Hysteresis Control of a (Ba/Sr)TiO$_3$ Based Actuator: a Comparison of Prandtl-Ishlinskii and Nonlinear Compensator Numerical Methods

A. Udrea, C. Lupu, D. Popescu

University “Politehnica” of Bucharest, Bucharest, 060042 Romania (e-mail: udrea.andreea@yahoo.com)

Abstract: The paper presents an experimental study on (Ba,Sr)TiO$_3$ based actuators hysteretic comportment. The purpose of the study is to model the nonlinearity introduced by the actuator’s material in positioning systems for better tracking performances and efficient numerical implementation. The classic and modified Prandtl-Ishlinskii techniques are used such that numerical control can be achieved, by using the inverse model of the nonlinearity. This method is compared to a proposed nonlinear compensator based control structure that uses the geometric characteristic of the process, a lookup table and a robust controller. The hysteretic comportment of the material is evaluated for the normal (symmetric hysteretic curve) and extended functioning domain (asymmetric hysteretic curve). The control methods are compared in terms of tracking error, number of operations and used memory from numerical implementation point of view. The results show that the proposed method needs lesser resources only in the case of extended functioning domain while the tracking errors for the two methods are in general of the same magnitude.

1. INTRODUCTION

Although sustained efforts to eliminate or at least reduce the effects of hysteretic phenomena onto controlled systems were made during the last decades, the traditional control approaches seem insufficient. The attention to the subject is increasing due to the large number of applications that are confronted with this aspect, actuators tracking performances being one of the interest problems.

Concerning the strategies for eliminating the hysteresis effects, they are generally based on constructing an inverse operator and using it in a feedforward structure (Kuhnen, 2001) or compensators based on inverses of hysteretic models obtained with fitting functions or neural networks – model-based control (Lupu, 2010; Nassirharand, 2006). In this context, there are a series of interest aspects that must be considered when choosing a strategy for hysteresis control: computational costs (number of operations done offline/online and used memory space); control performances (tracking error, stability analysis) and the necessary/available set of data for constructing the model.

The Prandtl-Ishlinskii (PI) models (Park, 2006; Shen, 2008; Wang, 2006) are very appealing for numerical control. It has a series of limitations concerning the compensator design due to non-inversability of the model for control under different conditions, but gives viable solutions for most of the applications confronting with this type of nonlinearity.

Another approach towards the hysteretic systems control is the use of compensator structures like inverse describing functions that approximate the nonlinearity (Nassirharand, 2006). Motivated by the extended usage of this strategy, we propose a combined feedforward-feedback control structure with nonlinear compensator, developed using the nonlinear geometric characteristic of the process and a classical robust control algorithm.

We want to determine the applicability of these two types of solutions to eliminate the hysteretic effect introduced by a Ba/SrTiO$_3$ based actuator in a positioning system. Ba/SrTiO$_3$ is a piezoceramic material with hysteretic proprieties largely investigated in the last 10 year. It is used for a large range of devices: actuators, oxygen sensors, magnetic memories, capacitors in microwave applications, etc.

Both methods target the material nonlinearity, not on the complete actuator nonlinearity following the idea that the proposed study can be adapted to other types of actuators and sensors and plants with specific parameters. The PI and proposed methods are compared in terms of tracking mean error and computational.

2. MATERIAL AND METHODS

2.1 Data acquisition

The used data are obtained by applying an electric field on the piezoceramic (Ba/SrTiO$_3$) probe and measuring the polarization. The polarization is directly proportional with the displacement (order of micrometers) so, in the absence of an accurate displacement measuring device, the measured polarization value is to/can be considered: $P = e/\Delta x$, where $P$ is the polarization; $e$ is the piezoelectric coefficient of the material and $\Delta x$ is the displacement.
The hysteretic comportment of the actuator was measured for the interest frequencies and input tensions that lead to a symmetric hysteretic characteristic - where the errors introduced by the nonlinearity are small - Fig. 1. c) and, also, for input values that lead to an asymmetric hysteretic curve (Fig. 1.f) and implicitly to larger errors.

