Adaptive Failure Compensation Control with Redundant Actuators

Gang Tao∗ Ruiyun Qi∗∗ Bin Jiang∗∗

∗ Department of Electrical and Computer Engineering, University of Virginia, USA (Email: gt9s@virginia.edu)

∗∗ College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, China
(ruiyun.qi@nuaa.edu.cn, binjiang@nuaa.edu.cn)

Abstract: Redundant actuators are commonly used for actuator failure compensation but bring new challenges for feedback control design as uncertain failures can introduce large structural, parametric and actuation uncertainties. Two key technical issues are associated with using redundant actuators: how to coordinate redundant actuators for effective feedback control, and how to adaptively compensate uncertain actuator failures. In this paper, we present a systematic overview of direct adaptive failure compensation based solutions to these issues for different types of control systems: state tracking and output tracking, using state feedback or output feedback, for linear and nonlinear systems.

Keywords: Actuator failure, adaptive control, parameter estimation, stability, tracking.

1. INTRODUCTION

System faults such as actuator/sensor failures, structural damage may drastically change system dynamics and introduce large structural uncertainties, resulting in performance degradation or even instability. To improve the survivability and reliability of safety-critical systems such as an aircraft system, effective fault-tolerant control techniques are crucial. Fault tolerance of dynamic systems can be achieved either from system robustness to faults such as component failures and structural damage uncertainties, or from controller reconfiguration in response to faults of a certain specific type (Boskovic [2002]).

In recent two decades, significant amount of research has already been done in this area. One typical group of approaches need to explicitly estimate the fault information. In those approaches, a fault detection and identification (FDI) unit is used and based on the fault information estimation, the controller is reconstructed to compensate the effects caused by the fault (Chen [1998], Chen [2007], Jiang [2006]). Typical FDI methods include residual generation techniques (Leuschen [2005]) and observer-based methods (Kabore [2001], Yang [2007]). Another group of approaches do not require explicit utilization of fault information. The control system can be well-designed to have the fault-tolerance capabilities (e.g. by properly choosing feedback gains or allocate control effort in several actuators) to compensate system uncertainties. Some faults can also be considered as certain kinds of uncertainties, which can be treated by robust control techniques (Boskovic [2002], adaptive control (Boskovic [1998], Tao [2004], Wang [2009], Boskovic [2010]), model-predictive control (Almeida [2010]) or intelligent fuzzy/neural control (Boskovic [2002], Yen [2004]). The survey papers (Boskovic [2005], Zhang [2008]) present the most recent bibliographical review of fault-tolerant control systems.

Among different fault-tolerant control approaches, the direct adaptive control approach has the key advantage that it can provide theoretically provable asymptotic tracking in addition to stability, in the presence of large parameter variation and uncertainties. Direct adaptive control designs can directly adapt fault uncertainties without using an explicit fault detection and identification unit. For a system with redundant actuators such as an aircraft system, direct adaptive control approaches can make effective use of the available actuation redundancy for failure compensation. When failure or damage occur, desired stability and tracking performance can still be achieved with adaptive cooperation of the remaining healthy actuators to provide failure compensation and dynamic control.

In this paper, we give a systematical overview of direct adaptive compensation of failures of redundant actuators in control of uncertain dynamic systems, in particular, a tutorial on two key technical issues: how to coordinate redundant actuators for effective feedback control under actuator failures, and how to adaptively compensate uncertain actuator failures. We also present some recent results on direct adaptive actuator failure compensation using an indirect model reference adaptive control approach. A full version of this paper is available from the authors.

In Section II, we formulate the control problems. In Section III, we address actuator failure compensation based on direct adaptive control including using state feedback for state tracking, state feedback for output tracking and output feedback for output tracking for both linear and
nonlinear systems. In Section IV, we discuss indirect adaptive control based actuator failure compensation.

2. PROBLEM STATEMENT

To formulate the problems of adaptive control using redundant actuators subject to uncertain failures, we consider the dynamic system (plant) with multiple actuators:

\[ \dot{x} = f(x) + \sum_{j=1}^{m} g_j(x)u_j, \quad y = h(x) \]  

(1)

where \( x \in \mathbb{R}^n \) is the state vector, \( y \in \mathbb{R} \) is the output signal, and \( u_i \in \mathbb{R} \) is the input component (actuator) which may fail during operation, and \( f(x) \in \mathbb{R}^n, g_j(x) \in \mathbb{R}^n \) and \( h(x) \in \mathbb{R} \) are some smooth (differentiable) functions.

