A New Approach for Deconvolution and Filtering of 3-D Microscopy Images

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Abstract: A new approach to the deconvolution and filtering of 3-D microscopy images is introduced in this paper. A state-space representation of the image is derived according to the assumption that the whole image can be modelled by an ensemble of smooth 3-D Gaussian random fields. Blurring and noise are then easily included in the representation. Making use of this model the image restoration is carried out by means of a Kalman-based minimum variance estimation algorithm. The reported simulation results show high performances of the proposed approach.

Keywords: image modelling; deconvolution; image restoration; optimal filtering.

1. INTRODUCTION

During last two decades biological and medical research moved towards molecular scale. As a consequence, a great deal of attention has been devoted to the development of advanced fluorescence techniques and 3-D optical microscopes (Vonesch et al. [2006]). In order to take full advantage of these instruments, several computational methods for image restoring were proposed with a significant impact in the field. Computational methods allow to overcame the resolution guaranteed by optical instruments by removing blurring and noise which generally degrade the microscopy images. The use of computational methods for enhance microscopy images is commonly referred as deconvolution microscopy. This paper is focused on 3-D (non blind) deconvolution methods.

Prior works (Tikhonov and Arsenin [1977], Preza et al. [1992]) suggest regularized minimizations of linear least square (LLS) functions to solve the problem. These methods are based on the use of a regularization term which reduces the contributions of high spacial frequencies to the image, where the noise component is dominant. Obviously, this also reduces the image sharpness. Therefore, the trade-off between image sharpness and noise amplification has to be determined by a suitable choice of the regularization parameter.

Regularized LLS methods may also estimate negative intensities in the restored image that generate artifacts. In order to solve this problem, nonlinear iterative constrained methods like the Tikhonov-Miller algorithm (ICTM) (van der Voort and Straeters [1995]) and the Carrington algorithm (Carrington et al. [1995]) minimize the sum of the squared error between the acquired image and the estimated image by incorporating a non-negativity constraint. This additive information allows these techniques to obtain good image restorations. Nevertheless, the deconvolution results and the convergence rate strongly depend on the regularization parameter (van Kempen and van Vliet [2000]) which need to be set by additional algorithms. Despite such drawbacks this class of method is widely used.

In order to improve the performance, in the maximum likelihood (ML) approaches (Richardson [1972], Holmes [1988], Sarder and Nehoray [2006]) the stochastic nature of noise is taken into account. The obtained methods are capable of restoring high noise images in the particular case of Poisson noise. The basic algorithms converges to noise asymptotically, requiring the introduction of regularization (e.g. Conchello and McNally [1996]) at the cost of further computational load. However, the ML methods are often used.

The ML estimation methods can be easily extended to the maximum a posteriori (MAP) ones (Trussel and Hunt [1979], Verveer and Jovin [1997]) based on realistic prior knowledge of the original image. The key idea is that the original image is a realization of a random field with a given probability distribution whose mean value is given by the prior knowledge of the image gray-level distribution. Solutions provided by these methods are stable and very accurate thanks to the use of a priori information about the original image which is generally paid with a significant computational load. In this paper it is proposed a new approach which starts from a little different point of view, based on very physical assumptions on the structure of the stochastic image model, without using any prior knowledge of the image gray-level distribution, as suggested in Germani and Jetto [1988] for 2-D images. In a natural way, these assumptions lead to define a consistent model for the “generating process” of the original image which is amenable to the implementation of a Kalman-based minimum variance estimation algorithm. Obviously, blurring and noise are suitably included within this model.
One of the key features of this new approach is that no minimization algorithm needs to be computed. The image is elaborated only once and solutions are stable. Such a method is a non-trivial extension of the 2-D image filtering algorithm introduced in De Santis et al. [1994]. A first example of this extension is proposed in Jetto [1999] without taking into account the blurring problem.

The paper is organized as follows. In Section 2 the image model is defined. In Section 3 the deconvolution and filtering algorithm is described. Numerical results are reported in Section 4. Finally, in Section 5 conclusions are summarized.

2. THE IMAGE MODEL

In this section we will first define a mathematical model of the original monochromatic 3-D image. Basing on the approach introduced in Germani and Jetto [1988] for 2-D images, we will consider the 3-D image as a 3-D signal which satisfies a few basic hypotheses. These hypotheses will be enough to define a consistent model for the “generating process” of the original image. Afterwards, we will model the 3-D image acquisition process taking into account the two main degradation effects due to the use of microscopes: blur and noise.