In the second case - Fig. 1. d) ,e) ,f) - the problem of extending the functioning domain for the material is considered. Increasing the polarization leads to larger displacements and an asymmetric characteristic.

In Fig. 1, the profound nonlinear characteristic of the material under a triangular command signal can be observed. The differences between the two cases (normal/extended functioning domain) are given by the (a) symmetry when considering the first bisector.

The data were acquired after the period when the loading curve was traversed. The sampling period was of 30 ms and 400 x-y pairs of data were stored for each measurement.

2.2. PI and MPI models and control solutions

The Prandtl-Ishlinskii hysteresis model is a weighted superposition of backlash operators (histerons). Each operator is described by a weight and a magnitude: $w_i$ and $r_i$. The sampling of the final model and inverse operator proposed by Kuhnen and Krejcí (2001) are presented next. We consider that the backlash operator is defined by the recurrent relation:

$$y(kT_s) = H_{wi} [x(kT_s), z_{0i}] = \max \{x(kT_s) - r_i, \min(x(kT_s) + r_i, y((k - 1)T_s))\}$$ \hspace{1cm} (1)

The hysteretic model is written as:

$$H[x(kT_s)] = \sum_{i=0}^{N} w_i H_{r_i} [x(kT_s), z_{0i}]$$ \hspace{1cm} (2)

where:
- $k = 0.. N$, $t_0=0$ and $t_N=kT_s$ ; $T_s$ - sampling period and $N+1$ sampled x-y data are used;
- $n$ - is the order of the model;
- weights vector - $w^T = (w_0, w_1, \ldots w_n)$;
- magnitudes vector - $r^T = (r_0, r_1, \ldots r_n)$ with
  $0=r_0<r_1<\ldots<r_n<\infty$;
- initial states vector - $z_0^T = (z_{00}, z_{01}, \ldots z_{0n})$;
- backlash operator vector:
  $$H_{r_i} [x, z_{0i}](t) = (H_{r_i} [x, z_{0i}](t) \ldots H_{r_i} [x, z_{0i}](t))$$

The inverted model, $H^{-1}$ - that gives the compensator is:

$$H^{-1}[y(kT_s)] = \sum_{i=0}^{N} w_i H_{r_i}^{-1} [y(kT_s), z_{0i}']$$ \hspace{1cm} (3)

with $w', r', z'$ to be determined as in (Udrea, 2010)

This model and inverse operator is recommended for hysteretic curves with symmetry - due to the superposition of backlash operators (symmetric) (Kuhnen, 2001). For more complex hysteretic nonlinearities (asymmetric curves), the modified Prandtl-Ishlinskii (MPI) model is better suited. This model overcomes the asymmetry by using one-side dead zone operators, characterized by the threshold $r_s$ and with the capability of catching the asymmetric features of the loops:

$$S[x, r_s] = \begin{cases} \max \{(x - r_i), 0\}, & r_s > 0 \\
\min \{(x - r_i), 0\}, & r_s < 0 \end{cases}$$ \hspace{1cm} (4)

These new operators are superposed on the backlash operators and the discrete model is obtained :

$$\Gamma[x(kT_s)] = \sum_{j=0}^{N} w_{j0} S_{0i} \sum_{i=0}^{N} w_{j0} H_{r_i} [x(kT_s), z_{0j}]$$ \hspace{1cm} (5)

where:
- in the brackets - $H_{r_0}$ stands for the corresponding backlash operator value at the specified moment;
- $n$ - the number of backlash operators; $2m+1$ - the number of dead zone operators; $n+2m+1$ – the order of the model;
- \( w_H / w_S \) weights vector (dimension \( n/2m+1 \)) for backlash/dead zone operators;
- \( r_H / r_S \) magnitudes vector (dimension \( n/2m+1 \)) for backlash/dead zone operators.

The inverted model - \( \Gamma^{-1} \) - is:

\[
\Gamma^{-1}[y(kT)] = \sum_{i=0}^{n} w_{H,i} H_y \sum_{j=m}^{n} w_{S,j} S_y y(kT_j), z_{i0} \]

(6)

with \( k = 0..N \) and \( r_{H}', w_{H}' \) and \( z_{i0}' \) to be determined as presented in (Udrea, 2010).

