The Emergence of Cooperative Leadership
from Homogenous Random Networks

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Abstract: We investigate the game learning skeleton on homogeneous random networks, where an individual plays the Prisoner’s Dilemma game with neighbors and the learning frequency during the steady state is counted as edge weight. The learning skeleton is defined as a communication kernel where each individual directs to the most frequently followed neighbor. We show that with the increase of the temptation to defect, the in-degree distribution of the learning skeleton varies from an exponential distribution to a power-law distribution, i.e., the scale-free property can emerge from underlying homogeneous random interaction networks. Moreover, we find that the longer the individual holds on the cooperative strategy, the more followers the individual has, and the easier the leader diffuses its behavior. Therefore, our work will shed some insight into designing reinforcement learning mechanisms in the social system to promote the emergence and persistence of cooperation.

Keywords: Social and behavioural sciences, Complex systems, Network topologies, Game theory

1. INTRODUCTION

In the past ten years, the complex networks theory has been rapidly developed and drawn attentions from different disciplines, including biology, physics, social sciences as well as engineering (Newman (2003); Wang & Chen (2003); Boccaletti et al. (2006); Barabási (2007)). A lot of interesting structural properties were disclosed, where the scale-free property may be one of the most important characteristics, i.e., the degree distributions of many real-world networked systems, ranging from biological and social systems to technological systems, follow a power-law form $P(k) \sim k^{-\gamma}$, which plays a crucial influence to the dynamical process taking place on them (Barabási & Albert (1999)). Various mechanisms have been proposed to understand the emergence of the scale-free property, such as the growth and preferential attachment mechanism (Barabási & Albert (1999)), the gradient mechanism (Toroczkai & Bassler (2004)), etc. In order to fully understand the complex networks, it is indispensable to address the interaction between topology and dynamics process as together (Barabási (2007)).

The evolutionary game dynamics provides a theoretical framework to characterize the evolution and maintenance of cooperation ubiquitously existing in biological, economic, social and engineering systems (Nowak (2006); Olfati-Saber et al. (2007)). In order to apprehend the conflict among the selfish individuals, the Prisoner’s Dilemma (PD) game as one of the most famous game models was proposed (Axelrod & Hamilton (1981)). In the PD game, the greedy individuals are tending to become defectors although the mutual cooperation can provide more benefit than the mutual defection. Therefore, many important mechanisms are studied to understand the establishment of cooperation in reality (Nowak (2006)).

The networked (spatial) reciprocity is one of the most important mechanisms to understand the secret behind altruistic behaviors among greedy individuals. The seminal work done by Nowak & May (1992) showed that cooperators on a regular lattice can group into compact clusters to defend the invasion of defectors. Furthermore, following the rapid development of complex networks theory, a lot of investigations have focused on the nontrivial connectivity patterns of networks for the maintenance of cooperation (G. Szabó & Fáth (2007)), including the regular lattice (Hauert & Doebeli (2004)), the small-world networks (Hauert & Szabó (2005); Vukov et al (20088); Rong et al. (2011)).
(2009)), the scale-free networks (Santos & Pacheco (2005); Santos et al. (2006); Gómez-Gardeñes et al. (2007); Rong et al. (2007); Payne & Epstein (2009); Chen et al. (2009)) as well as the adaptive networks (Hanaki et al. (2007); Tanimoto (2008); Helbing & Yu (2009); Segbroeck et al. (2010)). In the spatial game, an individual is not only playing game with neighbors, but also imitating advantage behaviors from neighbors. Individuals may follow their successful neighbors and meanwhile are followed by other losers, thereby forming a leadership leaning subgraph on the underlying interaction graph. Therefore, there are two kinds of graphs in the spatial game dynamics, one is the interaction graph characterizing who plays games with whom, the other is the learning graph depicting who selects the better behaviors from whom. Most previous researches assume the two graphs are identical and the different influence of the two kinds graphs for the cooperative dynamics is considerable to deeply study (Roca et al. (2006); Rong et al. (2010)).

In this paper we analyze the cooperative behaviors on a kind of homogeneous random networks where an individual has the same number of neighbors, and find that the learning graph has a heterogeneous structure different from the homogeneous interaction graph. The scale-free property can emerge from the learning graph to characterize the communication skeleton during the evolution process. This work will help to understand the origin of scale-free property from the cooperative dynamics process.

