Feedforward Control of Piezoactuator for Evaluating Cilia-Based Micro-Mixing

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Abstract: This work evaluates the use of different excitation waveforms (using a piezoactuator) to potentially improve biomimetic-cilia-based mixing in microfluidic applications. A challenge in such studies is that, at high frequencies, vibrations in the piezoactuator can distort the positioning achieved, and thereby, limit the ability to evaluate the effect of a desired excitation waveform on mixing. The main contribution of this work is to use feedforward control to account for the vibrational dynamics and avoid unwanted vibrations in the achieved positioning. Moreover, experimental results are presented to show that (i) mixing is substantially improved with the use of cilia when compared to the case without cilia and (ii) mixing with cilia can be further enhanced by using an asymmetric triangular excitation when compared to sinusoidal excitation.

Keywords: Microsystems: nano- and micro-technologies; Mechatronic systems

1. INTRODUCTION

Based on biological cilia systems, e.g., Brennen (1974); Tuck (1968); Ramia et al. (1993); Gueron and Levi-Gurevich (1998), biomimetic cilia-type compliant actuators have been proposed for mixing and manipulation in liquid environments, e.g., see Chen et al. (2010); Behkan and Sitti (2006); Alvarado and Youcef-Toumi (2006); Saif et al. (1999); Oh et al. (2009); Khatavkar et al. (2007); Dreyfus et al. (2005). Recent works have shown that sinusoidal excitation of the cilia-chamber’s position using a piezoactuator can lead to resonance vibrations in the cilia, which in turn, improve mixing in microfluidic devices (Oh et al. (2010); Kongthon et al. (2010, 2011)). This work aims to evaluate the use of different excitation waveforms of the cilia-chamber to potentially improve the cilia-based mixing. A challenge in such evaluation studies is that, at high frequencies, vibrations in the piezoactuator can distort the achieved positioning of the cilia-chamber, and thereby, limit the ability to evaluate the effect of a desired excitation waveform on mixing. The main contribution of this work is to use feedforward control to account for the vibrational dynamics and avoid unwanted vibrations in the achieved excitation waveform. Moreover, experimental results are presented to show that (i) the mixing-time (90%) is substantially reduced with the use of cilia when compared to the case without cilia by about 12 times from 176.4s to 14.9s (with sinusoidal excitation) and (ii) the mixing-time (90%) with cilia can be further reduced (about 2 times from 14.9s to 7.4s) by using an asymmetric triangular excitation when compared to the previous use of sinusoidal excitation in Oh et al. (2010); Kongthon et al. (2011).

Mixing can be improved by generating complex flows in the fluid to overcome the mixing-rate limits of laminar flows that are typical at the micro scale. For example, passive techniques such as grooves can be used to generate chaotic folding (and refolding) of the liquid as it flows past the grooves to improve mixing, e.g., in Stroock et al. (2002). Such flow-type mixing can be used when a sufficiently-large amount of sample is available to achieve the flow through the grooved channel. In contrast, if the amount of sample is limited, then batch-type mixing needs to be achieved in small chambers containing the sample. Batch mixing can be enhanced using a variety of actuation techniques such as high-frequency ultrasound excitation (Yaralioglu et al. (2004); Hawkes et al. (2004)) and time-varying external magnetic fields (Khatavkar et al. (2007); Grumann et al. (2005); Herrmann et al. (2006); Vifian et al. (2010)). In the current work, cilia are excited by relatively-low-frequency oscillations (compared to higher-frequency acoustic excitation) of the chamber containing the sample, which is advantageous for mixing samples that are susceptible to damage from high-frequency excitation and external magnetic fields. The current article shows that batch mixing can be substantially improved by using cilia when compared to the case without cilia. Moreover, it shows that the choice of the excitation waveform can influence the mixing rate in cilia-based devices.

2. THE POSITIONING PROBLEM

2.1 System Description

The displacement transverse to the length (x) of a cilium is excited by using a piezoactuator (Burleigh PZS200) as shown in Figure 1. The soft cilia are fabricated from Polydimethylsiloxane (PDMS) using a silicon mold; detailed information on cilia fabrication and material properties can be found in Oh et al. (2009). The dimensions of the silicon
mold used to fabricate the cilia are length \( L = 800 \mu m \), height \( H = 45 \mu m \), and width \( W = 10 \mu m \). The cilia and the fluid in the chamber are observed with an optical microscope and evaluated with an attached digital color CCD camera as well as captured still images (Pinnacle Studio Version 12). An example image of the resulting cilia vibration (due to the chamber oscillation) is shown in Figure 1(b).