Having the inverse operator, the hysteretic effect of the material is reduced (by linearization) in a simple feedforward structure. The linear part of the system can be classically controlled in various ways: PID, RST, etc.

As we stated in the introduction, one important aspect of the interest problem is represented by the computational costs. The offline (model and inverse model identification) number of operations, although important, is not of primary concern while the online volume of additions, multiplications and comparison is a constraint when dealing with real time implementation and a short sampling period. The online used memory space is also to be considered. In the case of closed loop control the previous values (number of operations and used memory) increase function of the controller order (example: for a PI controller 1 addition and 2 multiplications and 2 memory spaces).

### Table 1. Online number of operations/ sampling period - feedforward structure

<table>
<thead>
<tr>
<th>Method</th>
<th>Additions</th>
<th>Multiplications</th>
<th>Comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>( n+1 )</td>
<td>( n+1 )</td>
<td></td>
</tr>
<tr>
<td>MPI</td>
<td>( (n+1)(2m+1) )</td>
<td>( (n+1)(2m+1) )</td>
<td>( 3(n+1)(2m+1) )</td>
</tr>
</tbody>
</table>

### Table 2. Memory space

<table>
<thead>
<tr>
<th>Method</th>
<th>Memory space</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>( 3n+3 )</td>
</tr>
<tr>
<td>MPI</td>
<td>( 3n+1+2(2m+1)+2 )</td>
</tr>
</tbody>
</table>

2.3. Feedforward - feedback control structure with model based compensation

The model based compensation structure that we propose for comparison has some specific aspects that must be considered: determination of hysteresis model as geometrical (static) characteristic, construction of the inverse static characteristic to be embedded in the compensator and the classical robust control law design.

#### 2.3.1 Determination of the geometric characteristic

The determination of the geometrical characteristics of a hysteretic processes is based on several experiments (measures of \( u(k) \) and \( y(k) \)). The „increasing” and „decreasing” branches and inner loops branches of the final static characteristic are obtained by meaning the experimental data. The dispersion of the process trajectory can be found for each branch using (7):

\[
\sigma^2[n] = \frac{1}{n-1} \sum_{i=1}^{n} y^2[i], \; \forall n \in N \setminus \{1\}
\]

(7)

The dispersion expresses the influence of the superposing noise that action onto process, the process’s nonlinearity and modelling incertitude. Its value gives the necessary information for the design a robust control algorithm.

#### 2.3.2 Hysteresis compensator design

The classical way of constructing the compensator is based on inverting the hysteresis model. This inversion causes problems because we deal with a non bijective function. That is why we propose a method that does not effectively inverse the model.

The inverted model output \(-u(k)\) - depends on the value of the set point \(-r(k)\). This dependency is stored in a table. The values of \( p \) output values \( u(k) \) of the inverse model and the set point values \( r(k) \) - that drive the output at that specific value are saved; the indexing is made by the output (the models order is \( p \)). These values were determined using the halving procedure, by choosing output values positionated at equals intervals on the output domain of the process. This way, for a certain set point \( r(k) \), the algorithm looks up in the table for the value \( u(k) \) for which \( y(k) \)-the process output is the closest to the desired set point value. This pair of values gives the inverted model output for the current set point.

![Fig. 2 Construction of inverse model for a non bijective characteristic](image)

Each sampling period, the inverted model output is calculated using the set point value and the dependence table (linear fit):

\[
u(k) = y_N + \frac{y_{N+1} - y_N}{d} (x - x_N)
\]

(8)

For bijective characteristics, this approach can be easily applied, but most of the hysteretic processes (large distance between branches) can not be modelled this way. So, for the more complicated models, there will be more than one output
values for each given input. That means that we need a selector of a higher degree to choose between the multiple outputs and select the specific pair that represents the compensator output for the input at that specific moment.

The selector must take into consideration the actual increasing or decreasing, inner loop dynamic of the hysteretic process (process history). That is why, for each main or secondary branches, for each dynamic directions, we create a separate table. The selector will point in which table the pair - denoting the compensator - can be found by numerically evaluating the precedent and present position of the process.