Typical failures of the system actuators \( u_i(t) \), as described in (Tao [2004]), may be modeled as

\[ u_j(t) = \bar{u}_j(t) = \bar{u}_{j0} + \sum_{l=1}^{n_i} \bar{u}_{jl} f_{jl}(t), \quad t \geq t_j, \]  

(2)

for some \( t_j > 0, \ j \in \{1, 2, \ldots, m\} \), some unknown constants \( \bar{u}_{j0} \) and \( \bar{u}_{jl} \), and some known bounded signals \( f_{jl}(t), \ l = 1, \ldots, n_j \), and \( n_j \geq 1 \) (which depend on a certain application). Note that \( f_{jl}(t) \) can also be bounded functions of the system state variables: \( f_{jl}(t) = f_{jl}(x(t), t) \), and an unparametrizable term \( \delta_j(t) \) can be added to (2), which may be handled by robust control. For example, \( \bar{u}_j(t) = \bar{u}_{j0} \) represents an actuator such as an aircraft rudder or aileron stuck at an unknown value (position).

The linear version of the system (1) is

\[ \dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) \]  

(3)

where \( A \in \mathbb{R}^{n \times n}, B = [b_1, \ldots, b_m] \in \mathbb{R}^{n \times m}, C \in \mathbb{R}_1^{1 \times n} \) are unknown constant parameter matrices, \( u(t) = [u_1, \ldots, u_m]^T \in \mathbb{R}^m \) is the input vector whose components (actuators) may fail during system operation.

With the presence of possible actuator failures, the system input \( u(t) \) can be expressed as

\[ u(t) = (I_m - \sigma(t))v(t) + \sigma(t)\bar{u}(t) \]  

(4)

where \( v(t) \in \mathbb{R}^m \) is an applied control signal, \( \bar{u}(t) = [\bar{u}_1(t), \ldots, \bar{u}_m(t)]^T \), \( \sigma(t) = \text{diag}\{\sigma_1(t), \ldots, \sigma_m(t)\} \) with

\[ \sigma_j = \begin{cases} 1 & \text{if the } j \text{th actuator fails, i.e., } u_j = \bar{u}_j \\ 0 & \text{otherwise} \end{cases} \]  

(5)

This expression indicates that when actuators fail, some components of the applied control input \( v(t) \) cannot reach the system dynamics to deliver control effort.

The diagonal matrix \( \sigma(t) \) represents an actuator failure pattern, and for a particular application, all \( \sigma(t)'s \) of interest belong to a specified failure pattern set \( \Sigma \).

Problem I: Coordination of redundant actuators for effective control. For a base system in either of

\[ \dot{x} = f(x) + g_1(x)u_1, \quad y = h(x) \]  

(6)

\[ \dot{x} = Ax(t) + b_1u_1, \quad y = Cx, \]  

(7)

for which a feedback control law can be designed to achieve certain desired system performance, to expand its capacity to accommodate uncertain actuator failures, we can choose to add more actuators \( u_i, i = 2, 3, \ldots, m \), to the system. The question to answer is: how to coordinate such redundant actuators to the system, to ensure that when all healthy they help each other in their maximum effort to achieve the prespecified control objective, and when failed they can be replaced by each other in achieving the control objective (that is, their failure effect can be compensated by other healthy actuators)?

Problem II: Design of adaptive control laws for effective compensation of uncertain actuator failures. This problem deals with the case when the system and actuator failure parameters are unknown, that is, under the above solution existence conditions, we need to design the desired control algorithms to adaptively compensate the effects of actuator failure uncertainties, to achieve the desired control objective in the presence of system and failure parameter uncertainties.

The approach to be addressed in this paper is direct adaptive actuator failure compensation using parameter adaptation of a feedback controller structure (which is specified to be capable of accommodating all possible actuator failures whose failure patterns, values and time instants are all uncertain), without explicit failure detection. Such an approach can be realized based on either a direct adaptive control design or an indirect adaptive control design.

3. DIRECT ADAPTIVE CONTROL BASED ACTUATOR FAILURE COMPENSATION

In this section, we give a systematic study of the above stated problems: how to coordinate the redundant actuators and how to derive the desired nominal and adaptive control designs, for effective actuator failure compensation. We will address three types of designs: (i) state feedback for state tracking, (ii) state feedback for output tracking, and (iii) output feedback for output tracking, and cover both linear and nonlinear system cases.

