S_2 + \Lambda^T C_2^T C_2 \Lambda + \Phi
\]

\[
\hat{y}_2 = C_2 Z_2
\]

with \( Z_1 = [i_q \; \hat{R}_s]^T \quad \text{and} \quad Z_2 = [i_d \; \Omega]^T \) are the estimated state variables respectively of \( X_1 \) and \( X_2 \). \( p_1, p_2, p_3 \) are positive constants, \( S_1 \) and \( S_2 \) are symmetric positive definite matrices \( S_0(0) > 0, B(Z_1) = k \frac{p}{J} i_q \).

\[
K = \begin{bmatrix} -k_{c1} & -k_{c2} \\ -k_{c1} & \alpha \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix}
\]

with \( k, k_{c1}, k_{c2}, \alpha \) and \( \varpi \) are positive constants.

The second observer (6) is composed of two parts: the first part to estimate the state (\( i_d, \Omega \)) and the second part to estimate the load torque (\( T_l \)), by using the stator currents \( i_d \) and \( i_q \). Furthermore, \( S_1^{-1}C_1^T \) is the gain of observer (6) and \( \Xi \Lambda S_p^{-1} \Lambda^T C_2^T + \Gamma S_x^{-1} C_2^T \) and \( K C_2^T \) are the gains of observer (6). We can see that the gain of the observer (6) is spited in two terms. The first one, \( (\Xi \Lambda S_p^{-1} \Lambda^T C_2^T) \), is associated to the state estimation and depends on the solution of a Ricatti equation. The second one \( (\Xi \Lambda S_p^{-1} \Lambda^T C_2^T) \) is related to the identification parameter and depends on the solution of a differential equation. The solutions of these equations are dependent of the regularly persistence (richness of the signal) with respect to state and the parameter.

Remark 4. In equation (6) the term \( (B(Z_1)(y_1 - \hat{y}_1)) \) can be expressed as follows

\[
B(Z_1)(y_1 - \hat{y}_1) \equiv k \frac{p}{J} \psi f (i_q - \hat{i}_q) \equiv k (T_e - \hat{T}_e)
\]

where \( T_e \) and \( \hat{T}_e \) are respectively the "measured" and "estimated" electromagnetic torques.
Lemma 1. Assume that $v$ is a regularly persistent input for state affine system (3)-(4), and consider the following Lyapunov differential equation

$$
\dot{S}(t) = -\theta S(t) - A^T(v(t)) S(t) - S(t) A(v(t)) + C^T C
$$

with $S(0) > 0$, then

$$
\exists \theta_0 > 0, \forall \theta \geq \theta_0, \exists \alpha > 0, \beta > 0, t_0 > 0 : \forall t \geq t_0, \alpha I \leq S(t) \leq \beta I,
$$

where $I$ is the identity matrix.

It is worth mentioning that the conditions of observability loss have been stated in (Ezzat et al. (2010)), where the PMSM is unobservable for some input value (the rotor speed equal to zero). In the PMSM observability area, the inputs $v = (u, X_2)$ and $v = (u, X_1)$, for subsystem (3) and for subsystem (4) respectively, are regularly persistent and the convergence of the observer can be assured. However, in the unobservable region PMSM (under the conditions of zero speed), such inputs are "bad input" and the observer convergence is not guaranteed. The use of practical stability properties can solve this problem.

4. STABILITY ANALYSIS OF OBSERVER UNDER UNCERTAIN PARAMETERS

Under indistinguishable trajectories (unobservable area) the asymptotic convergence of any observer cannot not always be guaranteed because the observability properties are lost on these trajectories. Then, in such cases, it is necessary to analyze the stability of the observer and the closed loop system. The practical stability property, if satisfied, (LaskhmiKhanthan et al. (1990)) allows to establish that dynamics of the estimation error converge in a ball $B_r$ of radius $r$ ($x \in B_r \Rightarrow ||x|| \leq r$). If $r \to 0$ at $t \to \infty$, then the classical asymptotic stability is obtained.