2.2 The Positioning Problem

An oscillatory excitation \( u_c \) of the cilia-chamber is achieved by using a piezo-based positioning system (referred to as the piezoactuator in the following) as shown in Figure 1. The piezoactuator system is driven with input voltage \( V \), which is amplified to \( V_a \) before being applied to the piezoactuator as shown in Figure 2. The motion of the base of each cilium is also \( u_c \) since the ciliary-base is attached to the chamber through a relatively-stiff PDMS structure. To evaluate the mixing achieved with different types of excitation waveforms (i.e., time profile of \( u_c \)), the piezoactuator needs to position the chamber precisely along the desired waveform \( u_c \). At high frequencies (e.g., close to the resonance of the sloshing system at about 95Hz, Kongthon et al. (2011)), vibrations in the piezoactuator positioning system can lead to distortions in the achieved position trajectory (excitation waveform). The positioning problem is to correct for such vibration-caused distortions using control.

![Fig. 2. Block diagram of the positioning system. An input voltage \( V \) is amplified by \( K_{amp} \) to \( V_a \) before applying it to the piezoactuator (model \( G_p \)). The amplifier gain \( K_{amp} \) is adjusted to achieve a fixed amplitude excitation waveforms \( u_c \) of the cilia-containing chamber. Such fixed amplitude motion of the cilia-chamber enables the comparative evaluation of mixing with different excitation waveforms \( u_c \).](image)

3. PIEZOACTIONTOR MODEL

The model of the piezoactuator (\( G_p \) in Figure 2) is obtained experimentally. In particular, experimental frequency responses are obtained by using a dynamic signal analyzer (SRS Model SR785) at different input voltage \( V_a \) amplitudes as shown in Figure 3. A fit of the experimental response is obtained using MATLAB and the resulting model \( G_p \) is described by the following transfer function

\[
G_p(s) = \frac{u_c(s)}{V_a(s)} = k \prod_{i} \frac{s - z_i}{s - p_i}
\]

where the poles \( p_i \), zeros \( z_i \) and gain \( k \) are listed in Table 1. The model matches the experimentally obtained frequency responses till about 500Hz.

![Fig. 3. Experimental and model-based frequency response of piezoactuator \( G_p \) in Figure 2. Experimental responses are shown for two different input amplitude levels: the dashed line represents the response with input voltage \( V_a = 11.2V \) peak-peak; and the dotted line represents the response with \( V_a = 16V \) peak-peak. The response of the fitted model \( G_p \) (in Eq. 1) is represented by the solid line.](image)

<table>
<thead>
<tr>
<th>Poles, ( p_i )</th>
<th>((-23.95 \pm 235.56)i)(2\pi)</th>
</tr>
</thead>
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<tr>
<td>(-7.78 \pm 316.89i)(2\pi)</td>
<td></td>
</tr>
<tr>
<td>(-13.25 \pm 414.64i)(2\pi)</td>
<td></td>
</tr>
<tr>
<td>Zeros, ( z_i )</td>
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<td>(-3.01 \pm 376.16i)(2\pi)</td>
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<td>Gain, ( k )</td>
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Table 1. Parameters of piezoactuator model \( G_p \) in Eq. (1)

4. FEEDFORWARD CONTROL OF PIEZOACTIONTOR

The design of the excitation waveforms and the model-based feedforward input are discussed in this section.

4.1 Initial Choice of Excitation Waveforms

Two types of excitation waveforms are considered: a sinusoidal excitation (used in Oh et al. (2010); Kongthon et al. (2011)), and an asymmetric triangular excitation, e.g., as shown in Figure 4.