3.1 State Feedback for State Tracking

For state tracking, we need to define a reference model system to generate the reference state trajectory \( x_m(t) \). For \( x(t) \) to track \( x_m(t) \), the \( x \)-system and the \( x_m \)-system need to have certain structural similarity characterized by a so-called matching condition.

Design for Linear Systems

To drive the design conditions for the linear system \( \dot{x}(t) = Ax(t) + Bu(t) \), we start with the reference model system

\[ \dot{x}_m(t) = A_m x_m(t) + B_m r(t), \]  

(8)

where \( A_m \in \mathbb{R}^{n \times n}, B_m \in \mathbb{R}^n \) are constant with \( A_m \) being stable, and \( r(t) \in \mathbb{R} \) is a bounded reference input.

It can be verified that for the base linear system (7):

\[ \dot{x}(t) = Ax(t) + b_1 u_1(t), \]  

to match the reference model system (8) with the control law:

\[ u_1(t) = k_{11}^T x(t) + k_{21}^T r(t), \]  

(9)

that is, to make \( \dot{x}(t) = (A + b_1 k_{11}^T)x(t) + b_1 k_{21}^T r(t) \) = \( A_m x(t) + B_m r(t) \), the matching conditions

\[ A + b_1 k_{11}^T = A_m, b_1 k_{21}^T = B_m \]  

(10)
are needed. If we consider other individual base systems:
\[ \dot{x}(t) = Ax(t) + b_i u_i(t) \]
with the control law \( u_i(t) = k_i^T x(t) + k_{2i} r(t) \) (for the case when \( u_i = 0 \) for all \( j \neq i \), representing the case when all but one actuators fail and are stuck at zero input, one of possible operation mode with redundant actuators), we can reach the similar matching conditions: \( A + b_i k_{1i}^T = A_m, b_i^T k_{2i} = B_m \).

We have the answer to the redundant actuator coordination/placement and failure compensation problems:

**Proposition 1.** The state tracking problem is solvable if the actuation vectors \( b_i \) in (3) are chosen such that
\[ A + b_i^T k_{1i} = A_m, b_i k_{2i} = B_m, \]
for some constants \( k_{1i} \in R^n \) and \( k_{2i} \in R, i = 1, 2, \ldots, m \).

A key condition is that \( b_i k_{2i} = B_m, i = 1, 2, \ldots, m \), that is, for the state tracking control problem, all actuation vectors \( b_i \) should be parallel to each other—this is how the redundant actuators should be coordinated/placed, in order to solve the actuator failure compensation problem. Under such matching conditions (11), it can be verified that for any \( u_i(t) = \tilde{u}_i(t) \) in (2), \( t \geq t_j, j \in \{j_1, j_2, \ldots, j_p\} \subset \{1, 2, \ldots, m\}, 0 \leq p \leq m \), there exist piecewise constants \( k_{1i}^* \in R^n \) and \( k_{2i}^* \in R \) and piecewise parametrizable \( k_{3i}(t) \in R, i \in \{1, 2, \ldots, m\} \) but \( i \neq j_1, \ldots, j_p \), such that
\[ A + \sum_{i \neq j_1, \ldots, j_p} b_i k_{1i}^T = A_m, \sum_{i \neq j_1, \ldots, j_p} b_i k_{2i}^* = B_m \]
\[ \sum_{i \neq j_1, \ldots, j_p} b_i k_{3i}^*(t) + \sum_{j = j_1, \ldots, j_p} b_i \tilde{u}_i(t) = 0. \]

This implies that with the nominal gain matrix \( K_i = [k_{1i}, k_{2i}, \ldots, k_{1m}]^T \in R^{m \times n} \) and nominal gain vectors \( k_{2i} \) and \( k_{3i}^* \), \( k_{3i}(t) = [k_{31}(t), k_{32}(t), \ldots, k_{3m}(t)]^T \) with each \( k_{3i}^*(t) \) parametrized by some corresponding \( f_j(t) \) (for which \( k_{1j}, k_{2j}^* \) and \( k_{3j}^* \), \( j = j_1, \ldots, j_p \), can be arbitrary), such that the nominal control law
\[ v(t) = v^*(t) = K_i^T x(t) + k_{2i}^* r(t) + k_{3i}^*(t) \]
can make the closed-loop system as the desired \( \dot{x}(t) = A_m x(t) + B_m r(t) \), in the presence of actuator failures.