A priori stoking the models' values in tables in an inverted manner and using s selectors (s=2 in our case) - is easy to implement, is less computational costly than inverting other complicated models during run time and do not alter the control algorithm performances because this will be design with sufficient robustness (function of dispersion (7)).

A set of measurements are required for the determination of the geometric characteristic and dispersion maximal value, but this is the price for a fast real-time algorithm.

2.3.3 Control law design

The control algorithm’s duty is to eliminate the disturbances and differences between inverse model computed command and real process behaviour. A large variety of control algorithms can be used here, PID, RST (Landau, 1997), fuzzy etc. For this study we decided to use a RST algorithm (Fig. 4, without the compensator block), the R, S, T polynomials are:

\[ R(q^{-1}) = r_0 + r_1 q^{-1} + ... + r_n q^{-n} \]

\[ S(q^{-1}) = s_0 + s_1 q^{-1} + ... + s_n q^{-n} \]

\[ T(q^{-1}) = t_0 + t_1 q^{-1} + ... + t_n q^{-n} \]

and the linear identified process’s model:

\[ y(k) = \frac{B(q^{-1})u(k)}{A(q^{-1})} \]

This approach allows the users to verify, and if is necessary, to calibrate algorithm’s robustness. The next represents “disturbance-output” sensibility function.

\[ S_u(e^{j\omega}) = H_u(e^{j\omega}) = \frac{A(e^{j\omega})S(e^{j\omega})}{A(e^{j\omega})S(e^{j\omega}) + B(e^{j\omega})R(e^{j\omega})}, \quad \forall \omega \in R \]  (11)

At the same time, the negative maximum value of the sensibility function represents the module margin.

\[ \Delta M = -\max_{\omega \in \pi} |S_u(e^{j\omega})| \]  (12)

Based on this value, in an “input-output” representation, the process nonlinearity can be bounded inside of a “conic” sector, presented in Fig. 3, where \( a_1 \) and \( a_2 \) are calculated using next expression:

\[ \frac{1}{1-\Delta M} \geq a_1 \geq a_2 \geq \frac{1}{1+\Delta M} \]  (13)

![Fig. 3 Robust control design procedure](image)

Finally, if there is imposed that all nonlinear characteristics to be (graphically) bounded by the two gains, or that the gain limit is greater or equal to the process static characteristic maximal distance \( \Delta G \geq mg \), a controller that has sufficient robustness can be designed.

The analysis is performed graphically by using the modulus margin or using the gain margin and comparing it to the maximal dispersion of the mean static hysteretic characteristic computed for different frequencies (ranging on the frequencies domain of interest).

For identifying the process model, one must select an appropriate operating point. In general, it is chosen as the mean value of the domain. Based on the obtained linear model a robust controller is designed. In conformity with the multi-model control solution (Lupu, 2010), function of the hysteresis type, one, two or three controllers are recommended for each identified linear part.

2.3.4 Parallel control structure

We propose the following control structure; it is based on an inverse model anticipation module as compensator and is presented in Fig. 4, where \( u(k) \) is the sum between the command of the classic algorithm and the anticipated value of the inverted model output. This anticipation leads to a better tracking of the set point.

The experimental results shown in the next section confirm this affirmation.
3. RESULTS

3.1 Symmetric hysteretic curve modelled with PI backlash operator superposition

Considering the case where the hysteretic curve is symmetric due to the application of a positive electric field such that the reorientation of all the ferroelectric domains is not complete (unsaturated) - the curve evolution is presented in Fig. 5.

For this model, due to the symmetry, the superposition of backlash operators alone had been used. The models order is 10 (although good results were obtained starting with order 8) and 400 data were used to determine it.

In Fig. 5.a), the measured curve is plotted in red and the obtained model in blue. It can be observed that a good approximation has been achieved; the differences are greater in the upper region where the curve is not perfectly symmetric relatively to the first bisector. In Fig. 5.b) the model and the inverted model (giving the compensator) that is used to eliminate the nonlinearity are presented.