2.1 The Image Signal: Basic Assumptions

A 3-D image can be generally considered as a signal defined in a region of a three dimensional space which satisfies the following assumptions.

1. Smoothness assumption: the image is modeled by the union of disjointed subregions whose interior is regular enough to be well described by a function \( x : \mathbb{R}^3 \rightarrow [0, 1] \), of class \( C^\nu(\mathbb{R}^3) \).

2. Stochastic assumption: all the derivatives of order \( \nu + 1 \) of the 3-D signal are modeled by means of zero-mean independent Gaussian random fields.

3. Inhomogeneity assumption: the random fields representing the image process relative to different subregions are independent.

Assumptions 1 and 2 are based on the consideration that most images are composed of open disjoint subregions whose interior is regular enough to be well described as a finite support restriction of a smooth three-dimensional Gaussian process. The degree of smoothness depends on the particular considered image. The boundary of each subregion is constituted by the image edges, which represent sharp discontinuities in the distribution of the gray-level. Without embedding the information on edge locations, filtering could introduce spatial artifacts (e.g. blurring) on the estimated image. Assumption 3 means that no correlation can be assumed among pixels belonging to different subregions. This allows the edges to be directly taken into account by the image model. A stochastic image generating process can be obtained describing the gray-level discontinuities by a space-varying model, where only the information on edge location is needed. Consequently, the optimal restoration procedure will be guaranteed by the corresponding non-stationary Kalman filter. The information on edge locations can be approximately obtained by applying an edge-detector operator to a pre-deconvolved version of the image. This preliminary deconvolution can be provided by a regularized LLS method which is fast but not really accurate. Although the edge locations will be known only approximately, results prove that the new approach is robust with respect to these errors.

2.2 The Homogeneous Image Equation

We now describe an image by means of the gray-level signal together with its partial derivatives with respect to the spatial coordinates, up to a certain order. The vector so obtained is assumed as the state vector of the image model. Moreover, by assumptions 1 and 2, a stochastic relation between the states evaluated at two different points in the same subregion is obtained. Let us indicate by \( x(p) \) the value of the original image at the spacial coordinate \( p = [r, s, t]^T \), inside a smooth subregion. Because of the smoothness assumption, it is possible to define a state vector composed of the signal and its partial derivatives with respect to \( r, s, t \):

\[
X(p) = \left[ \frac{\partial^\nu x(p)}{\partial r^{\nu-\alpha} \partial s^{\alpha-\mu} \partial t^{\mu}} \right]^T, \quad \alpha = 0, \ldots, \nu; \quad \mu = 0, \ldots, \alpha
\]

(1)

If \( \nu \) is the maximum order of derivation, the dimension of \( X(p) \) is \((\nu + 3)(\nu + 2)(\nu + 1)/3! \triangleq n \). Let \( p(u) = p_0 + vu, \quad v = [\gamma, \beta, \delta]^T \) denote a parametric representation in \( u \) of a straight line passing through the point \( p_0 = [r_0, s_0, t_0]^T \). As a direct consequence of the state vector definition (1), the following equation can be written:

\[
\dot{X}(p(u)) = \gamma \left( \frac{\partial}{\partial r} X(p(u)) \right) + \beta \left( \frac{\partial}{\partial s} X(p(u)) \right) + \delta \left( \frac{\partial}{\partial t} X(p(u)) \right)
\]

(2)

for \( u \) in its range, i.e. \( -\infty \leq u \leq \infty \). Moreover, the dot denoting the derivative with respect to \( u \). Moreover, by direct computation we have:

\[
\frac{\partial}{\partial q} X(p(u)) = A_q X(p(u)) + BW_q(p(u)),
\]

(3)

with \( q = r, s, t \), and where:

- \( A_r, A_s, \) and \( A_t \) are commuting \( n \times n \) matrices composed by 0 and 1 elements in suitable positions;
- \( B \) is the following \( n \times m \) matrix, with \( m = (\nu + 2)(\nu + 1)/2 \):

\[
B = \begin{bmatrix} 0 & I_m \end{bmatrix},
\]

where the null block has dimension \((n-m) \times m \) and \( I_m \) denotes the identity matrix in \( \mathbb{R}^m \);
- the vectors \( W_r(p(u)), W_s(p(u)), \) and \( W_t(p(u)) \) have dimension \( m \) and are given by

\[
W_q(p(u)) = \left[ \frac{\partial^{\nu+1} x(p)}{\partial r^{\nu-\alpha} \partial s^{\alpha-\mu} \partial t^{\mu}} \right]^T, \quad \alpha = 1, \ldots, \nu; \quad \mu = 1, \ldots, \alpha
\]