Theorem 1. (LaskhmiKhanthan et al. (1990)). Assume that

i) $h_1, h_2$ are given such that $0 < h_1 < h_2$;
ii) $V \in C[\mathbb{R}^+ \times \mathbb{R}^n, \mathbb{R}^+]$ and $V(t, e)$ is locally Lipschitz in $e$;
iii) for $(t, e) \in \mathbb{R}^+ \times B_{h_2}$, $d_1(||e||) \leq V(t, e) \leq d_2(||e||)$ and

$$
\dot{V}(t, e) \leq \varphi(t, V(t, e))
$$

where $d_1, d_2 \in \mathbf{W}$ and $\varphi \in C[\mathbb{R}^+ \times \mathbb{R}; \mathbb{R}]$;
iv) $d_2(h_1) < d_1(h_2)$ holds.

Then, the practical stability properties of:

$$
i = \varphi(t, l), \quad l(t_0) = l_0 \geq 0,
$$

imply the corresponding practical stability properties of

$$
\dot{e} = f(t, e), \quad e(t_0) = e_0, \quad t_0 \geq 0.
$$

4.1 Stability analysis

Consider that the PMSM parameters are uncertain bounded with well-known nominal values. Then, equations (3-4) can be rewritten as

$$
\Sigma_1 : \begin{cases}
X_1 = A_1(y)X_1 + F_1(X_2) + \Phi_1(u) + \Delta A_1(y) + \Delta F_1(X_2) \\
y_1 = C_1 X_1
\end{cases}
$$

$$
\Sigma_2 : \begin{cases}
X_2 = A_2(y)X_2 + F_2(X_1, X_2) + \Phi_2(u) + \Phi_1 T_1 + \Delta A_2(y) + \Delta F_2(X_1, X_2) \\
y_2 = C_2 X_2
\end{cases}
$$

with $\Delta A_1(y)$, $\Delta A_2(y)$, $\Delta F_1(X_2)$ and $\Delta F_2(X_1, X_2)$ are the uncertain terms of $A_1(y)$, $A_2(y)$, $F_1(X_2)$, $F_2(X_1, X_2)$, respectively. It follows that the uncertain terms are represented as

$$
\Delta A_1() = \begin{bmatrix} 0 - i_q & 0 \\ 0 0 \end{bmatrix}, \quad \Delta F_1() = \begin{bmatrix} -\Delta \psi_f \Omega - \rho \Omega i_d \\ 0 \end{bmatrix}
$$

$$
\Delta A_2(y), \Delta F_2(X_2, X_1) \text{ can be written following a similar way}
$$

$$
\Delta A_2(y) = \begin{bmatrix} 0 p i_q \\ 0 - f_0 J \end{bmatrix}, \quad \Delta F_2() = \begin{bmatrix} -\Delta R_s i_d \\ p \Delta L_s \Delta \psi_f i_d \end{bmatrix}
$$

Considering the SMPMS physical operation domain $\mathcal{D}$, then there exist positive constants $\rho_1 > 0$, for $i = 1, ..., 4$; such that $||\Delta A_1(y)|| \leq \rho_1$, $||\Delta A_2(y)|| \leq \rho_2$, $||\Delta F_1(X_2)|| \leq \rho_3$, $||\Delta F_2(X_1, X_2)|| \leq \rho_4$. The parameters $\rho_i$, $i = 1, ..., 4$ are positive constants determined from the maximal values of $\Delta A_1(y)$, $\Delta A_2(y)$, $\Delta F_1(X_2)$ and $\Delta F_2()$ in the physical domain $\mathcal{D}$.

Let define the estimation errors as

$$
\epsilon_1 = X_1 - Z_1, \quad \epsilon_2 = X_2 - Z_2, \quad \epsilon_3 = T_1 - \hat{T}_1.
$$

From equations (5)-(6) and (9)-(10), one gets

$$
\dot{\epsilon}_1 = [A_1(y) - S_1^{-1} C_1^T C_1] \epsilon_1 + \Delta A_1(y) X_2 + F_1(X_2) - \dot{F}_1(X_2) \quad (12)
$$

$$
\dot{\epsilon}_2 = [A_2(y) - \varpi S_1^{-1} A^T C_1 C_1 \epsilon_1 + A_2(y) X_2 + F_2(X_2) - \dot{F}_2(X_2, X_1) - F_2(Z_2, Z_1) \quad (13)
$$

$$
\dot{\epsilon}_3 = -\varpi S_1^{-1} A^T C_1 C_1 \epsilon_1 - B(Z_1) C_1 \epsilon_1. \quad (14)
$$