![Fig. 5 Symmetric characteristic](image)

The performances of the compensator were tested by tracking the reference (red) that consists in two sinusoidal signals with different amplitudes and frequencies. The output of the system is plotted in blue in Fig. 6.c). The quadratic error is 1.13%. The closed loop results when using a PI controller are improved and the obtained error is under 0.25%.

Using the model based compensator parallel structure, the obtained error when using a 10 points defined characteristic for the compensator and an RST controller designed to assure robustness is of around 0.2% (Fig. 6).

So, for the two different methods the results in terms of tracking error are of the same magnitude. For this case (symmetric hysteresis) the needed memory space and number of operations are given next. For the PI method, for compensation 32 memory locations; 10 additions, 10 multiplications and 20 comparisons are needed. For the controller A 3 additions; 3 multiplications and 6 memory locations are used.

For the parallel control method the followings are needed - for compensation – 10 memory locations; 14 additions, 8 multiplications and 33 comparisons and for the RST controller: 6 additions, 7 multiplications and 14 memory locations.

In this case, the use of PI method is justified because all the obtained results are better than in the case of the second method.

3.2 Symmetric hysteretic curve modelled with PI backlash operator superposition and saturation operators.

This part of the paper refers to the hysteretic comportment of the material when extending the functioning domain. Higher values of the polarization and implicitly higher displacement are obtained. In Fig. 7. f), it can be observed that the nonlinearity is more pronounced and asymmetric relatively to the first bisector.

First, the model was created by using only backlash operators. In a second phase, the model is created using both backlash and dead zone operators. The two models and the obtained compensators performances will be compared.

For the first model, 10 backlash operators were used. In Fig. 7. a) the difference between the model and the measured data is significant. The asymmetry can not be captured by the symmetric backlash operators. Increasing the order of the model does not improve the results. The compensator designed based on this model reduce the tracking error – figure 7 c), but the error is larger than 3.5%. An additional
proportional integrator controller makes helps decreasing the error to 0.75%.

a) Real characteristic(red) vs. Model(blue)

b) Model and inverted model

c) Reference(red) vs. (blue) Compensated actuator output

d) Real characteristic vs. Model

e) Model inverted model

f) Reference (red) vs. (blue) Compensated actuator output.

Fig. 7. Asymmetric characteristic – backlash operators only: a) b) c); Asymmetric characteristic – backlash and dead zone operators: d) e) f)

PI method needs: for compensation – 32 memory locations; 3; 10 additions; 10 multiplications and 20 comparisons and for the controller – 3 additions; 3 multiplications and 6 memory locations.

For the MPI model, 15 backlash operators and 10 dead zone operators has been used – the complexity of the model was significantly increased. The modelling results improved – the asymmetry was captured by the asymmetric dead zone operators. Increasing their number leads to improved results, but increases the amount of operations that must be done on each sampling period, this way decreasing the numeric robustness of the compensator. The trajectory was better pursuit. The tracking error was reduced to 1.5%. An additionally PI controller leads to a tracking error of 0.17%.

MPI method needs: for compensation – 65 memory locations; 165 additions, 165 multiplications, 480 comparisons and for the controller – 3 additions; 3 multiplications and 6 memory locations.

The parallel control structure leads to errors of 0.28% and the used resources are: for compensation – 10 memory locations; 14 additions, 8 multiplications and 33 comparisons; for the controller – 6 additions; 7 multiplications and 14 memory locations.

So, for specific applications that have limited resources in terms of memory and operations / unit of time, the second method can be used, the tracking error being of the same magnitude order in both cases.

4. CONCLUSIONS

We present a study on two methods for hysteresis effects compensation in terms of online numerical applicability and performances. The studies were performed on data acquired from a Ba/SrTiO$_3$ (as actuator material) probe. The effect of the material nonlinearity is compensated. The two methods are comparable in tracking performances. The PI methods use less resources then the proposed parallel structure but the MPI is much more costly then these previous two methods. When dealing with asymmetric hysteretic characteristic and under physical numerical control constraints, the parallel structure has its advantages.

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