An adaptive failure compensation scheme meeting the desired control objective (that is, closed-loop stability and asymptotic tracking of \( x_m(t) \) by \( x(t) \), in the presence of up to \( m - 1 \) uncertain failures of the \( m \) actuators) can be developed using the adaptive version of (14):
\[ v(t) = K_i^T x(t) + k_{2i} r(t) + k_{3i} \]
where \( K_i(t), k_{2i} \) and \( k_{3i}(t) \) are the estimates of the unknown parameters/signals \( K_i^*, k_{2i}^* \) and \( k_{3i}^* \), which leads the tracking error \( e(t) = x(t) - x_m(t) \) to satisfy
\[ \dot{e}(t) = A_m e(t) + B_m \]
\[ \cdot \sum_{j \neq j_1, \ldots, j_p} 1/k_{2j}^2 \left( \tilde{k}_{ij}(t)x(t) + \tilde{k}_{2j}(t)r(t) + \tilde{k}_{3j}(t) \right) \]
for some parameters \( k_{2j}^2 \), where \( \tilde{k}_{ij} = k_{ij} - k_{ij}^* \) etc. Desirable stable adaptive laws for updating \( K_i(t), k_{2i}(t) \) and \( k_{3i}(t) \) can be developed to meet the desired control objective without the knowledge of when, which and how much the actuator failures are (Tao [2004]).

**Design for Nonlinear Systems** The similar solution can be derived for the nonlinear system \( \dot{x} = f(x) + \sum_{j=1}^m g_j(x)u_j \). We first define a nonlinear reference system model
\[ \dot{x}_m(t) = F_m(x_m(t)) + G_m(x_m(t))r(t), \]
for some functions \( F_m \) and \( G_m \in R^n \), and assume it a bounded-input bounded-state stable system. For the base system (6): \( \dot{x} = f(x) + g_1(x)u_1 \), we choose a feedback control law of the form
\[ u_1(t) = k_{11}^*(x(t)) + k_{21}^*(x(t))r(t), \]
for some functions \( k_{11}^*(x(t)) \in R^n \) and \( k_{21}^*(x(t)) \in R^n \). For the closed-loop system
\[ \dot{x} = (f(x) + g_1(x)k_{11}^*(x)) + g_1(x)k_{21}^*(x)r \]
to match the reference model system (17), we need \( f(x) + g_1(x)k_{11}^*(x) = F_m(x), g_1(x)k_{21}^*(x) = G_m(x) \).

Similarly, for all individual actuators \( u_i \), we have the answer to the redundant actuator placement and failure compensation problems for the nonlinear system (1):

**Proposition 2.** The state tracking problem is solvable if the actuation vectors \( g_i(x) \) in (1) are chosen such that
\[ f(x) + g_i(x)k_{1i}^*(x) = F_m(x), g_i(x)k_{2i}^*(x) = G_m(x), \]
for some functions \( k_{1i}^*(x) \) and \( k_{2i}^*(x) \), \( i = 1, 2, \ldots, m \).

Hence, a key condition, similar to the linear case, is that \( g_i(x)k_{2i}^*(x) = G_m(x), i = 1, 2, \ldots, m \), that is, for the state tracking control problem, all actuation vectors \( g_i(x) \) should be parallel to each other for each \( x \)—this is how the redundant actuators should be placed/coordinated, in order to solve the actuator failure compensation problem.

Development of such adaptive actuator failure compensation schemes may be done similar to the linear cases, with some potentially interesting and challenging new issues associated with parametrization of nonlinear functions \( f(x), g_i(x), k_{1i}(x), F_m(x), k_{2i}(x) \) and \( G_m(x) \), for parameter adaptation, which is left for further investigation.

### 3.2 State Feedback for Output Tracking

In this subsection, we consider the output tracking failure compensation problems which turn out to require much less matching conditions than that in (11) or (21). Such relaxed design conditions make output tracking designs suitable for more practical problems.