4.2 Tracking of the Excitation Waveforms

The sinusoidal excitation \( u_c \) is relatively easy to achieve. The input voltage \( V \) to the amplifier ( in Figure 2) was chosen as the same sinusoidal excitation waveform \( u_c \) in Figure 2 and the amplifier gain \( K_{amp} \) was adjusted till the output position of the piezoactuator \( u_c \) had the desired peak-to-peak amplitude of 20 \( \mu m \). On the other hand, if the idealized triangular profile in Figure 4(b) is applied...
4.3 Optimal Inversion

Optimal inversion developed in Dewey et al. (1998) can be used to achieve a compromise between the goal of exact tracking the desired trajectory and handling potential uncertainties in the model as shown in Devasia (2002). Most piezoactuators tend to have some frequency regions where plant uncertainty is unacceptably large — usually at high frequencies and close to the resonances. Moreover, the frequency response can be amplitude-dependent due to hysteresis nonlinearities as seen in Figure 3. The optimal inverse allows the inverse to be computed only in frequency regions where the uncertainty (in the model $G_p$) is “sufficiently” small. The optimal inverse $v_a = v_{a, opt}$ is obtained by minimizing the following cost function

$$J(V_a) = \int_{-\infty}^{\infty} \{V_a^*(j\omega)R(j\omega)V_a(j\omega) + E_P^2(j\omega)Q(j\omega)E_P(j\omega)\} d\omega,$$  \hspace{1cm} (2)

where $*$ denotes the complex conjugate transpose and $E_P = u_c - u_{c,d}$ is the positioning error with respect to a given desired excitation waveform $u_{c,d}$. The terms $R(j\omega)$ and $Q(j\omega)$ are real-valued, frequency-dependent weightings that penalize the size of the input $V_a$ and the positioning error $E_P$. If the model uncertainty is large at some frequency $\omega$, then the choice of weights $R(j\omega) > 0$ and $Q(j\omega) = 0$ results in a minimum cost with zero input $V_a(j\omega) = 0$, i.e., no tracking at that frequency.

The optimal inverse input $V_{a, opt}$ minimizing the cost function can be found, for the single-input-single-output (SISO) case as Dewey et al. (1998)

$$V_{a, opt}^*(j\omega) = \frac{G_p^*(j\omega)Q(j\omega)}{R(j\omega) + G_p^*(j\omega)Q(j\omega)G_p(j\omega)} u_{c,d}(j\omega) = G_{p, opt}^{-1}(j\omega)u_{c,d}(j\omega)$$ \hspace{1cm} (3)

and the time-domain signal for the feedforward input $V_a^*(t) = V_{a, opt}^*(t)$ is then obtained through an inverse Fourier transform of $V_{a, opt}^*(j\omega)$. The optimal inverse (in Eq. 3) can be expressed as

$$G_{p, opt}^{-1}(j\omega) = \frac{G_p^*(j\omega)Q(j\omega)G_p(j\omega)}{R(j\omega) + G_p^*(j\omega)Q(j\omega)G_p(j\omega)} G_p^{-1}(j\omega)$$ \hspace{1cm} (4)

Therefore, the optimal inverse can be considered as a frequency weighted inverse.

4.4 Simulation Results of Optimal Inversion

The optimal input $V_{a, opt}$ scaled by a factor $K_{opt}$, i.e.,

$$V_a = V_{a, opt} = V_{a, opt}^* K_{opt}$$

Fig. 4. Excitation waveforms $u_e$ for one time period $T$: (a) Sinusoidal excitation waveform; and (b) Idealized asymmetric triangular waveform.

Fig. 5. Simulation results using the piezoactuator model $G_p$ in Figure 2. (a) Input voltage $V_a$, which is a scaled version of the idealized asymmetric triangular excitation profile in Figure 4(b). (b) The corresponding piezoactuator output $u_e$.

Fig. 6. Input energy weight, $R(j\omega)$ and tracking-error weight, $Q(j\omega)$ used to find the optimal inverse of the piezoactuator model $G_p$. Choosing $R$ and $Q$: The cost function in Eq. (2) can be used to account for model uncertainty and piezoactuator constraints by appropriately choosing the input energy weight, $R(j\omega)$ and the tracking-error weight, $Q(j\omega)$. In particular, the weights can be chosen such that good tracking is achieved in the low-frequency range (till 100 Hz for the cilia-based mixing device) and to reduce the emphasis on tracking at high frequencies where the model tends to become less accurate. This implies that the tracking-error weight $Q$ should be larger than the input energy weight $R$ in the low-frequency range and vice versa in the high-frequency range, e.g., as shown in Figure 6.
to achieve the desired peak-to-peak amplitude of 20 \mu m of the corresponding excitation waveform \( u_{c,\text{opt}} \) is shown in Figure 7(a). The resulting output of the piezoactuator, i.e., the excitation waveform