Applying the transformation $\epsilon_2 = \epsilon_2 - \Delta \epsilon_3$, it yields $\dot{\epsilon}_2 = \dot{\epsilon}_2 - \Delta \epsilon_3 - \dot{\epsilon}_3$. Then the estimation error dynamics are given by

$$
\dot{\epsilon}_1 = [A_1(y) - S_1^{-1} C_1^T C_1] \epsilon_1 + \Delta A_1(y) X_2 + F_1(X_2) - \dot{F}_1(X_2)
$$

$$
\dot{\epsilon}_2 = [A_2(y) - \Gamma \epsilon_1 + (B' - K') \epsilon_1 + \Delta A_2(y) X_2 + F_2(X_2, X_1) + \dot{F}_2(X_2, X_2) + F_2(Z_2, Z_1)
$$

$$
\dot{\epsilon}_3 = -\varpi S_1^{-1} A^T C_1 C_2 \epsilon_3 - \varpi S_1^{-1} A^T C_1 C_2 \epsilon_3 - B' \epsilon_1.
$$

with $B' = B(Z_1) C_1, K' = K C_1 C_1$. Since $(u, X_2)$ and $(u, X_1)$ are regular persistent inputs for subsystems (9)-(10), respectively; and from Lemma 1, then there exist $t_0 \geq 0$ and real numbers $\eta_{\text{max}} > 0, \eta_{\text{min}} > 0$ which are independent of $\theta$, such that $V(t, \epsilon_i) = \epsilon_i^T S_i \epsilon_i (1 \leq i \leq 3)$.
∀t ≥ t0 \ \eta_{S_i}^{\text{min}} \ |\epsilon_i|^{2} \leq V(t,\epsilon_i) \leq \eta_{S_i}^{\text{max}} \ |\epsilon_i|^{2}. \quad (16)

**Theorem 2.** Consider the extended SPMSM dynamic model represented by (3)-(4). System (5)-(6) is an adaptive observer for system (3)-(4). Furthermore, the strongly uniformly practically stability of estimation error dynamics (15) is established.

**Sketch of Theorem 2 Proof.**
A Lyapunov function candidate is considered as \( V_o = V_1 + V_2 + V_3 \), where \( V_1 = \epsilon_1^2 S_1 \epsilon_1 \), \( V_2 = \epsilon_2^2 S_2 \epsilon_2 \) and \( V_3 = \epsilon_3^2 S_3 \epsilon_3 \). Taking the time derivative of \( V_o \) and using (5), (6) and (15), we have
\[
\dot{V}_o \leq -(\rho_1 - 2k_1k_0 - \mu_1) V_1 - (\rho_2 - \mu_2) V_2 - (\rho_3 - \mu_3) V_3,
\]
where \( \rho_i = \mu_i + \frac{\mu_i}{\varphi_i} \). \( \mu_i = \max(\mu_i, \mu_i) \).

Then, \( (\rho_i > 0) \) and \( \psi > 0 \), and \( \psi > 0 \), such that
\[
\psi V_1 + V_2 + V_3 > \psi V_1 + V_2 + V_3.
\]
So that
\[
\rho_i > 2k_1k_0 + \mu_1 \varphi_i, \quad \rho_i > \mu_1 \varphi_i, \quad \rho_i > \mu_1 \varphi_i.
\]

5. CONTROLLER

Given the measurement of the currents and the voltages, to design a control law in order to the speed of the motor tracks a desired reference (1) \( \Omega \rightarrow \Omega_{r\text{e}} \).

**Step 1**
Let be \( \xi = \Omega - \Omega_{r\text{e}} = \Omega_{r\text{e}} - \Omega_{r\text{e}} \) and the tracking error. Then, the time derivative, we have
\[
\dot{z}_1 = -p_{\psi} i_q - \frac{J}{\Omega} z_2 - \frac{J}{\Omega} \delta_{r\text{e}}.
\]
where \( \xi = \frac{\psi}{\Omega} \).