**Design for Linear Systems** In this case, we consider the linear system (3) subject to actuator failures (2):
\[ \dot{x}(t) = Ax(t) + Bu(t), \]
where \( y(t) = Cx(t) \). For output tracking, the reference model system is
\[ y_m(t) = W_m(s)r(t), \]
where \( P_m(s) \) is a stable monic polynomial of degree \( n^* \) and \( r(t) \) is a reference input. To derive the matching conditions needed for actuator failure compensation, we start with the nominal controller structure (14)
\[ v(t) = v^*(t) = K_i^T x(t) + k_{2i}^* r(t) + k_{3i}^* \]
and, for the failures $u_j(t) = \bar{u}_j(t)$, $j = j_1, \ldots, j_p$, to make the closed-loop system match the reference system (22), the following conditions are needed for $K_1^*, k_2^*$, and $k_3^*$:

$$C(sI - A - \sum_{j \neq j_1, \ldots, j_p} b_j k_1^{*T} - \sum_{j \neq j_1, \ldots, j_p} b_j k_2^{*-1} + \sum_{j = j_1, \ldots, j_p} b_j \bar{u}_j) = 0.$$  

(25)

We first consider the case when $u_j(t)$ is healthy but $u_j(t) = \bar{u}_j(t)$, for all $j \neq i$. The condition (24) becomes

$$C(sI - A - b_j k_1^{*T} - b_j k_2^{*2}) = W_m(s).$$  

(26)

equivalently, $(A, b_j, C)$ is minimum phase and has relative degree $n^*$, and (25) is satisfied with $k_3^* = 0$.

For the case when all actuators $u_i(t)$, $i = 1, 2, \ldots, m$, are healthy, we need to make sure their influence on the system output does not conflict to each other. Here, we consider the applications in which the redundant actuators have similar physical characteristics and are coordinated by

$$u_i(t) = \alpha_i v_0(t), \quad \alpha_i > 0, \quad i = 1, 2, \ldots, m,$$  

(27)

for some chosen parameters $\alpha_i$. When actuators have different physical characteristics, their redundant partners can be grouped up and their failure compensation needs to be achieved by multivariable adaptive designs (Tao [2004]).

Finally, for the case when there are $p$ actuator failures, that is, $u(t) = \bar{u}_j(t)$, $j = j_1, \ldots, j_p$, $0 < p < m$, $\{j_1, \ldots, j_p\} \subset \{1, 2, \ldots, m\}$, we have the system (3) as

$$\dot{x}(t) = Ax(t) + \sum_{i \neq j_1, j_2, \ldots, j_p} \alpha_i b_i v_0(t) + \sum_{j = j_1, j_2, \ldots, j_p} b_j \bar{u}_j(t)$$  

$$y(t) = Cx(t).$$  

(28)

Clearly, for the design of an adaptive failure compensation control scheme to handle all possible actuator failures with $j = j_1, \ldots, j_p$, for output tracking, we need:

**Assumption 1.** The systems $(A, \sum i \neq j_1, j_2, \ldots, j_p \alpha_i b_i, C)$ are minimum phase and have relative degree $n^*$, for all possible failure indices $j_1, \ldots, j_p$.  

To design $v_0(t)$, we choose the controller structure

$$v_0(t) = k_1^{*T}(t)x(t) + k_2^{*}(t)r(t) + k_3(t),$$  

(29)

where $k_1^* \in R^n$, $k_2^* \in R$, and $k_3 \in R$ are the estimates of some nominal parameters $k_1^{*0} \in R^n$ and $k_2^{*0} \in R$, and signal $k_3^*(t) \in R$. The closed-loop system can be given as

$$y(t) = y_m(t) + W_m(s) \left[ p^*(\hat{k}_1^{*0} x + \hat{k}_2^{*0} r + \hat{k}_3^{*0}) \right](t)$$  

$$+ W_m(s)[p^* k_3^*(t)](t)$$  

$$+ C(sI - A - b_j k_3^{*T})^{-1} \sum_{j = j_1, \ldots, j_p} b_j \bar{u}_j(t).$$  

(30)

Since the components of $\bar{u}_j(t)$ can be arbitrary signals, in view of (30), the desired matching condition was $C(sI - A - b_j k_3^{*T})^{-1} b_j = W_m(s)\beta_j^*$, for some $\beta_j^* \in R$, $j = j_1, \ldots, j_p$. This seemingly restrictive condition for $b$ and $b_j$ different can be ensured under some simple verifiable conditions given by the next lemma (Tao [2004]).

