A Criterion for Wheel Holding Analysis


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Abstract: This paper is the following of previous works on the suspension transfer function design. This paper deals with the wheel holding. First, it describes the theoretical criterion then it demonstrates the efficiency of this criterion on real car measurements.

Keywords: suspension, frequency domain, wheel holding, criterion, experiment.

1. INTRODUCTION

The main conflict for suspension is between comfort and safety. The suspensions filter the road to protect the passengers from vibrations. It also keeps the wheels in contact with the road. The second point is crucial because the wheels are the only parts of the vehicle in contact with the road. Therefore, it’s important to keep a maximum adhesion regardless the road conditions.

The aim of this paper is to propose a criterion on the wheel holding. This criterion would include the comparison of the level of the wheel holding between two suspensions and would allow to tune the suspension transfer to hold the wheel.

Before tuning a suspension, is necessary to evaluate the vertical dynamic of the car to choose the good model for the analysis and the synthesis of the transfer function for the wheel pulsation.

Then simulations must be made to validate the criterion on existing suspension first. It allows to compare the results given by the criterion and the one given by simulations. Secondly, this criterion is used to tune a suspension; the simulations allow to check the wheel response.

Once all these procedures are completed, we repeat the last step on real cars. Then, we compare the experimental results of all the different wheel holdings to get optimal settings.

2. SUSPENSION TRANSFERT ANALYSIS

2.1 Which synthesis model for the wheel holding?

The Figure 1 shows the Quarter-Car Model with two degrees of freedom and the associated analysis. The first loop is composed of the tire and the wheel. This loop represents the wheel dynamic which is usually around 13Hz. The second loop, which is composed of the suspension and the car body, describes the dynamics of the car body usually around 1Hz. The choice of 1Hz is due to the response of the human body to different frequencies. In fact, under a frequency of 1Hz human feels seasick and between 2.5 and 8Hz is the frequency scale where human’s heart, eyes etc. are the more sensitive (Griffin MJ et al., 1988). Therefore, 1Hz is the most appropriate frequency.

In the frequency domain the equation 3 and 4 become:

\[ \frac{Z_1(s)}{z(t)} = \frac{C_1(s)}{(m_1s^2 + C_2(s))} \cdot \]

(5)
The transfer function between the wheel and the road is thus:

\[
Z_1(s) = \frac{k_i + b_i s}{(m_1 s^2 + k_i + b_i s + C_1(s))} Z_0(s) + \frac{C_1(s)}{(m_1 s^2 + k_i + b_i s + C_1(s))} Z_2(s) .
\]  

(6)

The transfer function between the wheel and the road is thus:

\[
\frac{Z_2(s)}{Z_0(s)} = \frac{k_i (m_2 s^2 + C_1(s))}{m_2 m_0 s^4 + (b_0 m_1 s^2 + (C_1(s) (m_1 + m_2)) + k_i m_2) s^2 + k_0}.
\]  

(7)

To simplify the study, the transfer \( C_2(s) \) is equal to \( K \) (the equivalent stiffness of the suspension) and the damping of the tire is considered equal to zero. Thus, the equation (7) become:

\[
\frac{Z_2(s)}{Z_0(s)} = \frac{k_i (m_2 s^2 + K)}{m_2 m_0 s^4 + (b_0 m_1 s^2 + (K + k_i) m_2) s^2 + k_0 K}.
\]  

(8)

With the same hypotheses the transfer function between the car body and the road is:

\[
\frac{Z_2(s)}{Z_i(s)} = \frac{k_i}{Z_i(s)} \frac{Z_2(s)}{Z_0(s)} = \frac{k_i}{Z_i(s)} \frac{k_i (m_2 s^2 + K)}{m_2 m_0 s^4 + (b_0 m_1 s^2 + (K + k_i) m_2) s^2 + k_0 K} .
\]  

(10)

These two transfers have the same denominator which can be written as:

\[
m_0 m_2 s^4 + (Km_1 + (K + k_i)m_2) s^2 + k_i K = (s^2 - \omega^2_0)(s^2 - \omega^2_1),
\]  