Defining the following candidate Lyapunov function, \( V_1 = \frac{1}{2} z_1^2 \). Taking \( \delta_{r\text{e}} = -i_{r\text{e}} \beta_{r\text{e}} \) and \( \epsilon_2 = \frac{\psi}{\Omega} \).

Then, we obtain \( \dot{V}_1 = z_2 z_1 - w_1 z_2 \).

**Step 2**
From \( z_2 \), \( \xi = \frac{\psi}{\Omega} \).

Similarly, defining the following candidate Lyapunov function, \( V_2 = \frac{1}{2} z_2^2 \), whose time derivative is given by
\[
V_2 = \dot{V}_1 + \frac{1}{2} \dot{z}_2 = z_2 z_1 - w_1 z_2 + \dot{z}_2.
\]

Then, to force the stability, the control law can be computed by:
\[
v_q = \frac{1}{2} \left( -w_2 z_2 - z_1 + \beta_2 \right).
\]
\[
\dot{V}_{\text{oc}} \leq - (\delta + \vartheta) V_o + \mu \psi \sqrt{V_o} \\
- \vartheta' (\|z_\Omega\|^2 + \|z_\phi\|^2 + \|z_\phi\|^2)
\]

(24)

where

\[
\dot{V}_{\text{oc}} \leq - \eta V_{\text{oc}} + \mu \psi \sqrt{V_{\text{oc}}};
\]

(25)

with \( \eta = \min(\delta + \vartheta, \vartheta') \). Consider the following change of variable \( v_{\text{oc}} = 2 \sqrt{V_{\text{oc}}} \). The time derivative of \( v_{\text{oc}} \) satisfies the following inequality

\[
\dot{v}_{\text{oc}} \leq - \eta v_{\text{oc}} + \psi \mu
\]

(26)

and the solution of (26) is

\[
v_{\text{oc}} = v_{\text{oc}}(t_0) e^{-\eta(t-t_0)} + \frac{\psi \mu}{\eta} (1 - e^{-\eta(t-t_0)}).
\]

(27)

By using the same procedure such as the strongly uniformly practical stability proof of the observer, the strongly uniformly practical stability properties of (26) can be proved, that implies the corresponding strongly uniformly practical stability of the system. Hence, the estimation and the tracking errors of the closed-loop system converge towards a ball \( B_{h_{\text{oc}}} \) of radius \( h_{\text{oc}} \) with \( h_{\text{oc}} = \frac{\psi \mu}{\eta} \).

7. SIMULATION RESULTS

been simulated. The parameters of the SPMSM are given in (WebSetUp (2010)) where the industrial benchmark is described. See Figure (1–a) for the speed reference and Figure (1–b) for the load torque. The results shown in Figure (2–a) represent the measured speed with its reference for the nominal case and fully loaded motor. The tracking is very efficient as displayed by Figure (2–b) which shows the speed error (reference speed- measured speed). Only the estimated speed is supplied to the controller. It is clear that the observer has a good performance. Figure (4) and Figure (5) demonstrate the resistance and load torque estimation respectively for the nominal case. For the robustness test, a resistance variation of +50% and full load torque application is carried out. This case is represented by Figures (6), (7) and (8) for the speed, resistance and load torque estimation respectively. The results reveal the efficiency of the proposed observer-controller scheme.

REFERENCES


Fig. 4. Resistance estimation. a. Estimated resistance (Ohm) versus time (sec). b. Resistance estimation error (Ohm) versus time (sec).

Fig. 5. Load torque estimation. a. Estimated load torque (Nm) versus time (sec). b. Load torque estimation error (Nm) versus time (sec).

Fig. 6. At +50% $R_s$. a. Estimated speed (rad/sec) versus time (sec). b. Speed estimation error (rad/sec) versus time (sec).

Fig. 7. +50% $R_s$. a. Estimated resistance (Ohm) versus time (sec). b. Resistance estimation error (Ohm) versus time (sec).

Fig. 8. Load torque estimation at +50% $R_s$. a. Estimated load torque (Nm) versus time (sec). b. Load torque estimation error (Nm) versus time (sec).

International Workshop on Variable Structure Systems, Mexico City, Mexico, 26-28 June.