The remainder of this paper is organized as follows. In Section 2 we describe the evolutionary game dynamics and homogeneous random network model. In Section 3, we provide a detail analysis to disclose the underlying mechanism that why the cooperators can maintain in the random network. Conclusions will be given as Section 4.

2. NETWORK MODEL AND EVOLUTIONARY GAME DYNAMICS

In this paper we use the Prisoner’s Dilemma game as a metaphor to describe the interaction between two individuals. If they help each other, both of them will obtain the payoff $R$ as the reward of mutual cooperation. Otherwise they will get $P$ as the punishment for mutual defection. If a cooperator meets a defector, the former will obtain $S$ while the latter will receive $T$ as the temptation to defect. If the order of the four payoff parameters is $T \geq R \geq S \geq P$, it is best for selfish individuals to defect regardless of its opponent’s decision and they will fall into the dilemma that the benefit of mutual cooperation ($R$) obtains more than that of mutual defection ($P$). Following the pioneer work done by Nowak & May (1992), we choose $R = 1, P = S = 0, T = b (1.0 \leq b \leq 2.0)$, where $b$ is a tunable parameter of the temptation to defect.

In the networked(spatial) game theory, an individual $i$ occupies a node on a network, who plays the $PD$ game with its immediate connected neighbors and obtains the accumulated payoff $P_i$. In the social and biological systems each individual tends to select the more successful behaviors from its immediate neighbors. Therefore, after playing the $PD$ games, the individuals will compare their payoffs with neighbors and learn successful behavior. In this paper, we adopt the replicator dynamics when the individuals update their strategies synchronously (Santos & Pacheco (2005); Santos et al. (2006)), i.e., each individual $i$ randomly selects a neighbor $j$, and if they have different strategies, $i$ imitates $j$’s strategy with the probability $P_3(P_i - P_j)/bk_{\text{max}}$, where $k_{\text{max}}$ is the maximal degree between $i$ and $j$. Therefore, the individual $i$ has larger probability to learn its neighbor $j$’s current behavior if the latter owns more payoff than the former. There are two different processes in the spatial game, the interaction and the selection with neighbors. The interaction graph means who plays the game with whom, whereas, the learning graph implies who competes with whom for reproduction.

In this paper, we assume the interaction graph of individuals is homogeneous and random, which implies that the individuals have the same number of neighbors that are initially randomly selected and fixed during the strategy evolution process. The homogeneous random network can be obtained with the Maslov-Sneppen algorithm (Maslov & Sneppen (2002)): Start from a regular graph (such as the nearest-neighbor lattice), all edges are swapped as follows: (i) the different edge without used in the (ii) are randomly selected, and (ii) they are swapped forbidding duplicate connections and disconnected network. After all edges are randomly swapped without changing the degree distribution of the original network, we can get homogeneous random network with the low clustering property and the short distance between any pair of vertices (Rong et al. (2009)). Below we will analyze the cooperative behaviors on homogeneous random interaction graphs and disclose the scale emergence of the learning graph.

3. THE LEADERSHIP EMERGENCE OF GAME LEARNING SKELETON ON HOMOGENEOUS NETWORKS

In this paper, we assume initially each individual selects cooperative (C) or defective strategy with the equal probability. The frequency of cooperators $f_C$ is defined as the density of cooperators at the steady state. Moreover, the individuals in the steady state can be partitioned into three subsets: pure cooperators ($PC$), and pure defectors ($PD$), who always stably hold on their strategies, the fluctuating individuals, who intermittently change their strategies (Gomez-Gardeñes et al. (2007)). In this paper, the frequency of cooperators and pure strategies individuals is gotten by averaging over 20000 generations after the transient time of 10000 generation, where the system reaches the steady state. We grow the homogeneous random networks with 10000 nodes and 50 connections per node. Each data is averaged over 10 different homogeneous random networks with 4 runs for each network.