**Lemma 3.** For $(A, b)$ controllable, there exist constant $k_{10}^* \in R^n$, $k_{20}^* \in R$ and $\beta_j^* \in R$ such that

$$C(sI - A - b k_1^{*T})^{-1} b_j = W_m(s) \beta_j^*$$  

(31)

$$\text{if and only if the two systems } (C, A, \tilde{b}) \text{ and } (C, A, b_j) \text{ have the same relative degree } n^*.$$

To apply Lemma 3 to the system (30), we assume:

**Assumption 2.** The systems $(A, b_j, C)$ have relative degree $n^*$, for all $j = j_1, \ldots, j_p$.

Then, we can arrive at the error equation

$$y(t) - y_m(t) = W_m(s) \left[p^*(\hat{k}_1^{*0} x + \hat{k}_2^{*0} r + \hat{k}_3^{*0}) \right](t)$$  

(33)

which has the desirable form for adaptive control (note that the signal $k_{30}^*$ can be further parametrized).

Similar to the state tracking case, to derive the adaptive estimate of $k_{30}^*$, the base functions in $k_{30}^*$ need to be determined in advance, to cover all possible failure patterns $\sigma \in \Sigma$. For this purpose, we define the set \( \{l_1, l_2, \ldots, l_q\} \subset \{1, 2, \ldots, m\} \) such that for all possible failure indices $j_1, j_2, \ldots, j_p$, $p = 0, 1, \ldots, q < m$, it holds that $\{j_1, j_2, \ldots, j_p\} \subset \{l_1, l_2, \ldots, l_q\}$, and we assume that:

**Assumption 3.** The set $\{l_1, l_2, \ldots, l_q\}$ is known.

Then, we can choose a fully parametrized $k_{30}^*$ as

$$k_{30}^* = \sum_{j = l_1, l_2, \ldots, l_q} g_{jl} f_{jl}(t),$$  

(34)

for designing its adaptive version $k_{30}$ for (29).

We also have the answer to the redundant actuator placement/coordination and failure compensation problems:

**Proposition 4.** The output tracking problem is solvable if the actuation vectors $b_i$ in (3) with possible actuator failures $u_j(t) = \bar{u}_j(t)$, $j_1, j_2, \ldots, j_p$, are chosen such that the systems $(A, \sum i \neq j_1, j_2, \ldots, j_p \alpha_i b_i, C)$ are minimum phase and have relative degree $n^*$, and the systems $(A, b_j, C)$ have relative degree $n^*$, for all $j = j_1, \ldots, j_p$.

Under the proposition’s relative degree and minimum phase conditions, state feedback output tracking adaptive actuator failure compensation schemes can be developed to achieve the desired control objective, as shown in (Tao [2004]).

**Design for Nonlinear Systems** Consider the nonlinear system (1) subject to actuator failures: $\dot{x} = f(x) + \sum_{j=1}^m g_j(x)u_j$, $y = h(x)$. With (27), in the presence of $p$ actuator failures $u_j(t) = \bar{u}_j(t)$, $j_1, \ldots, j_p$, the system (1) becomes

$$\dot{x} = f(x) + \sum_{i \neq j_1, j_2, \ldots, j_p} \alpha_i g_i(x)v_0 + \sum_{j = j_1, j_2, \ldots, j_p} g_j(x)\bar{u}_j$$  

$$y = h(x).$$  

(35)
A feedback linearization based design can be developed for actuator failure compensation using state feedback for output tracking, based on the similar relative degree and minimum phase conditions:

Proposition 5. The output tracking problem is solvable if the actuation vectors \( g_i \) in (1) with possible actuator failures \( u_j(t) = \bar{u}_j(t), j_1, \ldots, j_p \), are chosen such that the systems \( (\dot{x} = f(x) + \sum_{j \neq j_1, \ldots, j_p} \alpha_j g_j(x)v_0, y = h(x)) \) are minimum phase and have relative degree \( n^* \), and the systems \( (\dot{x} = f(x) + g_j(x)u_j, y = h(x)) \) have relative degree \( n^* \), for all \( j = j_1, \ldots, j_p \).

The concepts of relative degree and minimum phase for nonlinear systems are the extension of that for linear systems and have the same physical meanings as that for linear systems. Adaptive state feedback actuator failure compensation designs are developed in (Tao [2004]) for the nonlinear system (1) in the following cases:

(i) \( f(x) \) and \( g_i(x), i = 1, 2, \ldots, m, \) are known, but the actuator failures are unknown, and

(ii) \( f(x) = f_0(x) + \sum_{i=1}^{m} \theta_i f_i(x) \) with \( f_i(x) \in \mathbb{R}^n \) known, \( i = 0, 1, 2, \ldots, l \), and \( \theta_i \in \mathbb{R} \) unknown, \( i = 1, 2, \ldots, l \), such that the system can be transformed into a strict-feedback form, and also the actuator failures are unknown.