(4)
The relations between the state vector $X_{i,j,k}$ at the pixel $(i,j,k)$ and the state evaluated at neighboring pixels for which $c_{i,j,k}^{(l)} = 1$ can be obtained by applying (6), with a suitable choice of $\gamma$, $\beta$, and $\delta$. The following component equations are derived:

$$c_{i,j,k}^{(0)} X_{i,j,k} = c_{i,j,k}^{(0)} \left( e^{A_1 \Delta t} X_{i,j,k-1} + W_{i,j,k}^{(0)} \right),$$  
$$c_{i,j,k}^{(1)} X_{i,j,k} = c_{i,j,k}^{(1)} \left( e^{A_2 \Delta t} X_{i,j-1,k} + W_{i,j,k}^{(1)} \right),$$  
$$c_{i,j,k}^{(2)} X_{i,j,k} = c_{i,j,k}^{(2)} \left( e^{A_3 \Delta t} X_{i-1,j,k} + W_{i,j,k}^{(2)} \right),$$  
$$c_{i,j,k}^{(3)} X_{i,j,k} = c_{i,j,k}^{(3)} \left( e^{-A_3 \Delta t} X_{i,j,1,k} + W_{i,j,k}^{(3)} \right),$$  
$$c_{i,j,k}^{(4)} X_{i,j,k} = c_{i,j,k}^{(4)} \left( e^{-A_2 \Delta t} X_{i+1,j,k} + W_{i,j,k}^{(4)} \right),$$

where

$$W_{i,j,k}^{(0)} = \int_0^1 \Delta e^{A_1 \Delta t (1-\tau)} B \cdot W_t ((i \Delta r, j \Delta s, (k-1 + \tau) \Delta t) \, d\tau,$$

$$W_{i,j,k}^{(1)} = \int_0^1 \Delta e^{A_2 \Delta t (1-\tau)} B \cdot W_s ((i \Delta r, (j-1 + \tau) \Delta s, k \Delta t) \, d\tau,$$

$$W_{i,j,k}^{(2)} = \int_0^1 \Delta e^{A_3 \Delta t (1-\tau)} B \cdot W_r ((i-1 + \tau) \Delta r, (j \Delta s, k \Delta t) \, d\tau,$$

$$W_{i,j,k}^{(3)} = \int_0^1 \Delta e^{-A_3 \Delta t (1-\tau)} B \cdot W_r ((i \Delta r, (j+1 + \tau) \Delta s, k \Delta t) \, d\tau,$$

$$W_{i,j,k}^{(4)} = \int_0^1 \Delta e^{-A_2 \Delta t (1-\tau)} B \cdot W_r ((i+1 + \tau) \Delta r, j \Delta s, k \Delta t) \, d\tau.$$
with \( q = r, s, t \), and where \( \Psi_r, \Psi_s, \) and \( \Psi_t \) are diagonal matrices such that
\[
E \left[ \tilde{W}_q(p) W_q^T(p) \right] = \Psi_q \delta(\|p - \overline{p}\|).
\]
Estimates of \( \Psi_r, \Psi_s, \) and \( \Psi_t \) can be obtained as functions of the image spectrum, as showed in Germani and Jetti [1988] for the 2-D case. From \((12)-(16)\) the following identities can be proved to hold
\[
W^{(3)}_{i,j,k} = -e^{-\Delta_i \Delta_j} W_{i,j+1,k},
\]
\[
W^{(4)}_{i,j,k} = -e^{-\Delta_i \Delta_j} W_{i+1,j,k}.
\]
The previous identities, obtained by assuming \( c^{(l)}_{i,j,k} = 1 \), imply the following relations among the covariance matrices of the Gaussian random fields \( W^{(l)}_{i,j,k} \), \( l = 1, 2, 3, 4 \)
\[
E \left[ W^{(3)}_{i,j,k} W^{(3)T}_{l,m,n} \right] = \delta_{i,l} \delta_{j,m} \delta_{k,n} e^{-\Delta_i \Delta_j} Q_s e^{-\Delta_k^2 \Delta_j},
\]
\[
E \left[ W^{(4)}_{i,j,k} W^{(4)T}_{l,m,n} \right] = \delta_{i,l} \delta_{j,m} \delta_{k,n} e^{-\Delta_i \Delta_j} Q_s e^{-\Delta_k^2 \Delta_j}.
\]
The statistics of the infinite three-dimensional Gaussian process corresponding to a generic smooth subregion is completely defined by \((17)-(20)\) and \((25)-(26)\). As a consequence, these equations define the statistics for any pixel of each subregion. Of course some of right-hand sides of \((23)\) and \((24)\) might lose their physical meaning in the presence of an edge, but the statistical meaning is retained, since it is related to the infinite three-dimensional random field. Therefore, relations \((25)-(26)\) are considered true even if some \( c^{(l)}_{i,j,k} = 0 \).