Figure 1 shows that the cooperator frequency $f_C$ as a function of the temptation to defect $b$ in homogeneous random networks. It is found that the cooperative behavior will monotonously decrease with the increase of $b$. Furthermore, we check that there are no pure cooperators. In contrast with $f_C$, the frequency of pure defector $f_{PD}$ is an increasing function of $b$ as showed in Fig. 1. When $b < 1.4$ there are no pure defectors and all nodes are fluctuating individuals. Whereas for higher value of $b$, the number of pure defectors will raise rapidly. However, the cooperative behavior on homogeneous random networks is
Fig. 1. The frequency of cooperators $f_C$ and pure defectors $f_{PD}$ as a function of the temptation to defect $b$ in the homogeneous random network with $N = 10000$ nodes and the degree $z = 50$. Each data is averaged over 10 different homogeneous random network with 4 runs for each network.

difficult to extinct with high temptation to defect (such as $b=1.8$), which is distinct from the well-mix situation that every individuals interacts equally with everyone else. It is showed in Fig. 2 that the number of cooperators will fluctuate around the equilibrium state after a transmit period. This implies that the spatial structure promotes the maintenance and emergence of cooperation. Here we will discuss the underlying mechanism that why the cooperators can maintain on the network.

We focus our attention on the structure of the game learning graph. Define the learning skeleton of spatial game from the underlying homogeneous random interaction network. From both social and biological perspectives, the individuals tend to hold the successful strategy, therefore, they will compare their current payoffs with neighbors and select the better behavior. We define the edge weight $w_{ij}$ as the accumulated times that an individual $i$ adopts

its neighbor $j$’s strategy during the steady state. The larger weight an edge has, the more times the source node learns the destination node’s behavior, and the more significant influence the latter to the former. Therefore, the edge weight characterizes the information flow of strategy learning. Specially, we obtain the game learning skeleton as follows: For node $i$ and its neighborhood $\Gamma_i$, the edge $w_{ix} = \max_{j \in \Gamma_i} \{w_{ij}\}$ is selected as a link on the game learning skeleton showed in Fig. 3. If a node has two or more neighbors with the same highest weight, then an edge with the highest weight is randomly selected. This implies that the edge directing to the most frequently imitated neighbors is selected into the learning skeleton. Repeating this operation, a game learning skeleton is generated where the out-degree of all nodes is one while the in-degrees may be different. We denote the in-degrees of the learning skeleton as $k_s$. A node with a larger in-degree of learning skeleton means it is a leader in the network who has many followers and puts the significant influence on its neighbors’ behaviors. There also exist the nodes whose $k_s$ are zero who mainly follow other’s behaviors. The game learning skeleton represents the communication kernel of the networked game system. Therefore, a question is spontaneously raised as what is the structural property of the game learning skeleton?

Here we investigate the degree distribution of the game learning skeleton and plot the cumulative in-degree distribution for different values of parameter $b$ in Fig. 4. Since there are nodes with zero in-degree of learning skeleton, we add one for every individuals’ $k_s$ and plot the cumulative in-degree distribution of $P(k > k_s + 1)$ instead of $P(k > k_s)$. It is showed that although the underlying game interaction graph is homogeneous, the in-degree distribution of the game learning skeleton is heterogeneous. When $b$ is relatively a small value (such as $b = 1.05$) which corresponds to a high cooperators’ frequency, the game learning skeleton follows an exponential in-degree distribution that means almost all individuals are imitated with the similar frequency in the steady state. Whereas, with the increase of $b$, the in-degree distribution of the game learning skeleton becomes heterogeneous and displays a
Fig. 4. Log-log plot of the cumulative in-degree distributions of the learning skeleton for different values of $b$. Since there exists the nodes whose in-degree of learning skeleton are 0, we plot $P(k > k_s + 1)$ vs $k_s + 1$. The inset shows the in-degree variance $\sigma^2(k_s)$ of the learning skeleton as a function of $b$. Each data is obtained in 10 different realizations of networks and 4 runs for each realization.