The design for the first case is based on feedback linearization, while the design for the second case is based on backstepping. In both cases, the actuator failure uncertainty is adaptively compensated to achieve output tracking.

The more general cases with more uncertainties in \( f(x) \) and \( g_i(x), i = 1, 2, \ldots, m, \) than that in a strict-feedback form, are still open for further study.

### 3.3 Output Feedback for Output Tracking

In this subsection, we discuss output feedback based adaptive actuator failure compensation. Using output feedback, the control objective is to achieve output tracking despite system and failure uncertainties.

**Design for Linear Systems**

For the linear system (3) with only output signal \( y(t) \) available for measurement, an output feedback controller, with the actuation scheme (27), has the structure

\[
v_0(t) = \theta_1^T \omega_1(t) + \theta_2^T \omega_2(t) + \theta_3^T \omega_3(t) + \theta_4^T t + \theta_4(t),
\]

where \( \theta_1(t) \in \mathbb{R}^{n-1}, \theta_2(t) \in \mathbb{R}^{n-1}, \theta_3(t) \in \mathbb{R}, \theta_4(t) \in \mathbb{R}, \) and \( \theta_4(t) \in \mathbb{R} \) are the estimates of some unknown parameters \( \theta_1, \theta_2, \theta_3, \theta_4 \), and signal \( \theta_4(t) \) which depend on the system and failure parameters, and \( \omega_1(t) = \frac{a(s)}{s^2} [v_0(t), \omega_2(t) = \frac{a(s)}{s^3} [y(t)], \) with \( a(s) = [1, s, \ldots, s^{n-2}]^T \) and \( \Lambda(s) \) being a monic stable polynomial of degree \( n - 1 \).

As shown in (Tao [2004]), in terms of the system (3):

\[
y(t) = \sum_{j=1}^{m} k_p Z_j(s)^{-1} u_j(t), \quad Z_j(s) = \frac{k_p Z_j(s)}{P(s)} \]

under Assumption 1, the parameters \( \theta_1^*, \theta_2^*, \theta_3^* \) and \( \theta_4^* = k_p^{-1} \) can be determined from

\[
\theta_1^T a(s) P(s) + (\theta_2^* + \theta_3^* + \theta_4^*) \Lambda(s) k_p Z_a(s) = \Lambda(s) P(s) - k_p Z_a(s) P_m(s)
\]

and under Assumption 2, the nominal version of \( \theta_4(t) \) is

\[
\theta_4^* = - \sum_{j=j_1, \ldots, j_p} P_a(s) k_p Z_j(s) [u_j(t)],
\]

for a polynomial \( P_a(s) \) of degree \( n^* - 1 \) such that \( \Lambda(s) - \theta_4^* P_a(s) \Lambda(s) P_a(s) \). The tracking equation becomes

\[
\theta^* P_m(s)[y - y_m(t)] = (\theta(t) - \theta^*)^T \omega(t),
\]

where, for \( \theta_4^* \) parametrized as \( \theta_4^* = \theta_4^* \omega_4^* \),

\[
\theta(t) = [\theta_1^T(t), \theta_2^T(t), \theta_3(t), \theta_4^T(t)]^T, \quad \theta^* = [\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*]^T,
\]

\[
\omega(t) = [\omega_1^*(t), \omega_2^*(t), y(t), r(t), \omega_3^*]^T.
\]

Based this error equation, adaptive laws for \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4(t) \) can be developed to achieve closed-loop signal boundedness and asymptotic tracking despite parameter and failure uncertainties (Tao [2004]). For an adaptive design, the term \( \theta_4^* \) in (38) needs to be fully parametrized, similar to that in (34) for the state feedback case.

Hence, we can conclude that Proposition 4 is also applicable to adaptive actuator failure compensation designs using output feedback for output tracking.

**Design for Nonlinear Systems**

In (Tao [2004]), output feedback based adaptive actuator failure compensation designs have been mainly developed for output-feedback nonlinear systems in the presence of actuator failures. For such system models which have an explicit relative degree structure, the relative degree and minimum phase conditions in Proposition 5 are also crucial for adaptive actuator failure compensation. It can be shown that Proposition 5 is also applicable to adaptive actuator failure compensation using output feedback for output tracking.