2.5 The Constitutive Equations

Assuming \( p_{i,j,k} > 0 \), we now exploit the component equations in order to derive a unique relation among \( X_{i,j,k} \) and the state evaluated at its \( p_{i,j,k} \) neighboring pixels. The case \( p_{i,j,k} = 0 \) (isolated pixels) will be considered separately. For convenience, let
\[
H_0 \triangleq e^{\Delta_i \Delta_j}, \quad H_1 \triangleq e^{\Delta_i \Delta_k}, \quad H_2 \triangleq e^{\Delta_i \Delta_j}, \quad H_3 \triangleq e^{\Delta_i \Delta_j}.
\]
In order to derive the constitutive equation, we manipulate \((7)-(10)\) in the following way. Let us consider one of \((8)-(10)\). It relates the state vector at the pixel \((i,j,k)\) with the state vector at the neighboring pixel \((i',j',k')\). We substitute this last with the right-hand side of \((7)\) suitably referred to the state vector \( X_{i',j',k'} \). Such a substitution can be carried out only when the considered version of \((7)\) has significance, i.e. when the corresponding coefficient \( c^{(0)}_{i',j',k'} \) is non-zero. In order to keep the significance of the considered component equation, the coefficient \( c^{(0)}_{i',j',k'} \) is included in the substitution. On the other hand, if the coefficient is zero we need to keep the correlation among the two pixel involved in the equation. Therefore, it seems natural to consider the substituting pixel as initial condition, namely, to assume \( X_{i',j',k'} = X^{(0)}_{i',j',k'} \), with \( X^{(0)}_{i',j',k'} \) externally imposed. Therefore, we obtain:
\[
c^{(0)}_{i,j,k} X_{i,j,k} = c^{(0)}_{i,j,k} (H_0 X_{i,j,k} + W_{i,j,k}),
\]
\[
c^{(1)}_{i,j,k} X_{i,j,k} = c^{(1)}_{i,j,k} (H_1 X_{i,j,k} + W_{i,j,k}),
\]
\[
c^{(2)}_{i,j,k} X_{i,j,k} = c^{(2)}_{i,j,k} (H_2 X_{i,j,k} + W_{i,j,k}),
\]
\[
c^{(3)}_{i,j,k} X_{i,j,k} = c^{(3)}_{i,j,k} (H_3 X_{i,j,k} + W_{i,j,k}).
\]
Let us denote by \( Z^{(l)}_{i,j,k} (l = 0, 1, \ldots, 4) \) each of the right-hand sides of \((27)\), therefore they can be rewritten in the compact form
\[
c^{(l)}_{i,j,k} X_{i,j,k} = c^{(l)}_{i,j,k} Z^{(l)}_{i,j,k}, \quad l = 0, 1, \ldots, 4.
\]
Each of these relations among \( X_{i,j,k} \) has significance, if \( c^{(l)}_{i,j,k} \) are non-zero coefficients. Therefore, we set the state vector \( X_{i,j,k} \) equal to the weighted mean of the multivariate variables \( Z^{(l)}_{i,j,k} \), i.e.
\[
X_{i,j,k} = \sum_{l=0}^{4} c^{(l)}_{i,j,k} Z^{(l)}_{i,j,k},
\]
with \( \Omega^{(l)}_{i,j,k} = \sum_{l=0}^{4} c^{(l)}_{i,j,k} \Omega^{(l)}_{i,j,k} \) and where the weight matrices \( \Omega^{(l)}_{i,j,k} (l = 0, \ldots, 4) \) are related to the inverse covariance matrices of \( Z^{(l)}_{i,j,k} (l = 0, \ldots, 4) \) taking into account the relations in \((17)-(20), (25), \) and \((26)\). Note that \( Z^{(l)}_{i,j,k} (l = 0, 1, \ldots, 4) \) depend on the state evaluated at the neighboring points belonging to the \((k - 1)\)-th slice.