power-law form with a sharp cutoff that is similar with the local gradient mechanism (Toroczkai & Bassler (2004)). The cutoff of $P(k > k_s + 1)$ is due to the degree of homogeneous random networks is 50. Moreover, we calculate the in-degree variance of the learning skeleton $\sigma^2(k_s)$ in the inset of Fig. 4 and find that it is a non-monotonously function of $b$. For low value of $b$, the homogeneous in-degree distribution of the learning skeleton leads to the low value of $\sigma^2(k_s)$. With the increase of $b$, $\sigma^2(k_s)$ rapidly raises, and the in-degree distribution of skeleton becomes heterogeneous. However, after a peak ($b = 1.4$), the value of $\sigma^2(k_s)$ decreases again with the emergence of the pure defectors and the decrease of the cooperative behavior. When $b = 1.8$, almost all individuals own small $k_s$ in the learning skeleton that are seldom learned by neighbors, leading to the small value of $\sigma^2(k_s)$. However, there exists a fat-tail with more heterogeneous power-law distribution $P(k > k_s + 1)$ when $b = 1.8$, which implies the scale-free property of the learning skeleton always exists when the cooperators coexist with the defectors.

In order to understand the mechanism of the emergence of the scale-free property in the learning skeleton, we further calculate the correlation between the average cooperative time of individuals and their in-degrees in the skeleton. Since all almost individual are fluctuating agents, we calculate how long an individual tends to hold on the cooperative strategy in the steady state. That is to say, we average the cooperation time of nodes during the steady state $T_C/T_s$, where $T_C$ counts how many steps an individual stays as cooperator and $T_s$ is the number of time steps of the steady state which is 20000 in this paper. In Fig. 5 we plot $T_C/T_s$ as a function of $k_s$, where $T_C/T_s$ is an increasing function of $k_s$ no matter $b$ is high or low. This implies that in the spatial game, the individuals tending to become cooperators are willing to become the leader who diffuse their behaviors to neighbors and locate on the centrality of the learning skeleton. Moreover, from Fig. 5 we find that all values of $T_C/T_s$ are below 1, which means none individuals are pure cooperators but fluctuating between the two strategies. When $b = 1.8$, the pure defectors with $T_C/T_s = 0$ occur in the low in-degree of skeleton, which implies the defective behavior is seldom imitated by neighbors. In the steady state, at each time step, some cooperators become defectors and vice versa, and the system maintains a dynamical balance, where cooperative behaviors are tending to be imitated and the cooperators are easier to become the leader on the game learning skeleton, and the followers are loser holding on the defective strategy.

Furthermore, we investigate the diversity of individuals’ influence, that means the times that their behaviors are imitated by their neighbors in the steady state, which is defined as the node strength, i.e., $S_i = \sum_{j \in \Gamma_i} \{w_{ij}\}$ for each individual $i$. The total node strength $S_{\text{sum}} = \sum_{i=1}^{N} S_i$. We measure the variance of node strength $\sigma^2(N \times S_i/\sum_{i=1}^{N} S_i)$ as a function of the temptation to defect $k_s$ in Fig.6. It is found that similar to the variance of $k_s$, there exists a middle region of $\sigma^2(N \times S_i/\sum_{i=1}^{N} S_i)$ where it reaches a peak and the distribution of the learning information is heterogeneous. Therefore, through the diversity of individuals’ influence, the cooperative behavior is persistent in the homogeneous networked system.

4. CONCLUSION

In this paper, we have studied the communication skeleton on the homogeneous random network using the Prisoner’s Dilemma game as the metaphor for competition dynamics. Through setting the learning frequency to the neighbors in the steady state as the edge weight, the learning skeleton with the maximum weight edges was obtained, which characterizes the game communication kernel among the individuals. We have investigated the structure of game
learning skeleton with the game interaction behaviors, and showed that with the increase of the temptation to defect, the in-degree distribution of learning skeleton varies from an exponential distribution to a power-law distribution. The individuals’ in-degrees of learning skeleton are positively correlated with the cooperative time in the steady state, i.e., the longer the individual holds on the cooperative strategy, the more followers the individual has in the skeleton, and the easier it diffuses its behavior. Therefore, although the frequency of cooperators will decrease with the increase of temptation to defect, the structure of leadership will also change during the evolving process, which promotes the existence of cooperative behavior in the network even with high value of the temptation to defect. Hence, the investigation of game learning skeleton will help researchers to understand the origin of network structure and the mechanism of cooperative dynamics. This will provide some clues to design an efficient cooperative protocol with competitive learning mechanism to optimize the performance of social networked systems.

Fig. 6: the variance of node strength $\sigma^2(N \times S_i/S_{sum})$ as a function of the temptation to defect $b$


REFERENCES