(11)

with:

\[
\omega_0^2 = \frac{[k_i m_2 + K (m_1 + m_2)] - [k_i m_2 + K (m_1 + m_2)]^2 - 4m_0 m_2 K i}{2m_0 m_1},
\]  

(12)

\[
\omega_1^2 = \frac{[k_i m_2 + K (m_1 + m_2)] + [k_i m_2 + K (m_1 + m_2)]^2 - 4m_0 m_2 K i}{2m_0 m_1} .
\]  

(13)

On tourism vehicle, the low pulsation form the car body dynamic and the high pulsation form the wheel dynamic. The two transfer functions between the car body and the road and between the wheel and the road without any damping are plotted in the Figure 2 with the parameters of a tourism car (tuned with a car body pulsation equal to 1Hz and a wheel pulsation equal to 15Hz as made by the manufacturers in the car industry) without any damping. The parameters of this Quarter-Car Model are summarized in the Table 1.

<table>
<thead>
<tr>
<th>data name</th>
<th>value</th>
<th>unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension stiffness</td>
<td>K</td>
<td>12 000 N/m</td>
</tr>
<tr>
<td>Tire stiffness</td>
<td>k1</td>
<td>350 000 N/m</td>
</tr>
<tr>
<td>Sprung mass of the quarter car</td>
<td>m2</td>
<td>300 kg</td>
</tr>
<tr>
<td>Unsprung mass of the quarter car</td>
<td>m1</td>
<td>40 kg</td>
</tr>
</tbody>
</table>

Table 1 : Vehicle data for tourism vehicle

One can note that the car doesn’t move around the wheel pulsation (gain of the transfer function \( Z_2(s)/Z_1(s) << 1 \)).

Considering the damping of the suspension and the one of the tire, the results are the same: the car doesn’t move around the wheel pulsation as presented on the Figure 3. For the tourism vehicle example, the equivalent damping of the suspension is 1131 Nm/s (to obtain a damping ratio of 0.3 usually used in the car industry) and the equivalent for the tire is 100 Nm/s as most of the tires for tourism vehicle.

Thus the study of the wheel dynamic can be simplified by using a one degree of freedom model as presented in the Figure 4.

The equivalent transfer function between the wheel and the road become:

\[
Z_1(s) = \frac{k_i + b_i s}{(m_1 s^2 + k_i + b_i s + C_1(s))} Z_0(s) .
\]  

(14)

The Figure 5 presents the wheel frequency response of the transfer function \( Z_2(s)/Z_0(s) \) of the tourism vehicle with damping. In blue is plotted the transfer function between the wheel and the road for the two degrees of freedom model (Figure 1) and in green, for the simplified model of the Figure 4. Near the wheel pulsation the simplified model has
the same shape as the complete model with the car dynamic. To conclude, the simplified model is a good model to synthesis the suspension for the wheel holding.

2.2 Criterion on the wheel holding

Previous studies have defined criterions in frequency domain around the car body pulsation (Moreau et al. 1999). The aim is to find a criterion on the wheel holding in order to tune the suspension transfer function $C_2(s)$ in frequency domain without doing any simulations.

An harmonic study is made on the simplified Quarter-Car Model of the Figure 4 around the wheel pulsation called $\omega_1$. A sinusoidal input is sent on the road:

$$z_0(t) = Z_0 \cos(\omega_1 t). \quad (15)$$

Because of the tire the wheel movement could be written:

$$z_1(t) = Z_1 \cos(\omega_1 t + \varphi_{10}(t)). \quad (16)$$

The general expression of the suspension force is equal to the sum of the two forces composing the suspension:

$$f_s(t) = f_{c}(t) + f_{d}(t) = C_2(j\omega_1)\cos(\arg(C_2(j\omega_1)))Z_1 \cos(\omega_1 t + \varphi_{10}(t)) \quad (17)$$

$$f_s(t) = C_2(j\omega_1)\sin(\arg(C_2(j\omega_1)))Z_1 \cos(\omega_1 t + \varphi_{10}(t)) \quad (18)$$

The general expression of the suspension force is equal to the sum of the two forces composing the suspension:

$$f_s(t) = \left|C_2(j\omega_1)\sin(\arg(C_2(j\omega_1)))\right|Z_1 \cos(\omega_1 t + \varphi_{10}(t)) \quad (19)$$