### 4. INDIRECT ADAPTIVE CONTROL BASED ACTUATOR FAILURE COMPENSATION

So far we have addressed adaptive actuator failure compensation problems using a direct adaptive control approach, that is, the feedback controller parameters are directly updated by some adaptive laws. In this section, we present designs for adaptive actuator failure compensation using an indirect adaptive control approach, that is, the system parameters and actuator failure parameters are estimated first and the feedback controller parameters are then calculated for control implementation.

Indirect adaptive control is based on adaptive estimation of the plant parameters and on-line calculation of the controller parameters. To derive an indirect adaptive actuator failure compensation control scheme, we express the system (3) as \( \dot{y}(t) = \sum_{j=1}^{m} \frac{Z_j(s)}{P(s)} u_j(t) \). With the actuation scheme (27) and under actuator failures (2), \( j \in \{j_1, j_2, \ldots, j_p\} \), the system becomes

\[
P(s)[y](t) = Z_\alpha(s)[v_0](t) + \sum_{j=j_1, \ldots, j_p} Z_j(s)[u_j](t)
\]
An indirect adaptive control based failure compensation scheme is designed also under Assumptions 1 and 2 stated in Section 3.2. This means that Proposition 4 is also applicable to such an indirect adaptive control design.

The nominal controller for output tracking and actuator failure compensation is the same as that in Section 3.3:

\[ v_0(t) = \theta_0^T \omega_1(t) + \theta_0^T \omega_2(t) + \theta_0^T y(t) + \theta_0^T r(t) + \theta_0^T (t) \]

with \( \theta_0^* \) satisfying (37) and (38) (with \( k_p = k_p = 1 \)).

**System and failure parametrization.** With Assumptions 1–3 of Section 3.2, we can express (41) as

\[
(s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0)[\tilde{y}] (t) \\
= (b_{n-n} s^{n-n} + \ldots + b_1 s + b_0)[\tilde{u}] (t) \\
+ \sum_{i=1}^{l_1} (b_{n-n} s^{n-n} + \ldots + b_0)[\tilde{u}] (t) 
\]

where, for \( i \in \{ l_1, l_2, \ldots, l_q \} \setminus \{ j_1, j_2, \ldots, j_p \} \), we consider \( \tilde{u}_i(t) \) in the form of (2) but with \( \tilde{u}_i = 0 \) for \( l = 0, 1, \ldots, n_j \) as \( u_i(t) \) is actually not failed but we estimate the parameters of \( \tilde{u}_i(t) \) for all \( i = l_1, l_2, \ldots, l_q \).

With details in (Tao [2011]), the failure terms can be expressed as

\[ y(t) = \theta_0^T \phi(t) \]

for some unknown parameter vector \( \theta^* \) and known regressor vector \( \phi(t) \), based on which an adaptive law can be designed to update the estimate \( \theta(t) \) of \( \theta^* \).

**Adaptive control design.** From the estimate \( \hat{\theta}(t) \), we can obtain the estimates of \( \hat{P}(s) \) and \( Z_a(s) \) to solve an adaptive version of (37) to obtain the estimates of \( \hat{\theta}_n^* \) in (42) and that of \( \hat{P}_a(s) \) in (38), and also the estimates of \( \hat{Z}_a(s) \) in (41) to construct an adaptive version of \( \hat{\theta}_a^* \), to implement an adaptive version of (42).

Such an adaptive control scheme has the desired feedback control and actuator failure compensation properties: all closed-loop system signals are bounded and \( \lim_{t \to \infty} (y(t) - y_m(t)) = 0 \), despite the actuator failure and system parameter uncertainties.

5. CONCLUDING REMARKS

Adaptive control systems studied in this paper have been tested and verified in our recent aircraft flight control system simulations (Tao [2004]), including Boeing 737 longitudinal model (elevator/stabilizer failure) and lateral model (rudder/aileron failure), Boeing 747 lateral model (rudder failure), DC-8 lateral model (aileron failure), F-18 wing dynamics model (aileron failure), Twin Otter longitudinal nonlinear model (elevator failure), a hypersersonic aircraft longitudinal nonlinear model (elevator failure).

Most existing results have been developed for single-output systems, and their extension to multiple-output systems is still open, and so is it for nonlinear systems. Moreover, the case when the basis functions \( f_j(t) \) in the actuator failure model (2) depend on the system state variables: \( f_j(t) = f_j(x(t), t) \), and are unbounded for unbounded \( x(t) \), needs to be studied.

REFERENCES