Equation \((28)\) is the unique relation among \( X_{i,j,k} \) we were looking for. It does not hold for isolated pixels. Indeed, in this case \((5)\) cannot be integrated along one of the five directions of Fig. 1. Moreover, for an isolated pixel \( p_{i,j,k} \) is equal to zero and the inverse matrix in \((5)\) cannot be defined. Therefore, we consider such pixels as initial conditions, namely, \( X_{i,j,k} = X^{(0)}_{i,j,k} \), as well done for the substitutions in \((27)\). Thus, \((28)\) is modified in the following way:
\[
X_{i,j,k} = \sum_{l=0}^{4} c^{(l)}_{i,j,k} \Phi^{(l)}_{i,j,k} Z^{(l)}_{i,j,k},
\]
where \( \rho_{i,j,k} \) is equal to zero if \( p_{i,j,k} \geq 1 \) or equal to 1 if \( p_{i,j,k} = 0 \), and
\[
\Phi^{(l)}_{i,j,k} = \sum_{l=0}^{4} c^{(l)}_{i,j,k} \Omega^{(l)}_{i,j,k} Z^{(l)}_{i,j,k}.
\]
Equation \((29)\) is referred to as constitutive equation (CE) of the sampled original image. For internal or boundary pixels it provides a relation between the state evaluated at the spatial point \((i, j, k)\) and the state evaluated at neighboring points belonging to the preceding slice. For isolated pixels \((29)\) resets the state. The form of the CE is identical for each image pixel, but its actual expression depends on the spatial position \((i, j, k)\) through the coefficients \( c^{(l)}_{i,j,k} \).
2.6 The Image Acquisition Process

Once introduced the original 3-D image model, we need to model of the acquisition process. As mentioned, our aim is to face two particular effects of degradation introduced by optical biomedical instruments: the blur and the noise.

Blurring can be mathematically modelled by convolution, and within this model the blur is characterized by the Point Spread Function (PSF). In this work we consider such a function as given (non blind approach). Moreover, we suppose the PSF to be spacelimited. Although theoretical PSFs are usually not spacelimited, in the real scenarios this is a reasonable hypothesis. Noise can be modelled by a zero-mean additive Gaussian discrete random field. Therefore, for the sampled image, we have

\[ y_{i,j,k} = \sum_{l=0}^{2d_i} \sum_{m=0}^{2d_j} \sum_{n=0}^{2d_k} h_{l,m,n} \cdot x_{i-1,j-1,k-1} + v_{i,j,k} \]  

(30)

where: \( y_{i,j,k} \) and \( h_{l,m,n} \) are the values of the sampled acquired image and the sampled PSF, respectively; \( 2d_i, 2d_j \) and \( 2d_k \) are the sizes of the discrete supports of the PSF on the three spacial directions; and \( v_{i,j,k} \) is the additive noise. Moreover, it is reasonable to assume the noise variance as given and the corrupting noise components at different points not correlated. In the following we will refer to function (30) as output function.

3. IMAGE FILTERING AND DECONVOLUTION

3.1 State-Space Realization of the Image

The CE (29) and the output function (30) can be exploited to derive a state-space realization of the sampled image amenable for the Kalman filter implementation.

We define the state vector

\[ X(k) = \begin{bmatrix} X_{k-d_i}^T & \cdots & X_{k-1}^T & X_{k+d_i-1}^T \end{bmatrix}^T, \]

(31)

where:

\[ X_{i,j,k} = \begin{bmatrix} X_{i,j,k}^T & X_{i,j+1,k}^T & \cdots & X_{i,j,M,k}^T \end{bmatrix}^T, \]

which consists of the overall information about the slices of the 3-D image belonging to the interval \([k-d_i, k + d_i - 1]\).

The state dimension is \( N \triangleq 2d_i d_j d_k \). The CE relates the information about a pixel belonging to a given slice with the information about the neighboring pixels belonging to the preceding slice. Therefore, the following equation hold

\[ X(k+1) = A(k)X(k) + B(k)X^0(k) + W(k), \]

(32)

where: \( A(k) \) and \( B(k) \) are two matrices of suitable dimensions which can be derived basing on the CE referred to the pixels belonging to the considered slices; \( X^0(k) \) is a vector carrying the initial condition for the state vector and \( \{W(k)\} \) is the state noise sequence. This last is related to the random fields defined in (12)-(16). Basing on the statistical properties described in section 2.4, it can be proved that \( \{W(k)\} \) is a sequence of zero-mean independent Gaussian random vectors (white noise sequence) whose covariance matrix can be easily computed taking into account both the random fields covariance matrices (17)-(19), (25) and (26) and the image configuration given by the values of coefficients \( c_{i,j,k}^{(l)} \).