Consequently, the dissipative effort of the tunable suspension $(18)$ must be at least equal to the reference to hold the wheel. Thus the equality between the equation 18 and 19 give the equation:

$$C_2(j\omega_1)\sin(\arg(C_2(j\omega_1))) = C_{2,ref}(j\omega_1)\sin(\arg(C_{2,ref}(j\omega_1))). \quad (20)$$

The improvement of the wheel holding implies a dissipative effort for the tunable suspension more important than the reference. In this case, the sign equal of the equation 20 becomes a sign superior or equal.

From the equation 20, it appears that the design of the suspension must integrate the wheel holding. The criterion on the wheel holding defined by this equation is not only a criterion on the gain but also a criterion on the phase of the suspension transfer function around the wheel pulsation. Knowing the reference transfer function, the right term of the equation 20 can be calculated. For example, the product between the gain and the phase of the transfer function $C_{2,ref}(s)$ (Figure 7) at the wheel pulsation is equal to :

$$C_{2,ref}(j\omega_1)\sin(\arg(C_{2,ref}(j\omega_1))) \approx 10^{(106.1/20)} \sin(83.61) = 106490. \quad (21)$$
Consequently, the left part of the expression 20 must be at least equal to 106,490 to have the same wheel holding than the reference. The problem is: there is one equation with two unknown variables (the gain and the phase of the tunable suspension transfer function around the wheel pulsation). One solution to this problem is to fix one parameter to find the other. This method is applied on the following part.

3. SIMULATION RESULTS

The objective of this part is to find and justify the criterion on the wheel holding. The chosen application is a new CRONE suspension developed by the PhD’s author (A. Rizzo et al. 2009a and b). This CRONE suspension is a suspension with a non-integer order transfer function for the suspension. This transfer function is developed to keep a correct filtering level of the road and in the same time to hold the car body. Only the results of the wheel holding are presented following the purpose of the study.

3.1 Models presentations

Different models were developed during this study. The First model is for the synthesis of the suspension. This model is a Quarter-Car Model to check the theoretical results. For simulation this model is completed with friction, non linearity and additional axle stiffness.

A complete model was created with the pumping (following vertical axis), the pitching (around y axis) and the rolling (around x axis) dynamic in order to validate the result of the Quarter-Car Model on a realistic vehicle. This model was developed in AMESIM®.

In addition of the complete car body movement, this validation model takes into account the friction, the non linearity, the axle additional stiffness, and all the property of the flow for hydraulic suspension.

3.2 Simulations results

The reference suspension considered is a hydractive suspension of the Citroën C5 in soft mode. The suspension to improve is the first tuning of the CRONE suspension, (P. Serrier et al. 2005) which didn’t take into account of the wheel holding. From this two transfer functions, two conclusions can be made. First the reference value for the criterion defined in the previous part is 38870. UNITÉS Then, from the transfer function of the first CRONE suspension, the product of the gain (93.3dB) and the sinus of the phase (15.9°) of the CRONE transfer function is 12667 following the theory described before, the wheel is two times less held than the reference.

The validation AMESIM® model allows to check the response of the wheel in function of the frequency road input. The input of the simulation is a chirp sinus from 0.1Hz to 30Hz with amplitude of 1mm which is exactly the same as the real bench for characterizing the car suspension. The frequency is proportioned to the time, that's why the wheel time response is an image of the wheel frequency response. The wheel responses from the validation model show that the wheel response of the first CRONE suspension at the wheel pulsation (around 13Hz) is two times important than the one of the reference. These results were predictable by using the criterion.

The criterion on the wheel define in the second part seems to be a good indicator of the wheel holding. In order to achieve
the development is necessary to validate this criterion on real experiment.