\[
u_{c,\text{opt}} = G_p V_{a,\text{opt}}
\]

is shown in Figure 7(b). To illustrate the importance of accounting for the vibrational dynamics, the response \( u_{c,\text{DC}} \) of the piezoactuator when the input \( V_a \) is chosen as a scaled version \( V_{a,\text{DC}} \) of the optimal output \( u_{c,\text{opt}} \), i.e.,

\[
V_{a,\text{DC}} = K_{\text{DC}} u_{c,\text{opt}}
\]

\[
u_{c,\text{DC}} = G_p V_{a,\text{DC}}
\]

is shown in Figure 7(b) where the constant (DC-type) gain \( K_{\text{DC}} \) is chosen to achieve the desired peak-to-peak amplitude of 20 \mu m of the corresponding excitation waveform \( u_{c,\text{DC}} \) shown in Figure 7(b).

Note that the achieved excitation waveform with the optimal inverse has a similar asymmetry to the response achieved with an idealized-triangular input \( V_a \) — however, the optimal inverse reduces the oscillations due to vibrations excited in the piezoactuator as shown in Figure 7(b). This reduction in vibrational effects is because the optimal inverse \( V_{a,\text{opt}} \) accounts for the vibrational dynamics. In contrast, without considering the vibrational dynamics (e.g., the pair \( V_{a,\text{DC}}, u_{c,\text{DC}} \)), the system still has vibration-caused distortions — although such distortion is less when compared to the idealized triangular input, which has substantially larger high-frequency content. In summary, the optimal-inversion approach results in an asymmetric excitation waveform, without substantial vibrational distortions, that can be used to evaluate the influence of the waveform on mixing.

\[
I_{\text{mix}}(t_k) = \frac{1}{I_{ss}} \left[ 1 - \frac{\sum_{p=1}^{P} |C_p(t_k) - C_p(t_N)|}{\sum_{p=1}^{P} |C_p(t_0) - C_p(t_N)|} \right]
\]

5. EXPERIMENTAL EVALUATION OF MIXING

The mixing of ink and water in an oscillating chamber is evaluated to show that (i) mixing is substantially improved with the use of cilia when compared to the case without cilia and (ii) mixing with cilia can be further enhanced by using an asymmetric triangular excitation when compared to a sinusoidal excitation.

5.1 Procedure for Mixing Experiments

The procedure of the mixing experiment with cilia is described below. The procedure for the mixing experiments without cilia (i.e., vibration only) is the same as the case with cilia except that cilia are not present in the chamber.

**Fig. 8. Schematic setup of cilia device for mixing experiments in a 3mm diameter chamber.**

Each mixing experiment (with and without cilia) was repeated seven times. For each mixing experiment, the initial image is captured after the addition of ink and after the thin PDMS sheet is used to cover the chamber but just before the start of the chamber oscillations with the piezoactuator. The last image is captured at the time instant \( t_N \) when, visually, the images do not change significantly. The time for the mixing index to reach and stay within 90% of its final value is used to quantify and comparatively evaluate the mixing performance with and without cilia for the two different waveforms. A CCD camera, attached to the microscope (see Figure 1(a)), is used to video-record the mixing process for evaluation — samples of the acquired images are shown in Figure 9.

5.2 Quantifying Mixing

The mixing index was quantified by comparing images from the video recording of the mixing process using a mixing index \( I_{mix} \) developed in Oh et al. (2010), which is a discrete version of the continuous time mixing index defined in Jeon et al. (2000). The mixing index, which is a measure of relative mixing, is initially 0 and approaches 1 when the fluids become fully mixed. It is given by
Asymmetric triangular excitation without cilia; and (b) Asymmetric triangular excitation with cilia.