We also define the output vector

\[ Y(k) = [y_{1,1,k} \ y_{2,1,k} \ \cdots \ y_{L,M,k}]^T \]

(33)

which collects the gray-level of the pixels belonging to the \( k \)-th slice of the acquired image. The output vector dimension is \( q \triangleq LM \). Basing on the output function, the following equation can be easily proved to hold

\[ Y(k) = CX(k) + V(k), \]

(34)

where \( C \) is a suitably dimensioned (block-) circulant matrix related to the sampled PSF and \( \{V(k)\} \) is a sequence of random zero-mean independent Gaussian vectors which simply collect the measure noise corrupting the gray-level of the pixels belonging to the \( k \)-th slice.

The state-space realization we were looking for is finally given by (32) and (34).

3.2 The Filtering Algorithm

Because the state noise \( \{W(k)\} \) is a white noise sequence, the state-space realization we derived in section 3.1 has a form amenable to the Kalman filtering implementation. Therefore, the image restoration can be realized by applying the standard non-stationary Kalman filtering algorithm to the space-variant linear system given by (32) and (34). Actually, the right algorithm to be used is the Kalman non-stationary smoother (Balakrishnan [1984]) since the causality assumed for the third spacial coordinate in section 2.2 is only artificial. In this sense, the use of the smoother allows to compensate the error introduced by the causality assumption.

The smoother provides the minimum variance estimate of the state vector \( X(k) \) for \( k = 1, \ldots, H \). The restored image can be so obtained by simply extracting the first component from the estimated state vectors. The acquired image can be used to initialize the filtering process and as initial condition for isolated pixels \( \{X^0(k)\} \).

Since blur and noise are both modelled by the state-space realization, the filtering process will simultaneously remove the noise and deconvolve the 3-D image.

4. NUMERICAL RESULTS

We implemented and tested the new approach on a range of synthetic 3-D images. Typical results are presented and discussed here. Fig. 2(a) shows a 3-D test image with a size of 64x64x64 pixels. Fig. 2(b) shows an example of blurred-noisy image (with SNR =10dB). Blurring is realized by a 3-D Gaussian PSF whose full width half maximum (HWFM) are 6 pixels along the first two spacial coordinates and 8 pixels along the third spacial coordinate. We added Gaussian noise to the blurred image with a SNR equal to 1, 10, 20, and 30dB. Moreover, we generated eight different realizations for each SNR level and results were averaged over these collections of experiments.

We first computed the edge-locations basing on images restored by a regularized LLS method (Tikhonov and Arsenin [1977], Preza et al. [1992]). Once the parameters \( c_{i,j,k}^{(l)} \) have been obtained, we implemented the Kalman non-stationary smoother (Balakrishnan [1984]) according to procedure briefly described in section 3.2. We chose a model order corresponding to the value \( v = 0 \) because the test image can be considered a piecewise constant image. In order to evaluate the performance of the new approach
we also realized the image estimation trough the ICTM method (van der Voort and Strasters [1995]). We searched the optimal value for the regularization parameter basing on the knowledge of the original image. Figures 2(c) and 2(d) show the results of the two restoration algorithms for the realization in Fig. 2(b). As the reader can note, our method clearly outperforms the ICTM one. Table 1 summarizes the numerical results which confirm the impression given by the human eye. Results are given in terms of improvement in SNR (restored image mean square error/blurred-noisy image mean square error), expressed in dB, for the new method (KBD) and the ICTM. Table 1 also provides results obtained through our method basing on the real edge-locations. We point out that the average amount of not corresponding edge locations among the real ones and those given by the pre-deconvolved image is equal to the 20% of the total number of real edges. Results show that our approach is robust with respect to the mentioned edge-location errors.

5. CONCLUSIONS

This paper introduces a new approach for deconvolution and filtering of 3-D images. Experimental results confirmed the merit of the approach by showing that high filter performances are really attainable for synthetic images. These results also promise similar performances for real images. Future works will be tough devoted to the analysis of the restoration performances for real microscopy images. Moreover, further activities will regard the computational aspect of the algorithm which can be optimized by exploit-

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ing the sparseness of most of the matrices involved in the computation.

REFERENCES