The aim is to use this criterion to tune the CRONE suspension transfer function around the wheel pulsation. As already explained, it is necessary to fix one of the gain or the phase of the CRONE transfer function to tune the transfer function. Here the gain of the CRONE suspension is fixed. In this case, the equation 20 become:

\[ |C2(j\omega)|\sin(\varphi) = 38870. \]  \hspace{1cm} (22)

With \( \varphi \) the phase around the wheel pulsation of the new CRONE transfer function to hold the wheel is:

\[ \varphi = \arcsin \left( \frac{38870}{|C2(j\omega)|} \right) = 60^\circ. \]  \hspace{1cm} (23)

This means that the phase of the new CRONE transfer function must be at least 60\(^\circ\) around the wheel pulsation. The new CRONE transfer function is tuned with this constraint, the value of the product gain by the sinus of the phase is 50965 which is a better wheel holding than the reference. Its representation in Bode diagram is on the Figure 11.

The Figure 12 presents the simulation results of the wheel for a chirp sinus road input like the real bench input. From these results, one can conclude that the new CRONE suspension improves the wheel holding from the first CRONE transfer function and verify the conclusion made using only the value of the criterion : the wheel holding is better with the new CRONE suspension than with the reference suspension.

4. EXPERIMENTAL RESULTS

4.1 Experimental process

In order to estimate the performance of a suspension on a real car, car manufacturers used bench with four cylinders. These cylinders can send on the wheel, sinus with different amplitude and different frequencies (Figure 13(a)). To measure the wheel holding, a sinus is send with a frequency varying from 0.1Hz to 30Hz and with amplitude of 1mm. Some accelerometers are on the centre of the wheel and another is on the cylinder (Figure 13(b)). They capture the acceleration of the wheel and the acceleration of the cylinder. Then a tool calculates the transfer function between the wheel and the road (cylinder) acceleration.

4.2 Vehicle data

The new CRONE suspension need some work to be realized so the experimental results are not exactly the new CRONE suspension. These suspensions are pseudo CRONE suspensions called S2 S3a and S3b. The structures of these hydraulic suspensions combine some capacitive and resistive cells based on the same principle than the CRONE suspension. The realization of this suspension is presented in the Figure 14 (b). The difference between the S2 and S3a is \( C_0 \) (K in the Table 2). The difference between the S3a and the S3b come from damping which are different as shown in the Table2.

Figure 13: Illustration of a bench for characterizing a performance of a car suspension (a) and Sensors Position on the wheel (b)

(a)

(b)

Figure 14: Hydraulic suspension structure measured on car

<table>
<thead>
<tr>
<th>data</th>
<th>S1</th>
<th>S2</th>
<th>S3a</th>
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</table>

Table 2 : Vehicle data for tourism vehicle
For each capacitive cell is associated a stiffness written $K_i$ where “$i$” is the capacitive cell suffix. $B_i$ are respectively the damping (resistive elements $R_i$) of each capacitive cell “$i$”. Finally the $B_{line}$ is the damping in the line between the two RC cells of the reference suspension. This damping is only on the realization of the reference suspension, not on the CRONE debased suspension. The reference suspension on the Figure 14 (a) is called S1.

Then in order to show that the criterion is independent of the suspension technology a vehicle with metallic suspension is characterized. This suspension is called M1. This metallic suspension has only one RC cell which is a metallic $R_1C_1$ cell. All these suspensions are realized on the same vehicle in order to always compare the same vehicle with the same tire, the same axle stiffness, etc. Then for each suspension, measurements are made on a 4 cylinders bench and these results are illustrated on Figure 15. This figure shows the gain of the transfer function between the wheel and the road. The rank of the suspension from the best wheel holding to the worst is: S3b, M1, S1, S3a, S2.

Thus the theoretical results (Figure 16) verify the experimental result (Figure 15) and the criterion described by the equation 20 allow to quantize the wheel holding and compare the level.

6. CONCLUSIONS

This paper presents a criterion to evaluate the level of a suspension on the wheel holding. After describing the theory, a criterion is set-up. Then some numeric simulations were realized to check this criterion. Finally, thanks to the collaboration with PSA Peugeot-Citroën, some measurements were made on a real car to validate the theory. The conclusion of these measurements show that the criterion found from a Quarter-Car Model is a good indicator of the wheel holding. Actually, the use of a Quarter-Car Model is not only a good analysis model for the low frequency, but it is also useful to study the wheel frequency. This paper demonstrated a simple way to analyze the wheel holding level of a car.

REFERENCES


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