The experimental results show that the mixing time is substantially reduced with cilia when compared to the case without cilia.

where \( [t_k]_{k=1}^N \) represents the different time instants when the images are evaluated, \( N \) is the total number of images, \( P \) is the number of pixels in each of the images, \( C_p(t_k) \) is the color of \( p^{th} \) pixel at time instant \( t_k \) and the normalization factor \( I_{ss} \) is given by

\[
I_{ss} = 1 - \frac{\sum_{p=1}^P |C_p(t_{N-1}) - C_p(t_N)|}{\sum_{p=1}^P |C_p(t_0) - C_p(t_N)|}.
\]

Each image used in this analysis is composed of an array of 352 pixels by 240 pixels, and the color of each pixel \( C \) is a vector of three values \( C = [R \: G \: B] \) that represents red, green, and blue (RGB) color with values between 0 and 255. Given any two pixel colors \( C_i \) and \( C_j \), the difference between them (used in Eqs. 7, 8) is defined as

\[
|C_i - C_j| = |R_i - R_j| + |G_i - G_j| + |B_i - B_j|.
\]

As the mixing progresses and reaches a steady state (i.e., as \( t_k \) approaches \( t_N \)), the difference between the color of the corresponding pixels in the images becomes small,

which tends to increase the values of \( I_{mix} \) from an initial value of zero towards one. The normalization factor \( I_{ss} \) in Eq. (8) uses the last two images to make the mixing index close to one when the mixing process reaches steady state and the images become similar. This is necessary because noise in the image (partly due to the oscillation) prevents the final value from reaching one without the normalization, i.e., when the normalization factor is chosen as \( I_{ss} = 1 \) in Eq. (7). To evaluate the mixing performance and determine the better input wave form. Two different wave pattern inputs (sinusoidal excitation, and asymmetric triangular excitation) are used to oscillate the mixing chamber for the case with cilia and the case without cilia.

### 5.3 Mixing Results and Discussions

The mixing time is substantially reduced with cilia when compared to the case without cilia. The reduction in mixing time with the cilia is visually observable in the images shown in Figure 9. For example, to reach the 90% mixing time (corresponding to the images on the 4th column), the mixing takes 9s with the cilia whereas it takes 89s for the case without the cilia along with the asymmetric excitation. To quantify the mixing rate, the variation of the mixing index (without and with cilia) are shown in Figures 10, 11 and the 90% mixing time for the different experiments are presented in Table 2. Without cilia, Table 2 shows that the mixing time to reach 90% mixing for the asymmetric triangle excitation is 93.4s and that for the sinusoidal excitation is 176.4s. In contrast, with cilia, Table 2 shows that the mixing time to reach 90% mixing for the asymmetric triangle excitation is substantially lower at 7.4s and that for the sinusoidal excitation is 14.9s. The experimental results show that the mixing time is substantially reduced with cilia when compared to the
<table>
<thead>
<tr>
<th>Excitation Wave Form</th>
<th>Oscillation Frequency (Hz)</th>
<th>Mixing Time without Cilia (s)</th>
<th>Mixing Time with Cilia (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinusoidal</td>
<td>96.4±1.27</td>
<td>176.3±18.3</td>
<td>14.9±3.25</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>95.7±1.38</td>
<td>93.4±37.5</td>
<td>7.4±1.15</td>
</tr>
</tbody>
</table>

Table 2. Mixing time(s) quantified by the time for the mixing index $I_{mix}$ in Eq. (7) to reach and stay within 90% of its final value, with and without cilia. The mean values ± the standard deviations of 7 experimental runs are shown.

case without cilia. Moreover, the mixing time is further decreased by applying the asymmetric triangular excitation input to oscillate the mixing chamber when compared to the case of the sinusoidal excitation input. In particular, (i) the mixing-time (90%) is substantially reduced with the use of cilia when compared to the case without cilia by about 12 times from 176.4s to 14.9s (with sinusoidal excitation) and (ii) the mixing-time (90%) with cilia can be further reduced (about 2 times from 14.9s to 7.4s) by using an asymmetric triangular excitation when compared to the previous use of sinusoidal excitation in Oh et al. (2010); Kongthon et al. (2011).

6. CONCLUSIONS

Feedforward control was used to achieve precision control of excitation waveforms applied to a novel biomimetic cilia-based mixer. Experimental results were presented to show that (i) the mixing-time is substantially reduced with the use of cilia when compared to the case without cilia by about 12 times (with sinusoidal excitation) and (ii) the mixing-time with cilia can be further reduced (about 2 times) by using an asymmetric triangular excitation when compared to the use of sinusoidal excitation. Based on these encouraging results, the ongoing efforts are focused on designing the excitation waveforms to optimize the mixing.

REFERENCES


